Black hole dynamics in non-asymptotically flat spacetimes

(Work in progress, Phys.Rev.D81:084052, Phys.Rev.D82:104037)

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2. Black holes in a box
3. Black holes in de Sitter
4. Black holes on a 5D cylinder
5. Final remarks

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Why numerical relativity

Study of systems with strong and dynamical gravitational fields

- Gravitational radiation
  - Astrophysics, gravitational wave astronomy
- Mathematical and theoretical Physics:
  - Cosmic censorship
  - Instabilities (Black hole interior, Myers-Perry)
- High-energy particle systems:
  - AdS/CFT correspondence;
  - Black hole production at the LHC;

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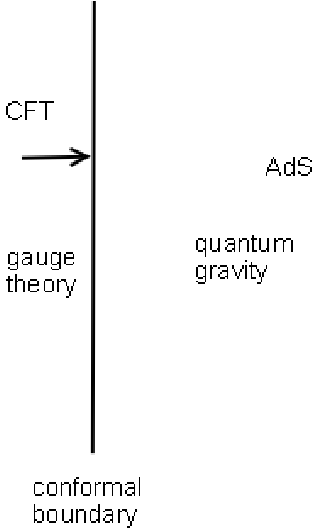
Motivation – AdS/CFT duality



Issues with numerical simulations in AdS:

- AdS is not globally hyperbolic
- The boundary plays an active role

$\rightarrow$ Toy model: standard general relativity in a “box”

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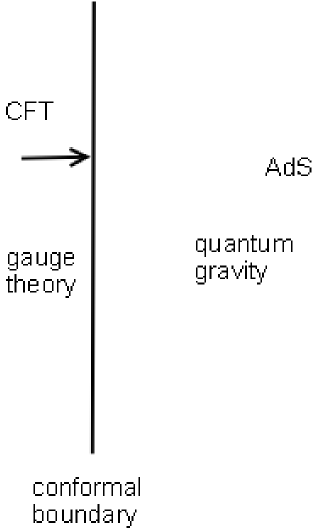

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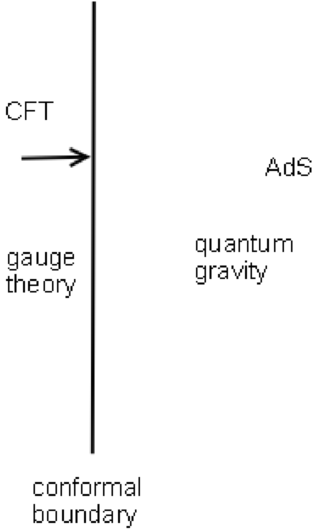

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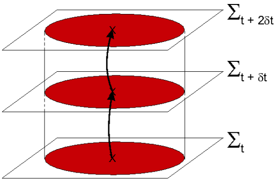
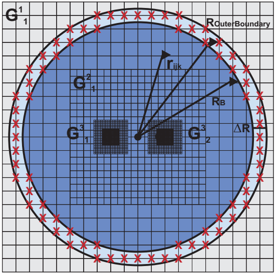
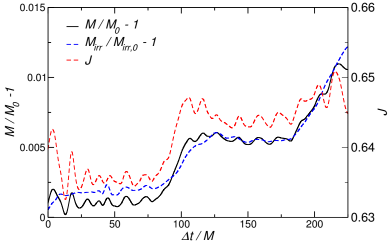
Black holes in a box Witek et al, Phys.Rev.D82:104037

- puncture initial data (equal-mass, non-spinning BHs)
- BSSN evolution scheme
- impose reflecting boundary conditions
- inspiraling BHB ⇒ spinning BH
  head-on collision ⇒ non-spinning BH





Results



Time evolution of the BH mass

Inspiral

- increasing mass and area of AH
- absorption of $\approx$ 15% of radiated energy per cycle

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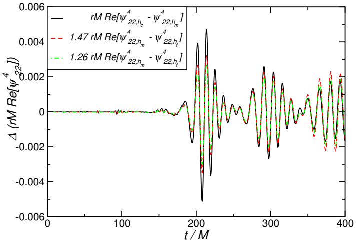
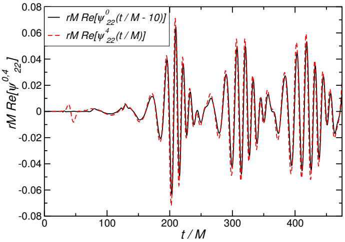


Convergence analysis



Real part of the $l = m = 2$ mode of $rM\Psi_0$ and $rM\Psi_4$

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- Accelerated bodies emit gravitational radiation
- Detected indirectly by measurements of the Hulse-Taylor binary system (1993 Nobel Prize)
- Interact weakly with matter ⇒ carry unique information about astronomical phenomena
  - ⇒ New window to the universe

- Observations suggest we live in an approximately de Sitter Universe;
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Formalism

Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0$$

Evolution equations

$$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

$$(\partial_t - \mathcal{L}_\beta) K_{ij} = -D_i \partial_j \alpha + \alpha \left({}^{(3)}R_{ij} - 2K_i{}^k K_{jk} + K_{ij} K - \Lambda \gamma_{ij} \right)$$

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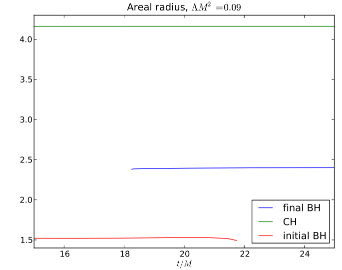

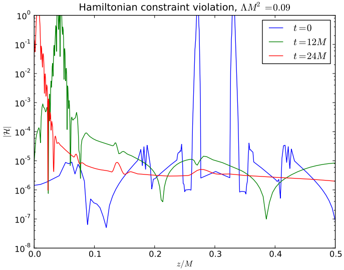

- Evolution is stable and the constraints are preserved;
- Successfully monitor the apparent horizons;

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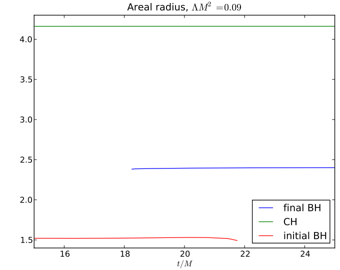

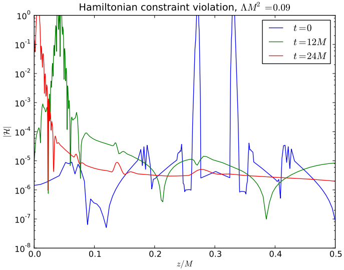

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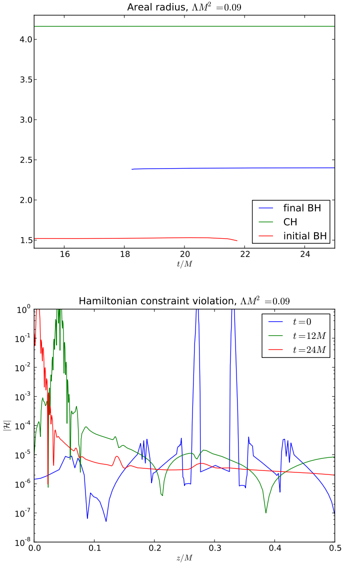

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Black holes in compact dimensions. . .

- arise in gauge/gravity duality and braneworld scenarios;
- have a richer phase structure and dynamics than in flat-space;
- analytical tools are capable of handling only a limited class of idealized scenarios;

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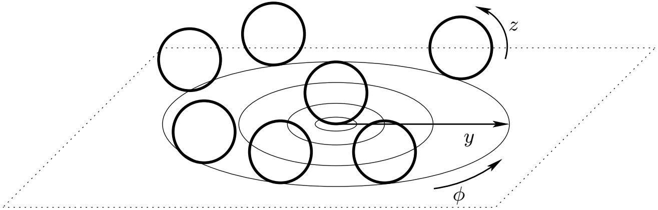
$D = 5$ black holes on a cylinder

In the absence of black holes, we have $\mathbb{M}^{1,3} \times \mathcal{S}^1$:

$$ds^2 = \underbrace{-dt^2 + dx^2 + dy^2 + y^2 d\phi^2}_{\mathbb{M}^{1,3}} + \underbrace{dz^2}_{\mathcal{S}^1}$$

$z \in [-L, L]$, $\phi \in [0, 2\pi[$



Formalism

Most general metric element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \lambda d\Omega^2_{D-4}$$

$\mu = 0, 1, 2, 3.$

D-dimensional vacuum Einstein equations imply

$$R_{\mu\nu} = \frac{D-4}{2\lambda}\left(\nabla_\mu \partial_\nu \lambda - \frac{1}{2\lambda}\partial_\mu \lambda \partial_\nu \lambda\right)$$

$$\nabla^\mu \partial_\mu \lambda = 2(D-5) - \frac{D-6}{2\lambda}\partial_\mu \lambda \partial^\mu \lambda$$

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Formalism

The resulting system is

$$(\partial_t - \mathcal{L}_\beta)\,\gamma_{ij} = -2\alpha K_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)\,K_{ij} = -D_i\partial_j\alpha + \alpha\left({}^{(3)}R_{ij} + KK_{ij} - 2K_{ik}K^k{}_j\right) - \alpha\frac{D-4}{2\lambda}\left(D_i\partial_j\lambda - 2K_{ij}K_\lambda - \frac{1}{2\lambda}\partial_i\lambda\partial_j\lambda\right)$$

$$(\partial_t - \mathcal{L}_\beta)\,\lambda = -2\alpha K_\lambda$$

$$\frac{1}{\alpha}(\partial_t - \mathcal{L}_\beta)\,K_\lambda = -\frac{1}{2\alpha}\partial^i\lambda\partial_i\alpha + (D-5) + KK_\lambda + \frac{D-6}{\lambda}K_\lambda^2 - \frac{D-6}{4\lambda}\partial^i\lambda\partial_i\lambda - \frac{1}{2}D^k\partial_k\lambda$$

$\rightarrow$ effective $3+1$ system with source terms

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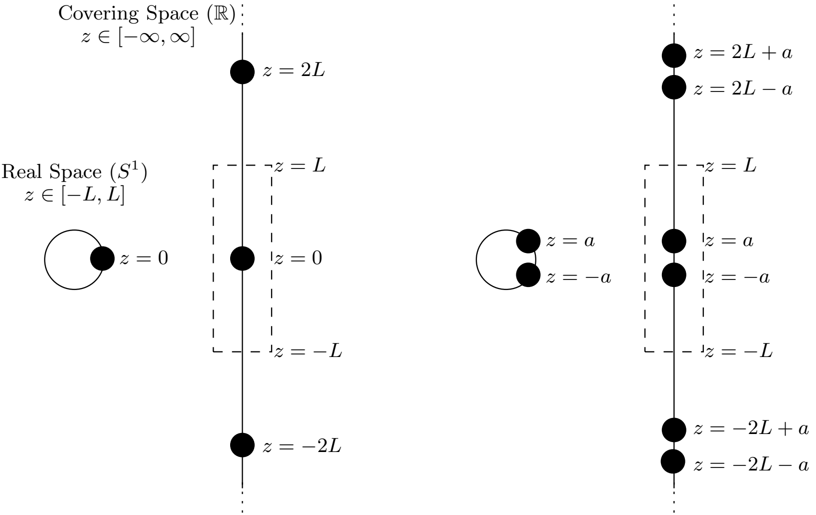
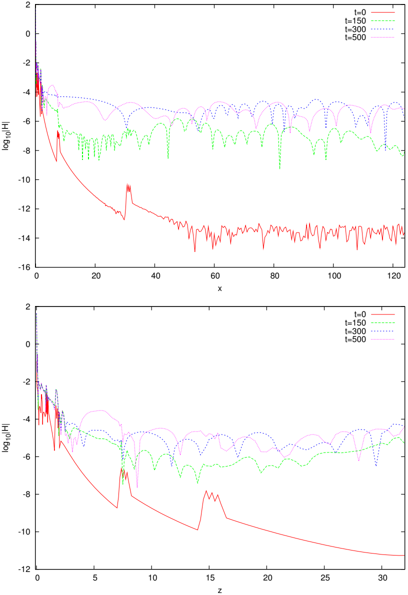
Initial data



Single black hole **Head on collision**

$L = 32$ single black hole evolution – constraint violation



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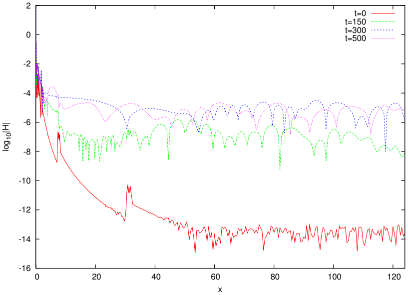

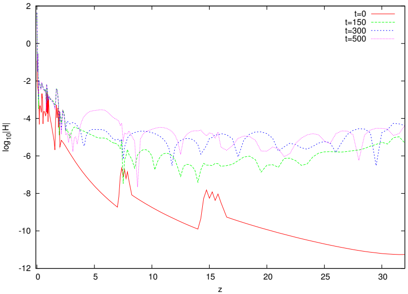

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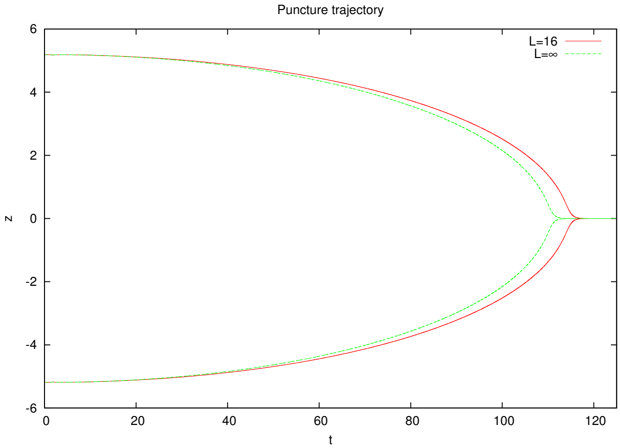
$L = 16$ head-on collision – trajectory



→ longer collision time for the cylindrical case

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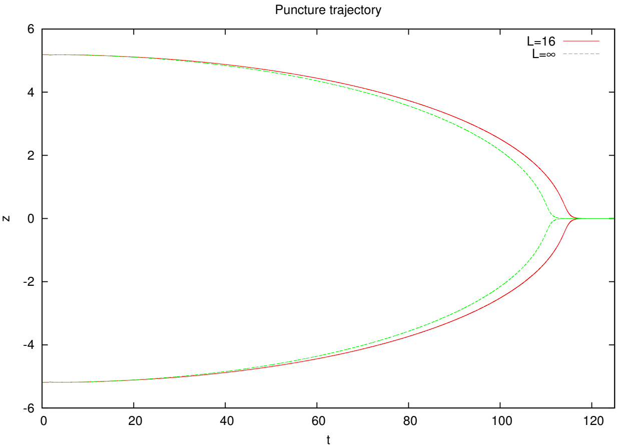

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Conclusions

- BH in a box
  - Studied stability of rotating BH and gravitational radiation in boxed spacetime;
  - Results consistent with expectations for wavepacket of radiation travelling back and forth
- BH in de Sitter
  - Evolved head-on collision of BHs in asymptotically de Sitter spacetime and monitored apparent horizons;
  - ToDo: Study formation of common apparent horizon as function of initial separation;
- $5D$ cylinder
  - Successfully evolved head-on collision of BHs in a $5D$ cylindrical spacetime using dimensional reduction procedure;
  - ToDo: deformation of the apparent horizon; radiated energy; smaller compactification radius;

The group

http://blackholes.ist.utl.pt/

the end