Black Holes and Compact Binaries in Alternative Theories

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August 31st, 2011
NR-HEP Workshop

Standing on the Shoulders of...

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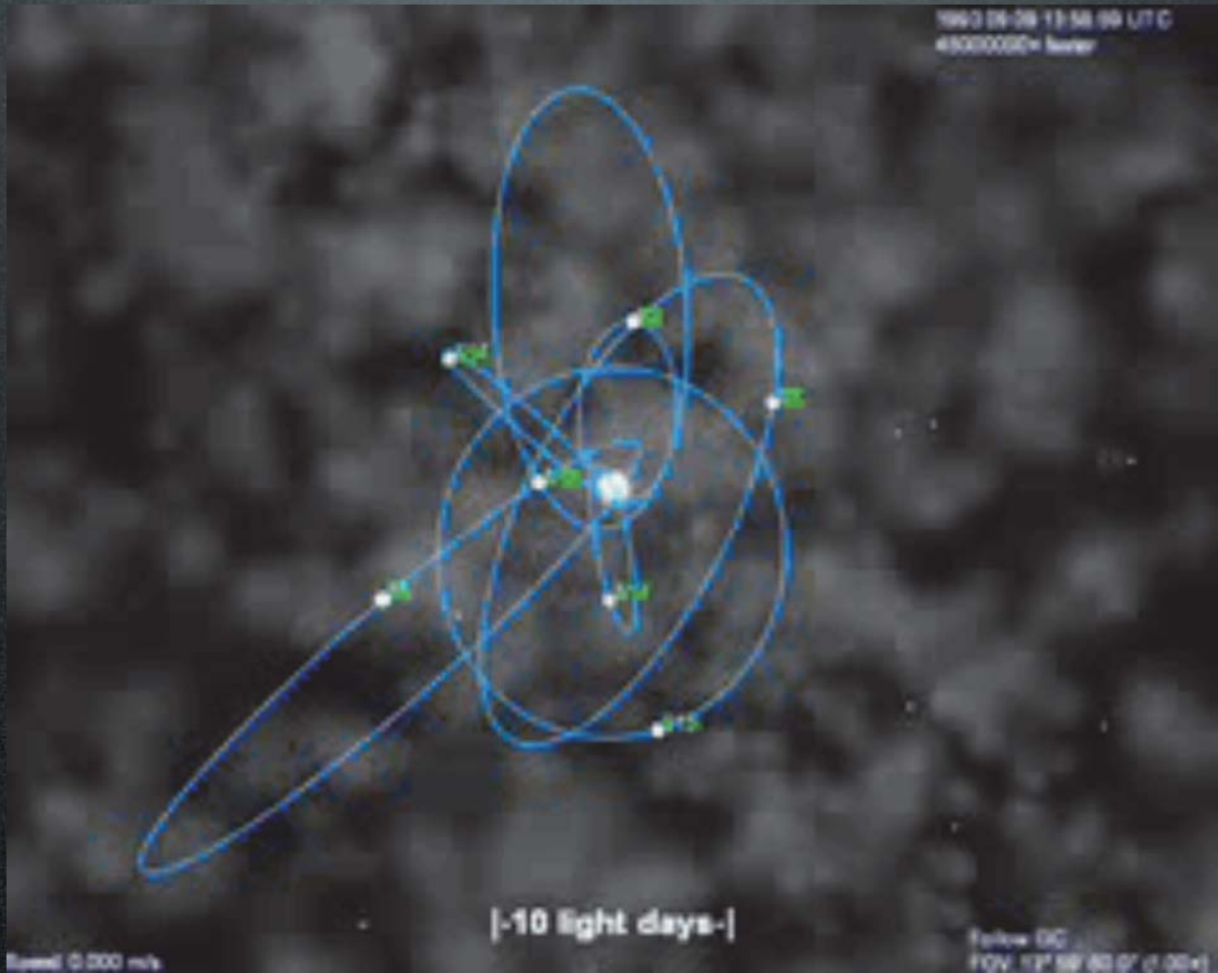
Do Black Holes Exist?

There is a dense object at the center of our galaxy, but how do we know that it can be completely described by the Kerr metric?



MPI movie

What else could it be? Can we test the Kerr hypothesis?

Road Map

I. Black Holes in Alternative Theories

II. Compact Binaries in Alternative Theories

III. Gravitational Wave Tests of Alternative Theories

I. Black Holes in Alternative Theories

Black Holes in General Relativity

No-Hair Theorems

All **stationary**, **vacuum** BH solutions to the **Einstein equations** have spherical horizon topology (Hawking)

All **stationary**, **vacuum**, BH solutions to the **Einstein equations** are static or axisymmetric (Hawking)

All **static**, **vacuum**, spherically symmetric BH solutions to the **Einstein equations** are Schwarzschild (Birhoff theorem, Israel)

All **stationary**, axisymmetric **vacuum** solutions to the **Einstein equations** are uniquely characterized by 2 parameters: M and S -> Kerr BHs are unique. (Carter)

Black Holes need not be Kerr if:

(i) They are not in vacuum (eg. plasma, accretion disk).
(ii) They are not stationary (eg. flows in disks).
(iii) They are not solutions to the Einstein equations.

Black Holes in Alternative Theories

Simplifications

We will focus on stationary, vacuum solutions to modified gravity field equations in four-dimensions.

We will search for “small” deformations away from GR solutions because we want (i) stable solutions, (ii) solutions that pass weak-field tests and (iii) analytic solutions.

We will study theories with curvature expansions of the form:

$$S = S_{GR} + S_{AT} + S_{Kin}$$

$$S_{GR} = \int d^4x \sqrt{-g}\, \kappa\, R \qquad\qquad S_{Kin} = \int d^4x \sqrt{-g}\, \beta\, (\partial_a \vartheta)(\partial^a \vartheta)$$

$$S_{AT} = \int d^4x \sqrt{-g}\, \left(\alpha_1 \vartheta R^2 + \alpha_2 \vartheta R_{ab} R^{ab} + \alpha_3 \vartheta R_{abcd} R^{abcd} + \alpha_4 \vartheta R_{abcd}\, {}^{*}R^{abcd} \right)$$

Theta is a spacetime function and the alpha’s are coupling constants.

Black Holes in DCSG

Spherically symmetric metrics are not modified in DCSG (Birkhoff theorem holds) but non-spherically symmetric ones are, coupling to parity breaking terms.

$$ds^2 = ds^2_{Kerr} + \frac{2Ma}{r}\sin^2\theta dt\, d\phi\, \left[\frac{5}{16}\left(\frac{\alpha_4^2}{\beta\kappa M^4}\right)\frac{M^3}{r^3}\left(1+\frac{12}{7}\frac{M}{r}+\frac{27}{10}\frac{M^2}{r^2}\right)\right]$$

$$\vartheta = \frac{5}{8}\frac{\alpha_4}{\beta}\frac{a}{M}\frac{\cos\theta}{r^2}\left(1+\frac{2M}{r}+\frac{18M^2}{5r^2}\right)$$

valid to O(a^2/M^2) and to O(alpha_4)

This modified gravity deformation corrects the dragging of inertial frames (the gravitomagnetic sector of the metric only).

A "hairy" solution in that it has a pseudo-scalar charge that acts as a magnetic dipole.

(Yunes and Pretorius '09)

Black Holes in EDGB

Spherically symmetric metrics are modified, Birkhoff Theorem does not hold (neither do the no-hair theorems).

$$g_{tt} = g_{tt}^{Schw} - \frac{1}{3}\frac{\alpha_3^2}{\beta\kappa}\frac{M^3}{r^3}\left(1 + \frac{26M}{r} + \ldots\right)$$

$$g_{rr} = g_{rr}^{Schw} - \frac{\alpha_3^2}{\beta\kappa}\left(1 - \frac{2M}{r}\right)^{-2}\frac{M^2}{r^2}\left(1 + \frac{M}{r} + \ldots\right)$$

$$\vartheta = \frac{\alpha_3}{\beta}\frac{2}{Mr}\left(1 + \frac{M}{r} + \frac{4M^2}{3r^2}\right)$$

A “hairy” solution in that it has a scalar charge that acts as an electric monopole.

(Yunes and Stein ’10)

Modified Gravity Bumpy Black Holes

Can we construct a metric that parametrically deviates from Kerr such that the previous metrics can be reproduced in some limit, while also approximately preserving the Kerr Killing symmetries?

$$\boxed{g_{ab} = g_{ab}^{Kerr} + h_{ab}}$$

where hab is given by some complicated functions of coordinates and bump coefficients. gab satisfies the Killing equations.

- The resulting metric is stationary and axially symmetric, but it does not solve the Einstein equations.
- This metric is valid for small deformations away from Kerr (hab<<1)
- With this, we can now study how EMRIs evolve in a parametrically deformed, bumpy spacetime.

(Vigeland, Stein and Yunes '10)

II. Compact Binaries in Alt. Theories

Compact Binaries in GR

Metric Perturbation

$$\boxed{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}} \qquad \boxed{G_{\mu\nu} = 8\pi T_{\mu\nu}} \quad \to \quad \boxed{\Box_\eta h_{\mu\nu} = \tau_{\mu\nu}[h^2]}$$

metric tensor

flat

GW metric pert.

Einstein tensor

Stress-Energy tensor

Flat-space, diff. wave op.

Annoying non-linearities

Acceleration

$$\boxed{\nabla^\mu \left(G_{\mu\nu} - 8\pi T_{\mu\nu}\right) = 0}$$

Assumptions

Four-Velocity

Pressure

Density

Perfect Fluid

$$\boxed{T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}}$$

Mass

Point Particles

$$\boxed{\rho = m\, \delta^3(x^i - y^i)}$$

Particle Location

Solve Perturbatively

orbital velocity

speed of light

$$\epsilon := \frac{v}{c} = \sqrt{\frac{m}{r}} \ll 1$$

total mass

orbital separation

An Example: Equations of Motion

$$a_1^i = -\frac{Gm_2 n_{12}^i}{r_{12}^2}$$

Leading term: Newtonian gravity.

$$+\frac{1}{c^2}\left\{\left[\frac{5G^2m_1m_2}{r_{12}^3}+\frac{4G^2m_2^2}{r_{12}^3}+\frac{Gm_2}{r_{12}^2}\left(\frac{3}{2}(n_{12}v_2)^2-v_1^2+4(v_1v_2)-2v_2^2\right)\right]n_{12}^i + \frac{Gm_2}{r_{12}^2}\left(4(n_{12}v_1)-3(n_{12}v_2)\right)v_{12}^i\right\}$$

1 PN (Relativity corrections)

$$\begin{aligned}
&+\frac{1}{c^4}\left\{\left[-\frac{57G^3m_1^2m_2}{4r_{12}^4}-\frac{69G^3m_1m_2^2}{2r_{12}^4}-\frac{9G^3m_2^3}{r_{12}^4}\right.\right.\\
&\quad+\frac{Gm_2}{r_{12}^2}\left(-\frac{15}{8}(n_{12}v_2)^4+\frac{3}{2}(n_{12}v_2)^2v_1^2-6(n_{12}v_2)^2(v_1v_2)-2(v_1v_2)^2+\frac{9}{2}(n_{12}v_2)^2v_2^2\right.\\
&\qquad\left.+4(v_1v_2)v_2^2-2v_2^4\right)\\
&\quad+\frac{G^2m_1m_2}{r_{12}^3}\left(\frac{39}{2}(n_{12}v_1)^2-39(n_{12}v_1)(n_{12}v_2)+\frac{17}{2}(n_{12}v_2)^2-\frac{15}{4}v_1^2-\frac{5}{2}(v_1v_2)+\frac{5}{4}v_2^2\right)\\
&\quad\left.+\frac{G^2m_2^2}{r_{12}^3}\left(2(n_{12}v_1)^2-4(n_{12}v_1)(n_{12}v_2)-6(n_{12}v_2)^2-8(v_1v_2)+4v_2^2\right)\right]n_{12}^i\\
&\quad+\left[\frac{G^2m_2^2}{r_{12}^3}\left(-2(n_{12}v_1)-2(n_{12}v_2)\right)+\frac{G^2m_1m_2}{r_{12}^3}\left(-\frac{63}{4}(n_{12}v_1)+\frac{55}{4}(n_{12}v_2)\right)\right.\\
&\qquad+\frac{Gm_2}{r_{12}^2}\left(-6(n_{12}v_1)(n_{12}v_2)^2+\frac{9}{2}(n_{12}v_2)^3+(n_{12}v_2)v_1^2-4(n_{12}v_1)(v_1v_2)\right.\\
&\qquad\qquad\left.\left.\left.+4(n_{12}v_2)(v_1v_2)+4(n_{12}v_1)v_2^2-5(n_{12}v_2)v_2^2\right)\right]v_{12}^i\right\}\\
&+\frac{1}{c^5}\left\{\left[\frac{208G^3m_1m_2^2}{15r_{12}^4}(n_{12}v_{12})-\frac{24G^3m_1^2m_2}{5r_{12}^4}(n_{12}v_{12})+\frac{12G^2m_1m_2}{5r_{12}^3}(n_{12}v_{12})v_{12}^2\right]n_{12}^i\right.\\
&\quad\left.+\left[\frac{8G^3m_1^2m_2}{5r_{12}^4}-\frac{32G^3m_1m_2^2}{5r_{12}^4}-\frac{4G^2m_1m_2}{5r_{12}^3}v_{12}^2\right]v_{12}^i\right\}
\end{aligned}$$

2 PN (more corrections …)

and craziness ensues...

$$+\frac{1}{c^6}\left\{\left[\frac{Gm_2}{r_{12}^2}\left(\frac{35}{16}(n_{12}v_2)^6-\frac{15}{8}(n_{12}v_2)^4v_1^2+\frac{15}{2}(n_{12}v_2)^4(v_1v_2)+3(n_{12}v_2)^2(v_1v_2)^2\right.\right.\right.$$
$$-\frac{15}{2}(n_{12}v_2)^4v_2^2+\frac{3}{2}(n_{12}v_2)^2v_1^2v_2^2-12(n_{12}v_2)^2(v_1v_2)v_2^2-2(v_1v_2)^2v_2^2$$
$$\left.+\frac{15}{2}(n_{12}v_2)^2v_2^4+4(v_1v_2)v_2^4-2v_2^6\right)$$
$$+\frac{G^2m_1m_2}{r_{12}^3}\left(-\frac{171}{8}(n_{12}v_1)^4+\frac{171}{2}(n_{12}v_1)^3(n_{12}v_2)-\frac{723}{4}(n_{12}v_1)^2(n_{12}v_2)^2\right.$$
$$+\frac{383}{2}(n_{12}v_1)(n_{12}v_2)^3-\frac{455}{8}(n_{12}v_2)^4+\frac{229}{4}(n_{12}v_1)^2v_1^2$$
$$-\frac{205}{2}(n_{12}v_1)(n_{12}v_2)v_1^2+\frac{191}{4}(n_{12}v_2)^2v_1^2-\frac{91}{8}v_1^4-\frac{229}{2}(n_{12}v_1)^2(v_1v_2)$$
$$+244(n_{12}v_1)(n_{12}v_2)(v_1v_2)-\frac{225}{2}(n_{12}v_2)^2(v_1v_2)+\frac{91}{2}v_1^2(v_1v_2)$$
$$-\frac{177}{4}(v_1v_2)^2+\frac{229}{4}(n_{12}v_1)^2v_2^2-\frac{283}{2}(n_{12}v_1)(n_{12}v_2)v_2^2$$
$$\left.+\frac{259}{4}(n_{12}v_2)^2v_2^2-\frac{91}{4}v_1^2v_2^2+43(v_1v_2)v_2^2-\frac{81}{8}v_2^4\right)$$
$$+\frac{G^2m_2^2}{r_{12}^3}\left(-6(n_{12}v_1)^2(n_{12}v_2)^2+12(n_{12}v_1)(n_{12}v_2)^3+6(n_{12}v_2)^4\right.$$
$$+4(n_{12}v_1)(n_{12}v_2)(v_1v_2)+12(n_{12}v_2)^2(v_1v_2)+4(v_1v_2)^2$$
$$\left.-4(n_{12}v_1)(n_{12}v_2)v_2^2-12(n_{12}v_2)^2v_2^2-8(v_1v_2)v_2^2+4v_2^4\right)$$
$$+\frac{G^3m_2^3}{r_{12}^4}\left(-(n_{12}v_1)^2+2(n_{12}v_1)(n_{12}v_2)+\frac{43}{2}(n_{12}v_2)^2+18(v_1v_2)-9v_2^2\right)$$
$$+\frac{G^3m_1m_2^2}{r_{12}^4}\left(\frac{415}{8}(n_{12}v_1)^2-\frac{375}{4}(n_{12}v_1)(n_{12}v_2)+\frac{1113}{8}(n_{12}v_2)^2-\frac{615}{64}(n_{12}v_{12})^2\pi^2\right.$$
$$\left.+18v_1^2+\frac{123}{64}\pi^2v_{12}^2+33(v_1v_2)-\frac{33}{2}v_2^2\right)$$
$$+\frac{G^3m_1^2m_2}{r_{12}^4}\left(-\frac{45887}{168}(n_{12}v_1)^2+\frac{24025}{42}(n_{12}v_1)(n_{12}v_2)-\frac{10469}{42}(n_{12}v_2)^2+\frac{48197}{840}v_1^2\right.$$
$$\left.-\frac{36227}{420}(v_1v_2)+\frac{36227}{840}v_2^2+110(n_{12}v_{12})^2\ln\left(\frac{r_{12}}{r'_1}\right)-22v_{12}^2\ln\left(\frac{r_{12}}{r'_1}\right)\right)$$
$$+\frac{16G^4m_2^4}{r_{12}^5}+\frac{G^4m_1^2m_2^2}{r_{12}^5}\left(175-\frac{41}{16}\pi^2\right)+\frac{G^4m_1^3m_2}{r_{12}^5}\left(-\frac{3187}{1260}+\frac{44}{3}\ln\left(\frac{r_{12}}{r'_1}\right)\right)$$
$$\left.+\frac{G^4m_1m_2^3}{r_{12}^5}\left(\frac{110741}{630}-\frac{41}{16}\pi^2-\frac{44}{3}\ln\left(\frac{r_{12}}{r'_2}\right)\right)\right]n_{12}^i$$
$$+\left[\frac{Gm_2}{r_{12}^2}\left(\frac{15}{2}(n_{12}v_1)(n_{12}v_2)^4-\frac{45}{8}(n_{12}v_2)^5-\frac{3}{2}(n_{12}v_2)^3v_1^2+6(n_{12}v_1)(n_{12}v_2)^2(v_1v_2)\right.\right.$$

3 PN (yet more corrections ...)

$$-6(n_{12}v_2)^3(v_1v_2)-2(n_{12}v_2)(v_1v_2)^2-12(n_{12}v_1)(n_{12}v_2)^2v_2^2+12(n_{12}v_2)^3v_2^2$$
$$+(n_{12}v_2)v_1^2v_2^2-4(n_{12}v_1)(v_1v_2)v_2^2+8(n_{12}v_2)(v_1v_2)v_2^2+4(n_{12}v_1)v_2^4$$
$$\left.-7(n_{12}v_2)v_2^4\right)$$
$$+\frac{G^2m_2^2}{r_{12}^3}\left(-2(n_{12}v_1)^2(n_{12}v_2)+8(n_{12}v_1)(n_{12}v_2)^2+2(n_{12}v_2)^3+2(n_{12}v_1)(v_1v_2)\right.$$
$$\left.+4(n_{12}v_2)(v_1v_2)-2(n_{12}v_1)v_2^2-4(n_{12}v_2)v_2^2\right)$$
$$+\frac{G^2m_1m_2}{r_{12}^3}\left(-\frac{243}{4}(n_{12}v_1)^3+\frac{565}{4}(n_{12}v_1)^2(n_{12}v_2)-\frac{269}{4}(n_{12}v_1)(n_{12}v_2)^2\right.$$
$$-\frac{95}{12}(n_{12}v_2)^3+\frac{207}{8}(n_{12}v_1)v_1^2-\frac{137}{8}(n_{12}v_2)v_1^2-36(n_{12}v_1)(v_1v_2)$$
$$\left.+\frac{27}{4}(n_{12}v_2)(v_1v_2)+\frac{81}{8}(n_{12}v_1)v_2^2+\frac{83}{8}(n_{12}v_2)v_2^2\right)$$
$$+\frac{G^3m_2^3}{r_{12}^4}\left(4(n_{12}v_1)+5(n_{12}v_2)\right)$$
$$+\frac{G^3m_1m_2^2}{r_{12}^4}\left(-\frac{307}{8}(n_{12}v_1)+\frac{479}{8}(n_{12}v_2)+\frac{123}{32}(n_{12}v_{12})\pi^2\right)$$
$$\left.\left.+\frac{G^3m_1^2m_2}{r_{12}^4}\left(\frac{31397}{420}(n_{12}v_1)-\frac{36227}{420}(n_{12}v_2)-44(n_{12}v_{12})\ln\left(\frac{r_{12}}{r'_1}\right)\right)\right]v_{12}^i\right\}$$
$$+\frac{1}{c^7}\left\{\left[\frac{G^4m_1^3m_2}{r_{12}^5}\left(\frac{3992}{105}(n_{12}v_1)-\frac{4328}{105}(n_{12}v_2)\right)\right.\right.$$
$$+\frac{G^4m_1^2m_2^2}{r_{12}^6}\left(-\frac{13576}{105}(n_{12}v_1)+\frac{2872}{21}(n_{12}v_2)\right)-\frac{3172}{21}\frac{G^4m_1m_2^3}{r_{12}^6}(n_{12}v_{12})$$
$$+\frac{G^3m_1^2m_2}{r_{12}^4}\left(48(n_{12}v_1)^3-\frac{696}{5}(n_{12}v_1)^2(n_{12}v_2)+\frac{744}{5}(n_{12}v_1)(n_{12}v_2)^2-\frac{288}{5}(n_{12}v_2)^3\right.$$
$$-\frac{4888}{105}(n_{12}v_1)v_1^2+\frac{5056}{105}(n_{12}v_2)v_1^2+\frac{2056}{21}(n_{12}v_1)(v_1v_2)$$
$$\left.-\frac{2224}{21}(n_{12}v_2)(v_1v_2)-\frac{1028}{21}(n_{12}v_1)v_2^2+\frac{5812}{105}(n_{12}v_2)v_2^2\right)$$
$$+\frac{G^3m_1m_2^2}{r_{12}^4}\left(-\frac{582}{5}(n_{12}v_1)^3+\frac{1746}{5}(n_{12}v_1)^2(n_{12}v_2)-\frac{1954}{5}(n_{12}v_1)(n_{12}v_2)^2\right.$$
$$+158(n_{12}v_2)^3+\frac{3568}{105}(n_{12}v_{12})v_1^2-\frac{2864}{35}(n_{12}v_1)(v_1v_2)$$
$$\left.+\frac{10048}{105}(n_{12}v_2)(v_1v_2)+\frac{1432}{35}(n_{12}v_1)v_2^2-\frac{5752}{105}(n_{12}v_2)v_2^2\right)$$
$$+\frac{G^2m_1m_2}{r_{12}^3}\left(-56(n_{12}v_{12})^5+60(n_{12}v_1)^3v_{12}^2-180(n_{12}v_1)^2(n_{12}v_2)v_{12}^2\right.$$

$$+174(n_{12}v_1)(n_{12}v_2)^2v_{12}^2-54(n_{12}v_2)^3v_{12}^2-\frac{246}{35}(n_{12}v_{12})v_1^4$$
$$+\frac{1068}{35}(n_{12}v_1)v_1^2(v_1v_2)-\frac{984}{35}(n_{12}v_2)v_1^2(v_1v_2)-\frac{1068}{35}(n_{12}v_1)(v_1v_2)^2$$
$$+\frac{180}{7}(n_{12}v_2)(v_1v_2)^2-\frac{534}{35}(n_{12}v_1)v_1^2v_2^2+\frac{90}{7}(n_{12}v_2)v_1^2v_2^2$$
$$+\frac{984}{35}(n_{12}v_1)(v_1v_2)v_2^2-\frac{732}{35}(n_{12}v_2)(v_1v_2)v_2^2-\frac{204}{35}(n_{12}v_1)v_2^4$$
$$\left.\left.+\frac{24}{7}(n_{12}v_2)v_2^4\right)\right]n_{12}^i$$
$$+\left[-\frac{184}{21}\frac{G^4m_1^3m_2}{r_{12}^5}+\frac{6224}{105}\frac{G^4m_1^2m_2^2}{r_{12}^6}+\frac{6388}{105}\frac{G^4m_1m_2^3}{r_{12}^6}\right.$$
$$+\frac{G^3m_1^2m_2}{r_{12}^4}\left(\frac{52}{15}(n_{12}v_1)^2-\frac{56}{15}(n_{12}v_1)(n_{12}v_2)-\frac{44}{15}(n_{12}v_2)^2-\frac{132}{35}v_1^2+\frac{152}{35}(v_1v_2)\right.$$
$$\left.-\frac{48}{35}v_2^2\right)$$
$$+\frac{G^3m_1m_2^2}{r_{12}^4}\left(\frac{454}{15}(n_{12}v_1)^2-\frac{372}{5}(n_{12}v_1)(n_{12}v_2)+\frac{854}{15}(n_{12}v_2)^2-\frac{152}{21}v_1^2\right.$$
$$\left.+\frac{2864}{105}(v_1v_2)-\frac{1768}{105}v_2^2\right)$$
$$+\frac{G^2m_1m_2}{r_{12}^3}\left(60(n_{12}v_{12})^4-\frac{348}{5}(n_{12}v_1)^2v_{12}^2+\frac{684}{5}(n_{12}v_1)(n_{12}v_2)v_{12}^2\right.$$
$$-66(n_{12}v_2)^2v_{12}^2+\frac{334}{35}v_1^4-\frac{1336}{35}v_1^2(v_1v_2)+\frac{1308}{35}(v_1v_2)^2+\frac{654}{35}v_1^2v_2^2$$
$$\left.\left.\left.-\frac{1252}{35}(v_1v_2)v_2^2+\frac{292}{35}v_2^4\right)\right]v_{12}^i\right\}$$
$$+\mathcal{O}\left(\frac{1}{c^8}\right).$$

[Blanchet 2006, Liv Rev Rel 9, 4, Eq. (168)]

Impressed yet ... ?

Compact Binaries in Alt. Theories

$$\boxed{G_{\mu\nu} + C_{\mu\nu} = 8\pi T_{\mu\nu}}$$

Start with the modified field equations

$$\boxed{\Box_\eta h_{\mu\nu} = \tau_{\mu\nu}[h^2] + \sigma_{\mu\nu}[h^2, \partial^n h]}$$

Linearize about Minkowski

$$\boxed{\Box_\eta \delta h_{\mu\nu} = \tau_{\mu\nu}[h^2_{GR}] + \sigma_{\mu\nu}[h^2_{GR}, \partial^n h_{GR}]}$$

Linearize about GR

and now you can use the same PN tools as always to solve the above wave equations (see eg. the DIRE approach or dim regularization).

But be careful!!

The point-particle description of BHs works in GR (in part due to the Birkhoff theorem), but this need not be so in Alternative Theories. In fact, usually one must compensate for violations of this description.

(Yagi, Stein, Yunes and Tanaka '10)

Some Preliminary Results

$$\vartheta = \frac{\alpha_3}{\beta} \frac{2}{Mr} \left(1 + \frac{M}{r} + \frac{4M^2}{3r^2}\right)$$

$$\vartheta = \frac{5}{8} \frac{\alpha_4}{\beta} \frac{a}{M} \frac{\cos\theta}{r^2} \left(1 + \frac{2M}{r} + \frac{18M^2}{5r^2}\right)$$

Going back to EDGB and DCSG, recall that the scalar fields act like an electric monopole and a magnetic dipole.

Based on this, one expects the scalar fields to radiate like an electric-type monopole (-1PN) and like a magnetic-type dipole (2 PN). This was confirmed through the methods described previously.

- In both cases, this radiation modifies the rate of change of the energy-momentum lost by the binary, strongly impacting waves.

- In the DCSG case, not including the spin-dependent background scalar field leads to a huge suppression of the modification.

(Yagi, Stein, Yunes and Tanaka '10)

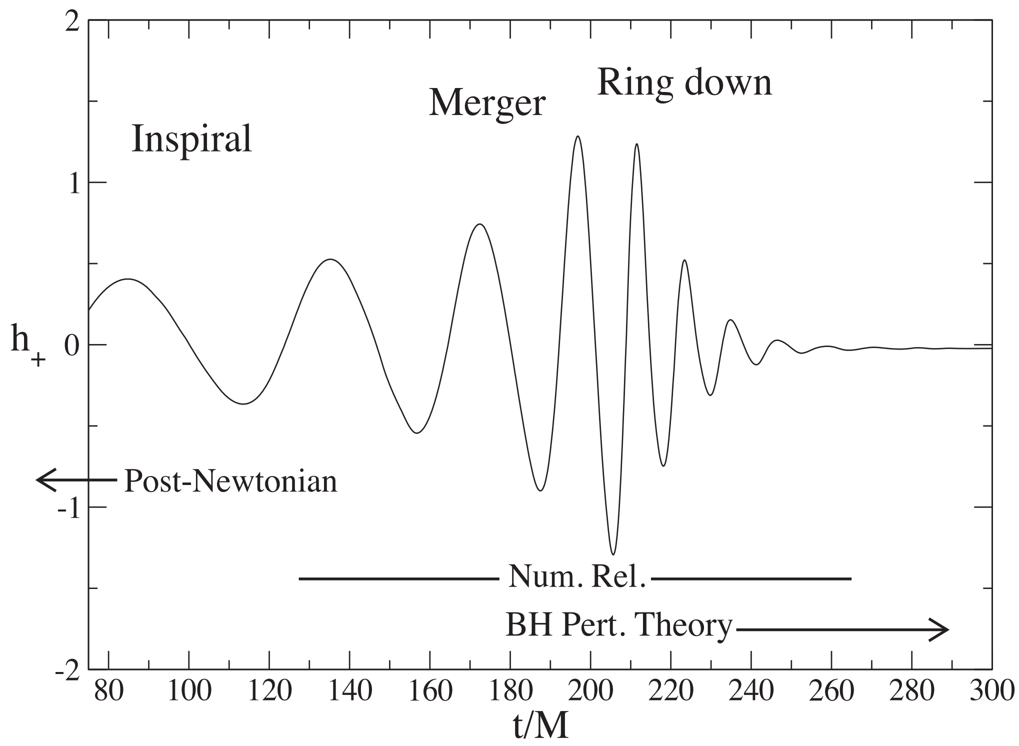
III. Gravitational Wave Tests

Gravitational Waveform in GR

$$\boxed{h_\times(t) \propto \frac{\mu}{R} \cos\iota \, [MF(t)]^{2/3} \cos 2\phi + \ldots}$$

gravitational wave | reduced mass | distance to the source | inclination angle | total mass | orbital freq. | orbital phase

Gravitational Waves contain information about the system that generates them.



Parameterized post-Einsteinian Framework (ppE)

(Yunes & Pretorius '09)

You cannot test GR by assuming GR templates a priori

Promote the response function to a non-GR response, with parameters that control "well-motivated" deformations

Extremely Simple Eg:
Inspiral ppE template

GR: $(\alpha, a, \beta, b) = (0, a, 0, b)$

BD: $(\alpha, a, \beta, b) = (0, a, \beta_{BD}, -7/3)$

PV: $(\alpha, a, \beta, b) = (\alpha_{CS}, 1, 0, b)$

$$\boxed{\tilde{h} = \tilde{h}_{\rm GR}\,(1 + \alpha\,\eta^c\,u^a)\,e^{i\,\beta\,\eta^d u^b}}$$

ppE parameters

Match filter with this new response function and let the data decide what these ppE parameters are.

(Yunes and Pretorius '09)

Questions for ppE

Given a GW detection, how sure are we it was a GR event? Statistically significant anomalies in the signal?

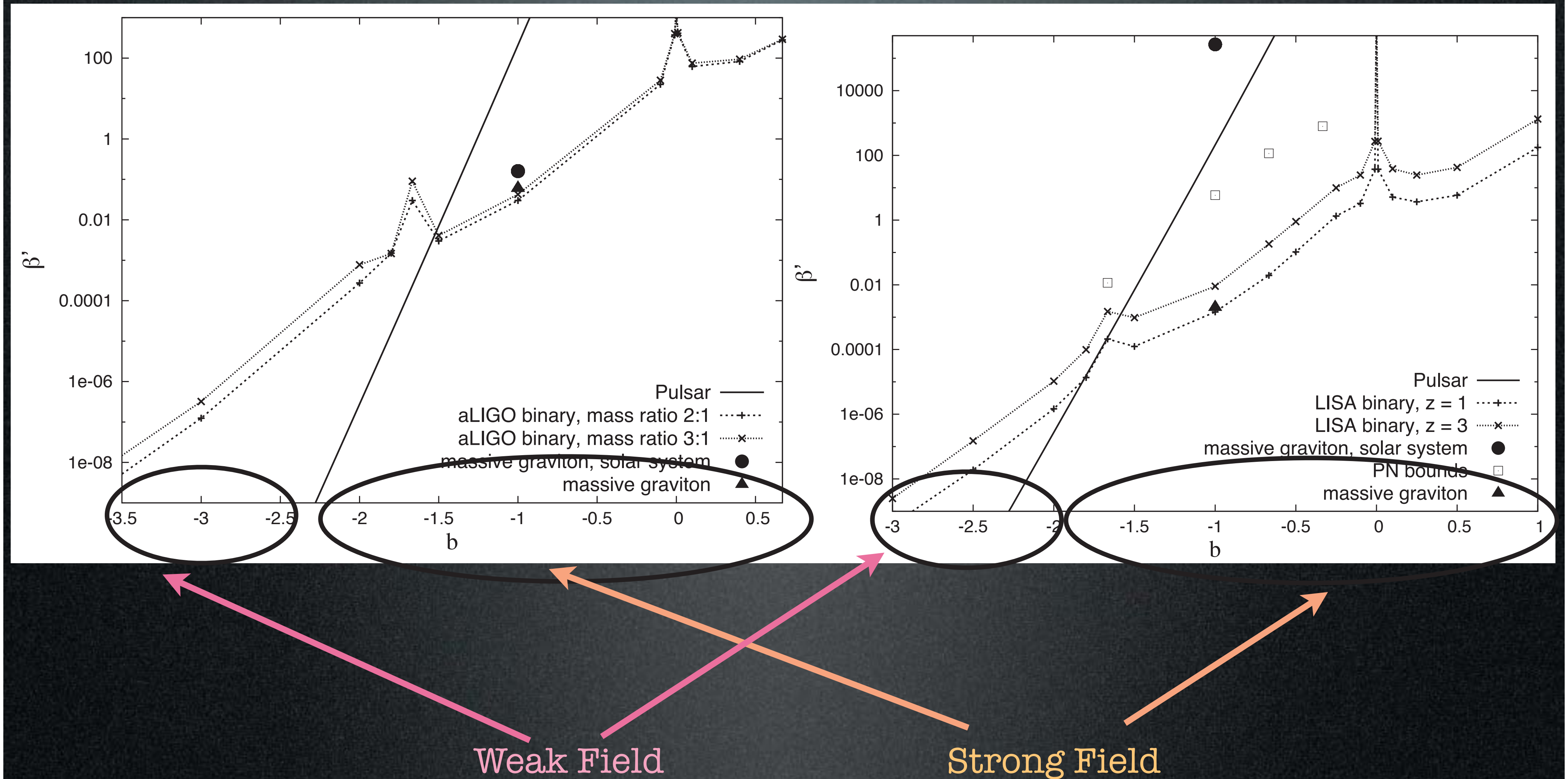
Can we test for deviations from/consistency with GR, without explicitly building templates banks for all conceivable theories?

How would we mischaracterize the universe if GR was **close** but **not quite** the correct theory of nature? ("fundamental bias")

Templates/ Theories	GR	ppE
GR	Business as usual	**Quantify** the likelihood of GR being the underlying theory describing the detected event, within the class of alt. theories captured by ppE
Not GR	Understand the bias that could be introduced filtering non-GR events with GR templates	Measure deviations from GR characterized by non-GR ppE parameters.

Constraining Phase Deviations

GR Signal/ppE Templates, 3-sigma constraints, SNR = 20



$$\tilde{h} = \tilde{h}_{\rm GR}\left(1 + \alpha f^{a}\right) e^{i\beta f^{b}}$$

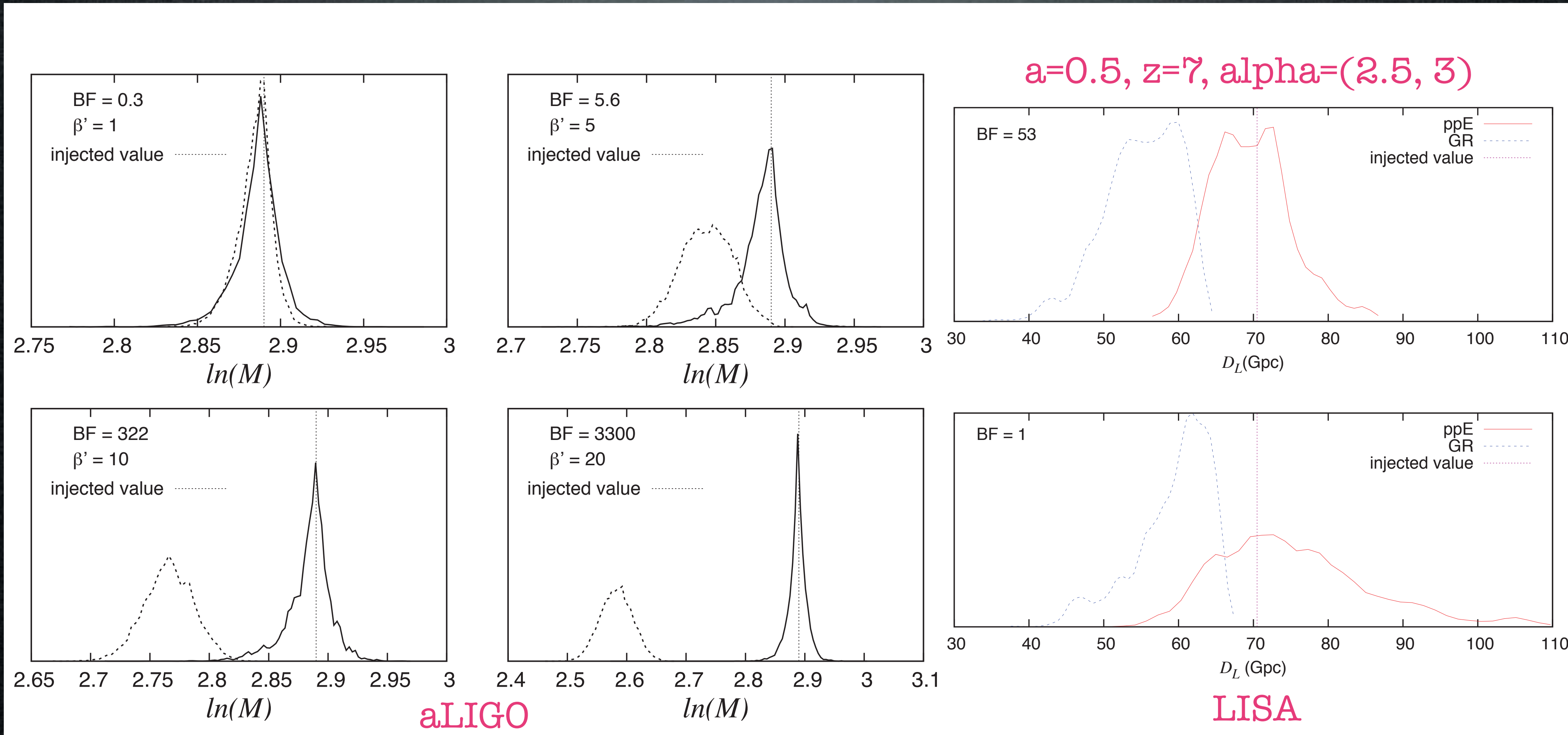
(Yunes and Hughes, PRD 82 (2010))

(Cornish, Sampson, Yunes & Pretorius (2011))

Parameter Bias

Non-GR Signal/GR and ppE Templates, SNR = 20

Non GR injection, extracted with GR templates (dashed) and ppE templates (solid). GR template extraction is “wrong” by much more than the systematic (statistical) error -> “Fundamental Bias”



(Cornish, Sampson, Yunes & Pretorius, 2011)

Conclusions

Black holes need not be described by the Kerr metric in alternative theories of gravity, even if these are stationary and axisymmetric.

Several analytic examples exist (EDGB, DCSG) for non-Kerr, “hairy” BH solutions in alternative theories.

Compact binary evolution is also modified in alternative theories of gravity, not only due to strong-field modifications to the individual BHs, but also due to new dynamical effects.

A new (ppE) waveform parameterization has been proposed to capture generic, model-independent GR deformations.

The full exploit of GW astrophysics will require the strong collaboration between relativists, astrophysicists, data analysts & high-energy theorists. The future is around the corner!

But What Theory Do We Pick?

A Minimal (?) Set of Criteria:

1. Weak-Field Consistency (existence and stability of physical solutions, satisfaction of precision tests).
2. Strong-Field Inconsistency (deviations only where experiments cannot currently rule out modifications)

It's not easy to fool Mother Nature! (Wald)

Other Nice Criteria:

3. Well motivated from fundamental physics.
4. Well-posed theory ?? This is hard to do...

A Proposed Recipe

(1) For low SNR sources, GWs are buried in noise. **Construct Templates** and extract via **matched filtering**, assuming GR is right. After all, Solar System/Binary Pulsar Tests have confirmed GR in the weak-field limit, so the early inspiral must be right.

(2) Go back to your data and study whether you have missed something or whether the data is consistent with GR:

If it is consistent

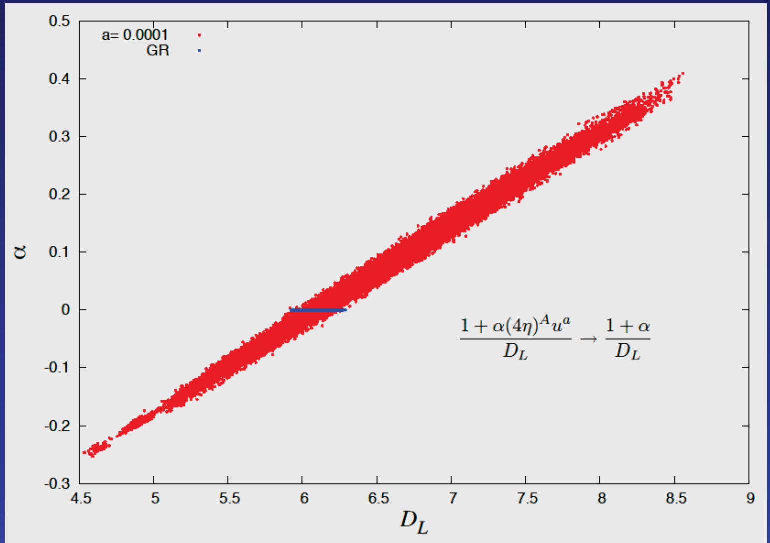
Test GR
Place a constraint on how large Phase and Amplitude deviations could be given uncertainties.

If it is not consistent

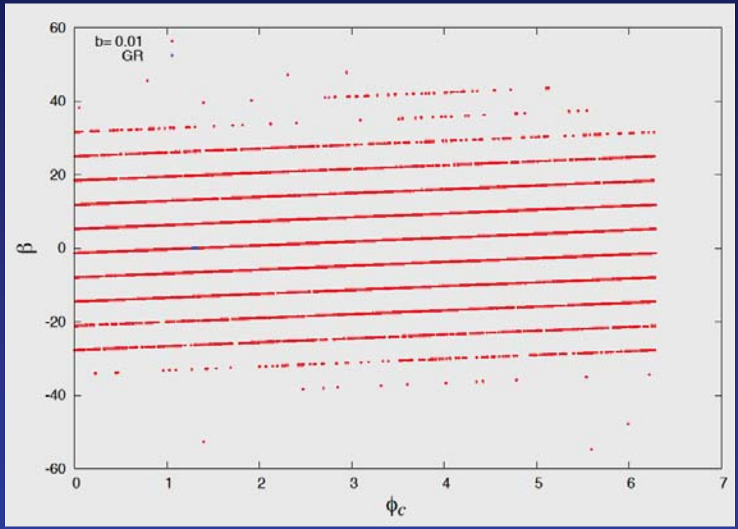
Cross-Correlate with other detectors to eliminate inst. and astroph. artifacts

Characterize any Phase or Amplitude deviation. Trace back to a specific modification to GR.

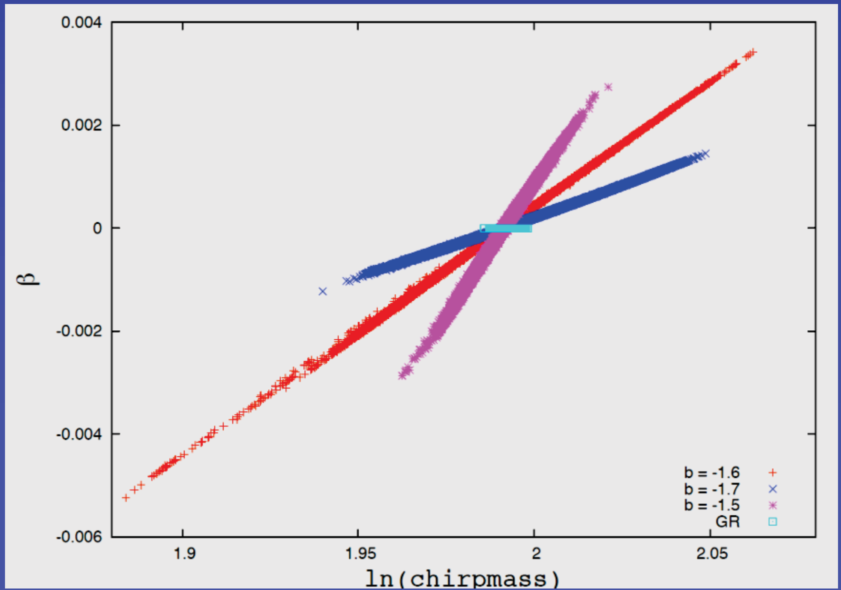
Unavoidable Correlations



peak at *a=0* is degeneracy between luminosity distance and effective α (LISA example)



peak at *b=0* is degeneracy between phase of coalescence and β (LISA example)

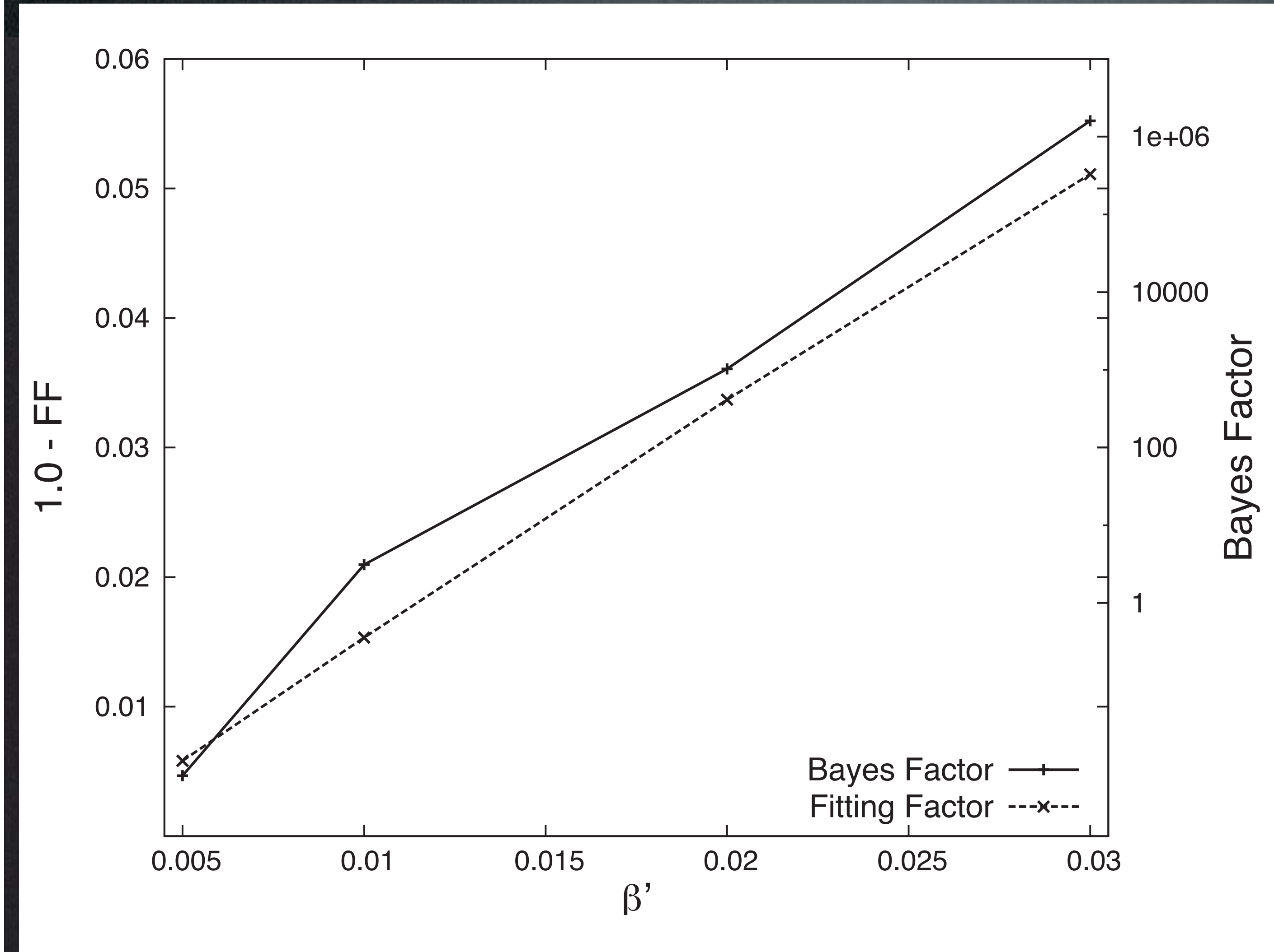


bump at *b=-5/3* (PN value) is a partial degeneracy between chirp mass and β (LIGO example)

Identifying GR Deviations

Non-GR Signal/ppE Templates, aLIGO, SNR = 20

Filter an injected ppE signal (a,alpha,b,beta)=(a,0,-1.25,0.1) with a ppE template family. For a given beta', integrate the posterior to find the evidence or Bayes factor or odds Ratio.



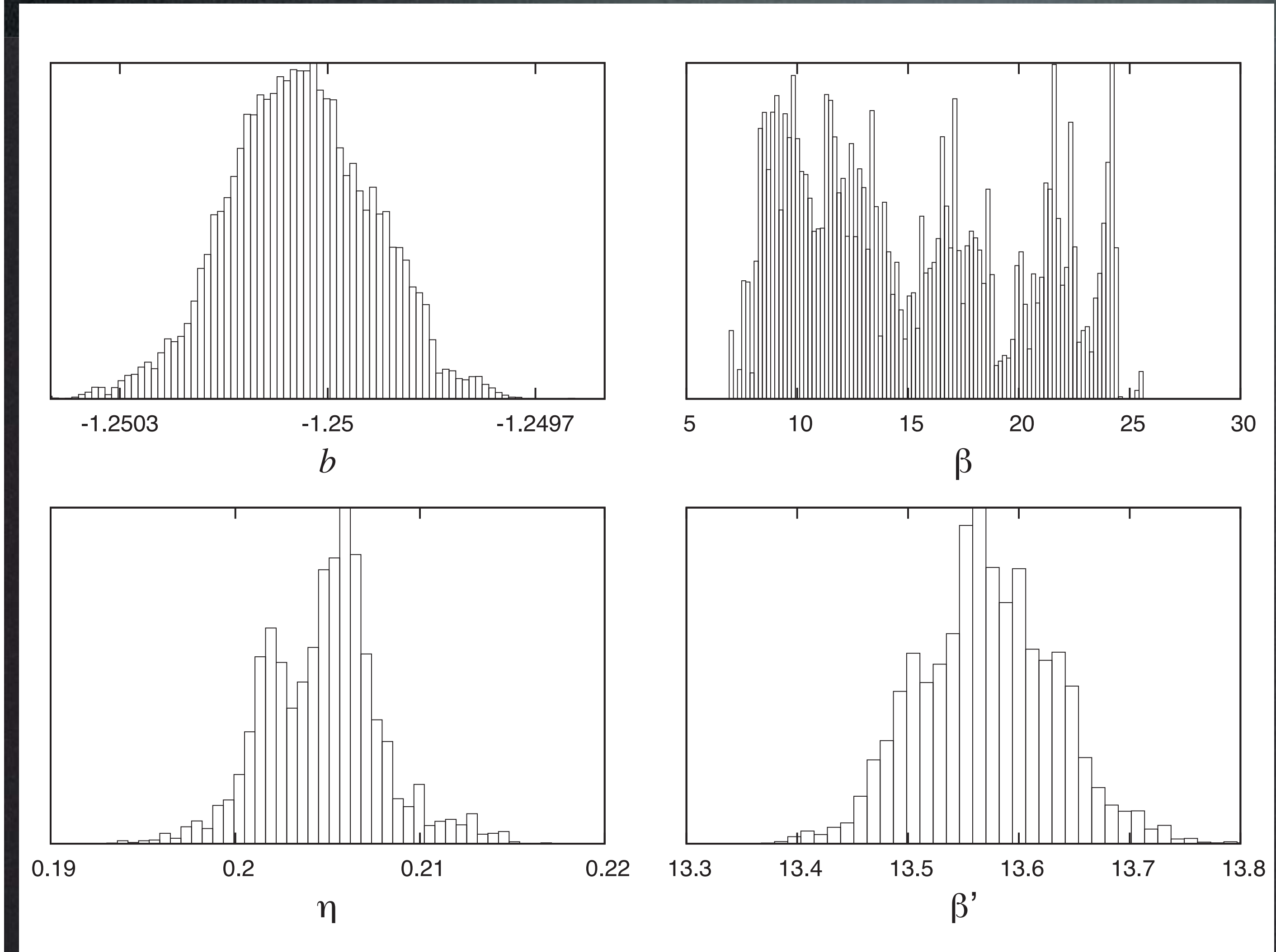
Eg. if beta' = 0.175, there would be a 100:1 odds that GR is wrong.

(Cornish, Sampson, Yunes & Pretorius, 2011)

Identifying GR Deviations

Non-GR Signal/ppE Templates, aLIGO, SNR = 20

Filter an injected ppE signal (b,beta,eta,beta')=(-1.25,10.0,0.204,13.57) with ppE templates. The marginalized posterior for beta shows a preference away from GR.



With a single detection, you cannot break the eta and beta degeneracy, so you can constrain beta'

(Cornish, Sampson, Yunes & Pretorius, 2011)