Black hole collisions in higher dimensional spacetimes

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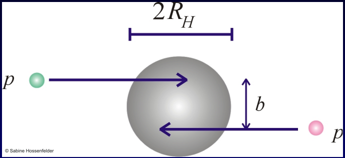
Outline

High Energy Collision of Particles

- above the Planck scale:
  gravity is dominant interaction
  $\Rightarrow$ classical description



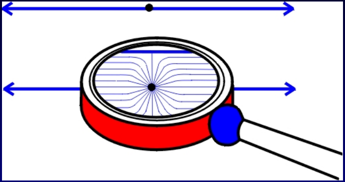
Consider particle collsions with $E = 2\gamma m_0 c^2 > M_{Pl}$

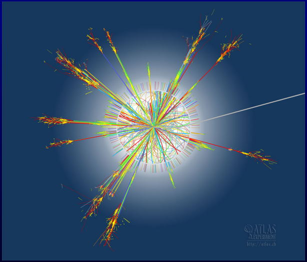
- Hoop - Conjecture (Thorne '72)
  $\Rightarrow$ black hole formation, if circumference of particle $< 2\pi r_S$
- Collisions of shock waves (Penrose '74, Eardley & Giddings '02)
  $\Rightarrow$ black hole formation if $b \leq r_S$
- numerical evidence in ultra relativistic collision of boson stars (Choptuik & Pretorius '10)
  $\Rightarrow$ black hole formation if boost $\gamma_c \geq 2.9$

$\Rightarrow$ black hole formation in high energy collisions of particles

TeV gravity

- in $D = 4$:
  $m_{EW} \sim 10^3\,GeV$, $M_{Pl} \sim 10^{19}\,GeV$
  $\Rightarrow$ "hierarchy problem"





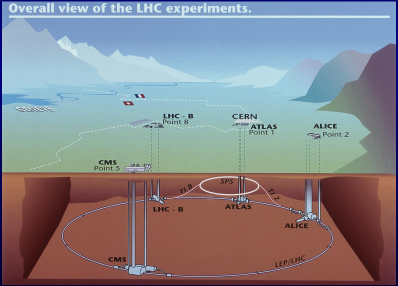
- higher dimensional theories of gravity
  - large extra dimensions
    (Arkani-Hamed, Dimopoulos & Dvali '98, Dvali, Gabadadze & Porrati '00)
  - warped extra dimensions
    (Randall & Sundrum '99)
- in $D > 4$: lowering of Planck scale
  $\Rightarrow M_{Pl} \sim TeV$

TeV gravity

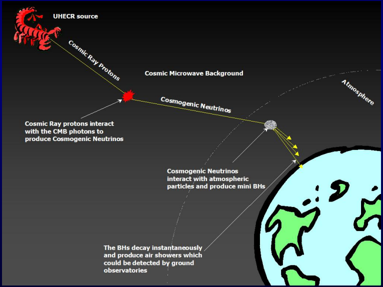
TeV gravity scenarios

$\Rightarrow$ signatures of black hole production in high energy collision of particles

- at the Large Hadron Collider
- in Cosmic Rays interactions

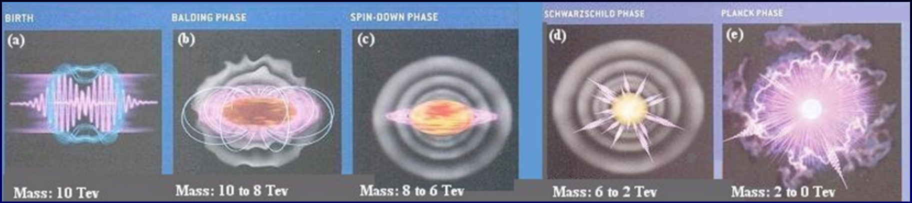


http://lhc.web.cern.ch/lhc/



http://www.phy.olemiss.edu/GR/

Life cycle of Mini Black Holes



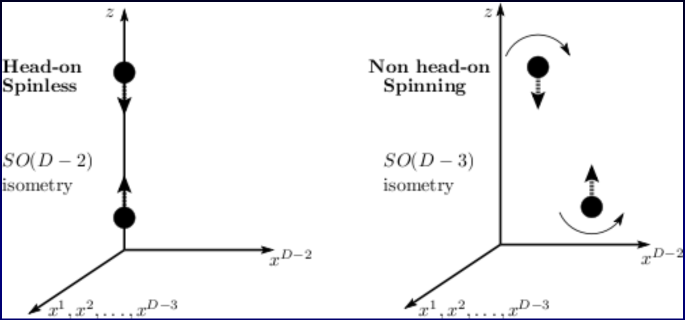
1. Formation
   - lower bound on BH mass from area theorem (Yoshino & Nambu '02)
2. Balding phase: end state is Myers-Perry black hole
3. Spindown phase: loss of angular momentum and mass
4. Schwarzschild phase: decay via Hawking radiation
5. Planck phase: $M \sim M_{Pl}$

Goal: more precise understanding of black hole formation
$\Rightarrow$ compute mass and spin of final black hole
$\Rightarrow$ input to event generators (TRUENOIR, Catfish, BlackMax, Charybdis2)
Toy model: black hole collisions in higher dimensions

Numerical Relativity in $D > 4$ Dimensions

- Yoshino & Shibata, Phys. Rev. **D80**, 2009,
  Shibata & Yoshino, Phys. Rev. **D81**, 2010
- Okawa, Nakao & Shibata, Phys. Rev. **83**, 2011
- Lehner & Pretorius, Phys. Rev. Lett. **105**, 2010
- Sorkin & Choptuik, GRG **42**, 2010; Sorkin, Phys. Rev. **D81**, 2010
- Zilhão et al., Phys. Rev. **D 81**, 2010,
  Witek et al, Phys. Rev. **D82**, 2010.

Numerical Relativity in D Dimensions



- consider highly symmetric problems
- dimensional reduction by isometry on a (D-4)-sphere

general metric element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \lambda(x^\mu) d\Omega_{D-4}$$

Numerical Relativity in D Dimensions

D dimensional vacuum Einstein equations $G_{AB} = R_{AB} - \frac{1}{2} g_{AB} R = 0$ **imply**

$$
\begin{aligned}
{}^{(4)}T_{\mu\nu} &= \frac{D-4}{16\pi\lambda}\left[\nabla_\mu\partial_\nu\lambda - \frac{1}{2\lambda}\partial_\mu\lambda\partial_\nu\lambda - (D-5)g_{\mu\nu} + \frac{D-5}{4\lambda}g_{\mu\nu}\partial_\alpha\lambda\partial^\alpha\lambda\right], \\
\nabla^\mu\nabla_\mu\lambda &= 2(D-5) - \frac{D-6}{2\lambda}\partial^\mu\lambda\partial_\mu\lambda
\end{aligned}
$$

$\Rightarrow$ $4D$ Einstein equations coupled to scalar field

$\Rightarrow$ different higher dimensions manifest in scalar field

Formulation of EEs as Cauchy Problem in $D > 4$

- 3+1 split of spacetime ${}^{(4)}\mathcal{M} = \mathbb{R} + {}^{(3)}\Sigma$ (Arnowitt, Deser, Misner '62)
  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \left(-\alpha^2 + \beta_k \beta^k\right) dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j$
- 3+1 split of 4D Einstein equations with source terms
  $\Longrightarrow$ Formulation as initial value problem with constraints (York 1979)

Evolution Equations

$$\begin{aligned}
\partial_t \gamma_{ij} &= -2\alpha K_{ij} + \mathcal{L}_\beta \gamma_{ij} \\
\partial_t K_{ij} &= -D_i D_j \alpha + \alpha \left({}^{(3)}R_{ij} - 2K_{il}K^l_j + K K_{ij} \right) + \mathcal{L}_\beta K_{ij} \\
&\quad -\alpha \frac{D-4}{2\lambda} \left(D_i D_j \lambda - 2K_{ij} K_\lambda - \frac{1}{2\lambda} \partial_i \lambda \partial_j \lambda \right) \\
\partial_t \lambda &= -2\alpha K_\lambda + \mathcal{L}_\beta \lambda \\
\partial_t K_\lambda &= -\frac{1}{2} \partial^l \alpha \partial_l \lambda + \alpha \left((D-5) + K K_\lambda + \frac{D-6}{\lambda} K_\lambda^2 \right. \\
&\quad \left. - \frac{D-6}{4\lambda} \partial^l \lambda \partial_l \lambda - \frac{1}{2} D^l D_l \lambda \right) + \mathcal{L}_\beta K_\lambda
\end{aligned}$$

Wave Extraction in $D > 4$

Generalization of Regge-Wheeler-Zerilli formalism by Kodama & Ishibashi '03

Master function

$$\Phi_{,t} = (D-2) r^{(D-4)/2} \frac{2rF_{,t} - F^r_t}{k^2 - D + 2 + \frac{(D-2)(D-1)}{2} \frac{r_S^{D-3}}{r^{D-3}}}\,, \qquad k = l(l+D-3)$$

Energy flux & radiated energy

$$\frac{dE_l}{dt} = \frac{(D-3)k^2(k^2-D+2)}{32\pi(D-2)} (\Phi^l_{,t})^2\,, \qquad E = \sum_{l=2}^{\infty} \int_{-\infty}^{\infty} dt \frac{dE_l}{dt}$$

Momentum flux & recoil velocity

$$\frac{dP^i}{dt} = \int_{S_\infty} d\Omega \frac{d^2 E}{dt d\Omega} n^i\,, \qquad v_{recoil} = \left| \int_{-\infty}^{\infty} dt \frac{dP}{dt} \right|$$

Numerical Setup

- use Sperhake's extended LEAN code (Sperhake '07, Zilhão et al '10)
  - 3+1 Einstein equations with scalar field
  - Baumgarte-Shapiro-Shibata-Nakamura formulation with moving puncture approach
    dynamical variables: χ, $\tilde{\gamma}_{ij}$, K, $\tilde{A}_{ij}$, $\tilde{\Gamma}^i$, ζ, K_ζ
  - modified puncture gauge

$$\begin{aligned}\partial_t \alpha &= \beta^k \partial_k \alpha - 2\alpha(\eta_K K + \eta_{K_\zeta} K_\zeta) \\ \partial_t \beta^i &= \beta^k \partial_k \beta^i - \eta_\beta \beta^i + \eta_\Gamma \tilde{\Gamma}^i - \eta_\lambda \frac{D-4}{2\zeta} \tilde{\gamma}^{ij} \partial_j \zeta\end{aligned}$$

- Brill-Lindquist type initial data

$$\psi = 1 + r_{S,1}^{D-3}/4r_1^{D-3} + r_{S,2}^{D-3}/4r_2^{D-3}$$

- measure lengths in terms of r_S with

$$r_S^{D-3} = \frac{16\pi}{(D-2)A^{S^{D-2}}} M$$

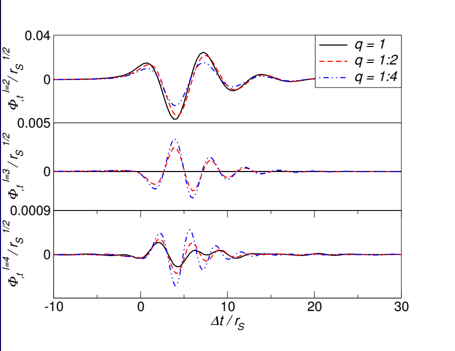
Unequal mass head-on

in $D = 5$ dimensions

Phys. Rev. **D 83**, 2011

Unequal mass head-on in $D = 5$

Modes of $\Phi_{,t}$

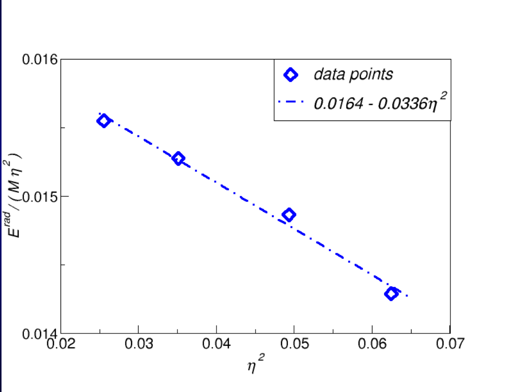


- consider mass ratios

$$q = r_{S,1}^{D-3} / r_{S,2}^{D-3} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$$

q	$E/M(\%)$	$E_{l=2}(\%)$	$E_{l=3}(\%)$	$E_{l=4}(\%)$
1/1	0.089	99.9	0.0	0.1
1/2	0.073	97.7	2.2	0.1
1/3	0.054	94.8	4.8	0.4
1/4	0.040	92.4	7.0	0.6

Unequal mass head-on in $D = 5$ - radiated energy



- $E/M \sim \eta^2$ (M.Lemos '10, MSc thesis, http://blackholes.ist.utl.pt/)
- fitting function

$$\frac{E}{M\eta^2} = 0.0164 - 0.0336\eta^2 \, ,$$

- within $< 1\%$ agreement with point particle calculation (Berti et al, 2010)

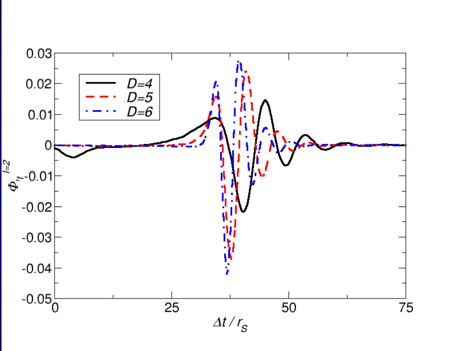
Equal mass head-on

in $D = 4, 5, 6$ dimensions

Phys. Rev. **D 82**, 104014 (2010)
work in progress

Equal mass head-on in $D = 6$ (preliminary results)

The good news is: We can do it !!!



- Key (technical) issues:
  - modification of gauge conditions
  - modification of formulation
- increase in E/M with D
  $\Rightarrow$ qualitative agreement with PP calculations
  (Berti et al, 2010)

D	$r_S\,\omega(l=2)$	$E/M(\%)$
4	$0.7473 - \mathrm{i}\,0.1779$	0.055
5	$0.9477 - \mathrm{i}\,0.2561$	0.089
6	$1.140 - \mathrm{i}\,0.304$	0.097 [1]

[1] preliminary

Head-on collisions

of boosted black holes

Zilhão et al (work in progress)

Initial data for boosted BHs in $D > 4$ (Zilhão et al)

- construct initial data by solving the constraints
- D-dimensional metric element

$$ds^2 = -\alpha^2 dt^2 + \bar{\gamma}_{ab}\left(dx^a + \beta^a dt\right)\left(dx^b + \beta^b dt\right)$$

- assumption: $\bar{\gamma}_{ab} = \psi^{\frac{4}{D-3}}\delta_{ab}\,,\ \bar{K} = 0\,,\ \bar{K}_{ab} = \psi^{-2}\hat{A}_{ab}$
- constraint equations

$$\partial_a \hat{A}^{ab} = 0\,, \quad \hat{\triangle}\psi + \frac{D-3}{4(D-2)}\psi^{-\frac{3D-5}{D-3}}\hat{A}^{ab}\hat{A}_{ab} = 0\,, \quad \text{with } \hat{\triangle} \equiv \partial_a\partial^a$$

- analytic ansatz for $\hat{A}_{ab}$ $\rightarrow$ generalization of Bowen-York type initial data
- elliptic equation for ψ $\rightarrow$ puncture method (Brandt & Brügmann '97)

$$\psi = 1 + \sum_{i=1}^{N}\frac{\mu_{(i)}}{4r_{(i)}^{D-3}} + u$$

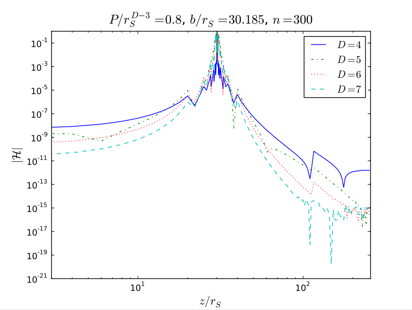
$\Rightarrow$ Hamiltonian constraint becomes

$$\hat{\triangle}u + \frac{D-3}{4(D-2)}\hat{A}^{ab}\hat{A}_{ab}\psi^{-\frac{3D-5}{D-3}} = 0$$

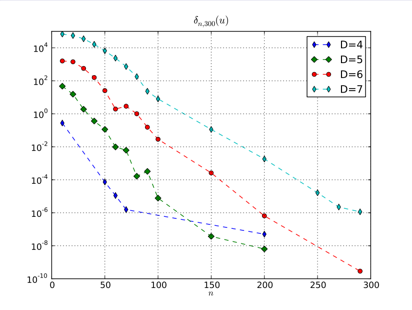
Initial data for boosted BHs in $D > 4$ - Results

- extension of the TwoPunctures pseudo-spectral solver (M. Ansorg et al, '04)
- initial data for punctures at $z/r_S = \pm 30.185$ with $P/r_S^{D-3} = 0.8$

Hamiltonian constraint



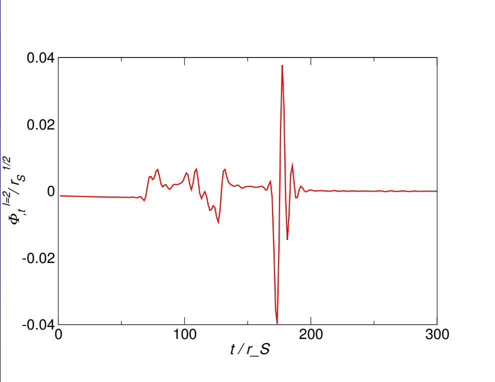
Convergence analysis



Head-on of boosted BHs in $D = 5$ (preliminary results)

- evolution of puncture with $z/r_S = \pm 30.185$ with $P/r_S^{D-3} = 0.4$

$l = 2$ mode of $\Phi_{,t}$



Conclusions and Outlook

- consider highly symmetric (black hole) spacetimes
- dimensional reduction by isometry
  $\Rightarrow$ formulation of D dimensional vacuum Einstein's equations as a scalar-tensor field theory in $D = 4$
- evolution of unequal mass head-on collisions in $D = 5$ with $q = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$
  $\Rightarrow$ extrapolation to PP limit shows good agreement with PP calculations
- evolution of equal mass head-ons in $D = 4, 5, 6$
- evolution of boosted BHs in $D = 5$
- ToDo:
  - numerical simulations of black hole collisions in $D \geq 7$
  - high energy collisions of BHs in $D \geq 5$
  - ...

Thank you!

http://blackholes.ist.utl.pt