Imprint of Alternative Theories

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alternative theories of gravity

- Up to now, there exists no experiment purported inconsistency of Einstein's theory.
- General relativity (GR) is definitely a beautiful theory of gravitation.
- However, we may have alternative approaches to explain all gravitational phenomena.
- We have also faced on some fundamental unknowns in the Universe such as dark energy and dark matter, which might be solved by new theory of gravitation.
- The candidates as *an alternative gravitational theory* should satisfy at least three criteria for viability; (1) *self-consistency*, (2) *completeness*, and (3) *agreement with past experiments*.

metric theory

- *Metric theories* should satisfy the following principles;
  - spacetime possesses a metric
  - metric satisfies the equivalence principle
- Metric theories of gravity differ from each other in their laws for the generation of the metric.
  - In *GR*, the metric is generated directly by the stress-energy of matter and of nongravitational fields.
  - In *Dicke-Brans-Jrdan thoery*, matter and nongravitational fields generate a scalar field ϕ; then ϕ acts together with the matter and other fields to generate the metric, while "long-range field" ϕ CANNOT act back directly on matter.

(1) Despite the possible existence of long-range gravitational field in addition to the metric in various metric theories of gravity, *the postulates of those theories demand that matter and non-gravitational fields be completely oblivious to them*.

(2) The only gravitational field that enters the equations of motion is the metric.

→ Thus, the metric and equations of motion for matter become the primary entities for calculating observable effects.

post-Newtonian limit

- The comparison of metric theories of gravity with each other and with experiments becomes particular simple, when one takes the *slow-motion* & *weak-field limit*. This approximation, known as the "*post-Newtonian limit*", is sufficiently accurate to encompass most solar-system tests.
  - In fact, the solar-system has weak gravity, the matter that generates solar-system gravity moves slowly, and has small internal energy, such as $|U| < 10^{-6}$, $v^2 < 10^{-7}$, $\Pi < 10^{-6}$.
- Each metric theory has its own post-Newtonian approximation.
- Despite the great differences between metric theories themselves, *their post-Newtonian approximations are very similar*.
  - Metric predicted by nearly every metric theory of gravity has the same structure.
  - It can be written as an expansion about the Minkowski metric in terms of dimensionless gravitational potentials of varying degrees of smallness, which are constructed from the matter variables.

Parameterized Post-Newtonian (PPN) formalism

- "Order of smallness" is determined according to the rules;
  $U \sim v^2 \sim \Pi \sim p/\varrho \sim O(\varepsilon)$, $v^i \sim |d/dt| \sim |d/dx| \sim O(\varepsilon^{1/2})$, and so on.
- A consistent post-Newtonian limit requires determination of
  $g_{00} \sim O(\varepsilon^2)$, $g_{0i} \sim O(\varepsilon^{3/2})$, and $g_{ij} \sim O(\varepsilon)$.
- The only way that one metric theory differs from another is in the numerical values of the coefficients that appear in front of the metric potential.
- The PPN inserts parameters depending on the theory in place of these coefficients. In order to indicate general properties of metric theories of gravity, *one need TEN parameters*.
- By using these parameters, one can identify the metric theories of gravity.
  - One set of values makes the PPN formalism identical to the PN limit of GR.
  - Another set of values makes it the PN limit of Dicke-Brans-Jordan theory, etc…

PPN parameters

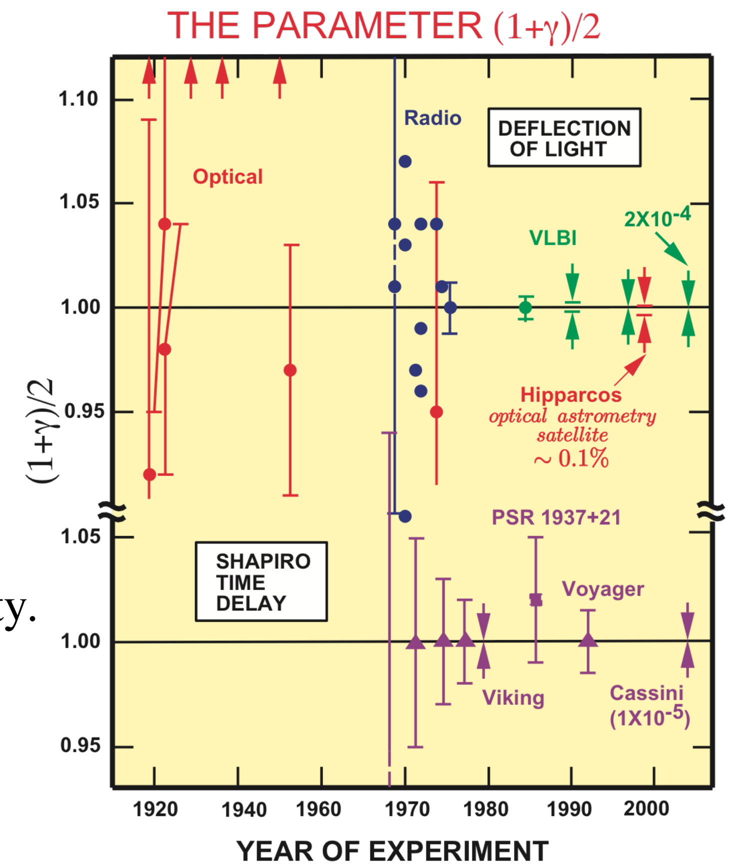
parameter	What it measures relative to GR
γ	How much space-curvature produced by unit rest mass ?
β	How much "nonlinearity" in the superposition law for gravity ?
ξ	Preferred-location effects ?
$\alpha_1, \alpha_2, \alpha_3$	Preferred-frame effects ?
$\alpha_3, \varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4$	Violation of conservation of total momentum ?

- The parameters γ and β are used to describe the "classical" tests of GR, and in some sense the most important.
- The parameter ξ is non-zero in any theory of gravity that predicts preferred location effects such as a galaxy-induced anisotropy in the local gravitational constant.

- In GR, (γ, β)=(1, 1) and the other parameters are zero.
- In scalar-tensor, the only non-zero parameters are γ and β.
- Fully conservative theories have three PPN parameters (γ, β, ξ).

tests of the parameter γ

- **Deflection of light**
  - 1995 VLBI: $(1+\gamma)/2 \sim 0.9996 \pm 0.0017$.
  - 2004 VLBI: $(1+\gamma)/2 \sim 0.99992 \pm 0.00023$.
  - → equivalently $\gamma-1=(-1.7 \pm 4.5) \times 10^{-4}$.
- **Time delay of light**
  - 1976 Viking: in a 0.1% measurement.
  - 2003 Cassini: $\gamma-1=(2.1 \pm 2.3) \times 10^{-5}$.
  - → $(1+\gamma)/2$ must be within at most 0.0012% of unity.
  - → Scalar-tensor theories must have $\omega > 40000$ to be compatible with this constraint.



C.M.Will (2006)

test of the parameter β

- **perihelion shift of Mercury**
  - $d\omega/dt = 42.''98\ [(2+2\gamma-\beta)/3 + 3 \times 10^{-4} \times J_2/10^{-7}]$ (arcsec./centuly)
  - $J_2 = (2.2 \pm 0.1) \times 10^{-7}$: determined from the data of heliosesmology

  → one obtains the PPN bound $|2\gamma-\beta-1| < 3\times10^{-3}$.

current limits on the PPN parameters

parameter	effect	limit	remarks
γ-1	Time delay	2.3×10^{-5}	Cassini tracking
	light deflection	4×10^{-4}	VLBI
β-1	perihelion shift	3×10^{-3}	$J_2=10^{-7}$ from helioseismology
	Nordtvedt effect	2.3×10^{-4}	$\eta_N=4\beta-\gamma-3$ assumed
ξ	Earth tides	10^{-3}	gravimeter data
α_1	orbital polarization	10^{-4}	Lunar laser ranging
		2×10^{-4}	PSR J2317+1439
α_2	spin precession	4×10^{-7}	solar alignment with ecliptic
α_3	pulsar acceleration	4×10^{-20}	pulsar $\mathrm{d}P/\mathrm{d}t$ statistics
η_N	Nordtvedt effect	9×10^{-4}	Lunar laser ranging
ς_1	---	2×10^{-2}	combined PNN bounds
ς_2	binary acceleration	4×10^{-5}	$\mathrm{d}^2P/\mathrm{d}t^2$ for PSR 1913+16
ς_3	Newton's 3rd law	10^{-8}	lunar acceleration
ς_4	---	6×10^{-3}	$6\varsigma_4 = 3\alpha_3+2\varsigma_1-3\varsigma_3$ assumed

In the strong gravitational field ??

- In the weak-field regime of gravity, many experiments say the validity of GR.
- While, the gravitational theory is *still unconstrained* in the strong-field regime of gravity.
- However, owing to the development of the technology, it is becoming possible to observe the compact objects with high accuracy.
- These observations could be used to make a constraint in the gravitational theory.
  - red shift
  - quasi-periodic oscillations (QPOs)
  - gravitational waves (GWs)

scalar-tensor theory (ST) of gravity

- A natural alternative to GR is the scalar-tensor theory (ST), in which gravity is mediated by long-range scalar fields in addition to the usual tensor field present in Einstein's theory.
- ST can be obtained from the low energy limit of string theory or/and other gauge theories.
- The existence of scalar fields is crucial in explaining the accelerated expansion phases of the universe, e.g., inflation and quintessence.
- Still, it is not clear how the scalar fields couple to gravity.
- A basic assumption is that the scalar and gravitational fields are coupled to matter via an "effective metric" $g_{\mu\nu} = A^2(\phi) g^*_{\mu\nu}$.
  - *Brans-Dicke theory*; $A(\phi)=\exp(\alpha_0\phi)$, $\alpha_0^2=(2\omega+3)^{-1}$

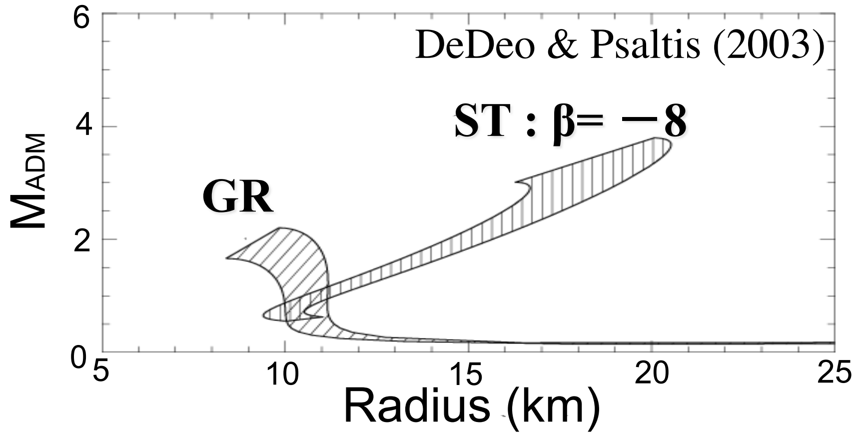
    → solar system experiments set strict limits as $\omega > 40000$, $\alpha_0^2 < 10^{-5}$.
  - *Damour & Esposito-Farese*; $A(\phi)=\exp(\alpha_0\phi + \beta\phi^2/2)$

    → Harada found that "*spontaneous scalarization*" is possible for $\beta<-4.35$.

basic equations in ST

- Metric; $g_{\mu\nu}=A(\phi)^2 g^*_{\mu\nu}$, where $A(\phi)=\exp(\beta\phi^2/2)$
- Field equations;
  - $G^*_{\mu\nu} = 8\pi G^* T^*_{\mu\nu} + 2(\phi_{,\mu}\phi_{,\nu} - g^*_{\mu\nu} g^{*\alpha\beta}\phi_{,\alpha}\phi_{,\beta}/2)$
  - $\Box^*\phi = -4\pi G^*\alpha(\phi) T^*$
- Perfect fluid;
  - $T_{\mu\nu} = (\varrho+P)U_\mu U_\nu + P g_{\mu\nu}$

- The stellar properties change dramatically!!

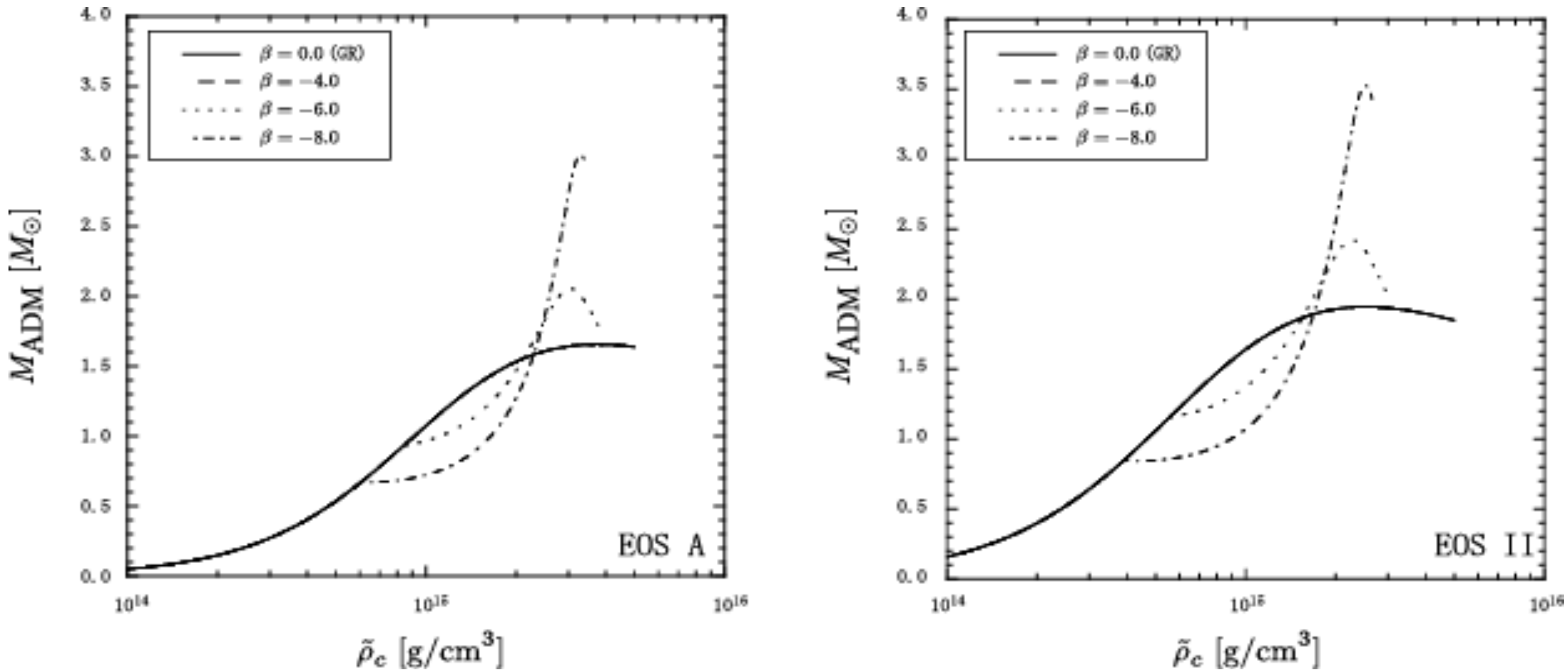


possibility to distinguish the ST from the GR

- DeDeo & Psaltis (2003) : redshift of emitted X & γ rays.
- DeDeo & Psaltis (2004) : QPOs
- HS & Kokkotas (2004) : GWs related to the fluid oscillations
- HS & Kokkotas (2005) : GWs related to the spacetime oscillations

stellar properties in ST

- Relation between the ADM mass and the central density, as varying the parameter β.



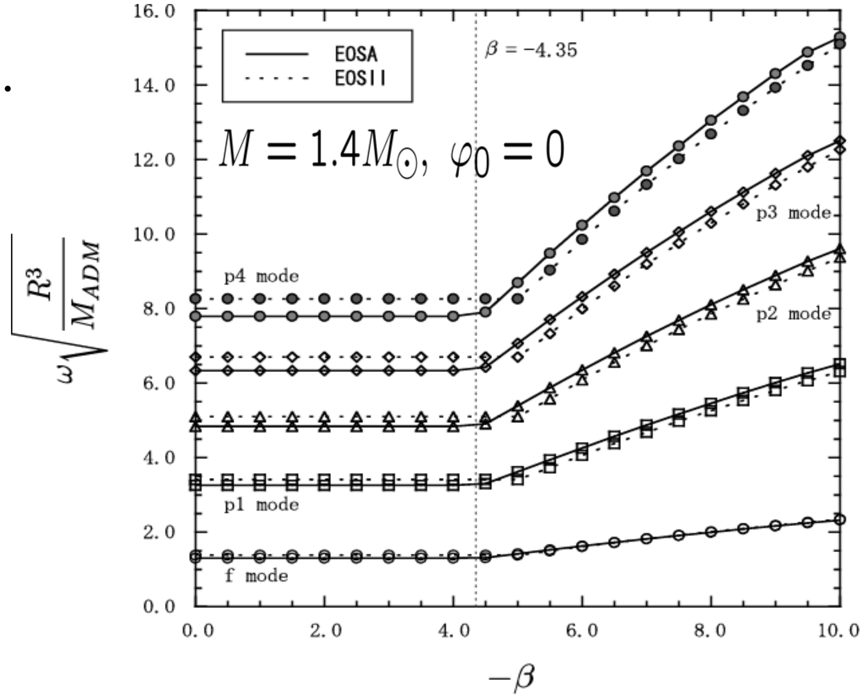
HS & Kokkotas (2004)

fluid oscillations in ST (1)

- Dependence of the frequencies on β.
- For β < -4.35, frequencies dramatically are changing. (spontaneous scalarization)
- Higher overtones depend strongly on β.
- For the n-th order mode,

$$\frac{\partial \omega_n}{\partial(-\beta)} \approx \frac{n}{4}$$

which is almost independent of EOS.

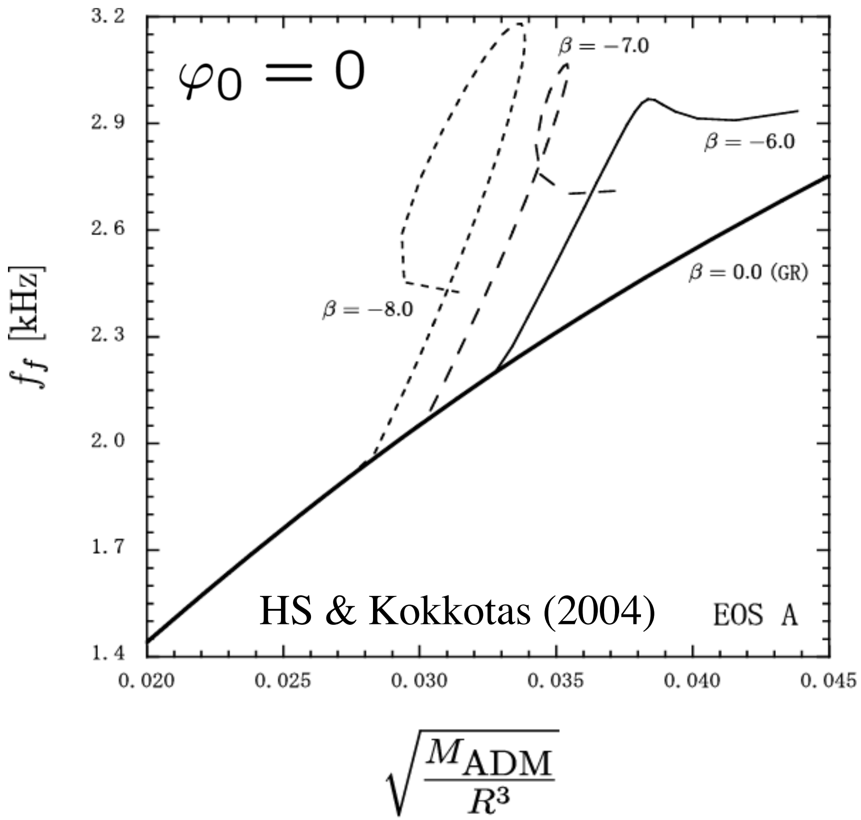


HS & Kokkotas (2004)

fluid oscillations in ST (2)

- Frequencies of *f* mode oscillations as a function of average density
  - In GR, frequencies can be expressed as a function of average density, which is *almost independent* of the EOS.
- In the case of the existence of ϕ, depending on the value of β, the frequencies become 30~50% larger.

→ Observations of GWs might reveal the existence of scalar field.



spacetime oscillations in ST (1)

- Metric perturbations; $g_{\mu\nu} = g^{(0)}{}_{\mu\nu} + h_{\mu\nu}$

$$\tilde{h}^{(-)}_{\mu\nu} = \sum_{l=2}^{\infty}\sum_{m=-l}^{l}\begin{pmatrix} 0 & 0 & -h_{0,lm}\sin^{-1}\theta\partial_\phi & h_{0,lm}\sin\theta\partial_\theta \\ 0 & 0 & -h_{1,lm}\sin^{-1}\theta\partial_\phi & h_{1,lm}\sin\theta\partial_\theta \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{pmatrix} Y_{lm},$$

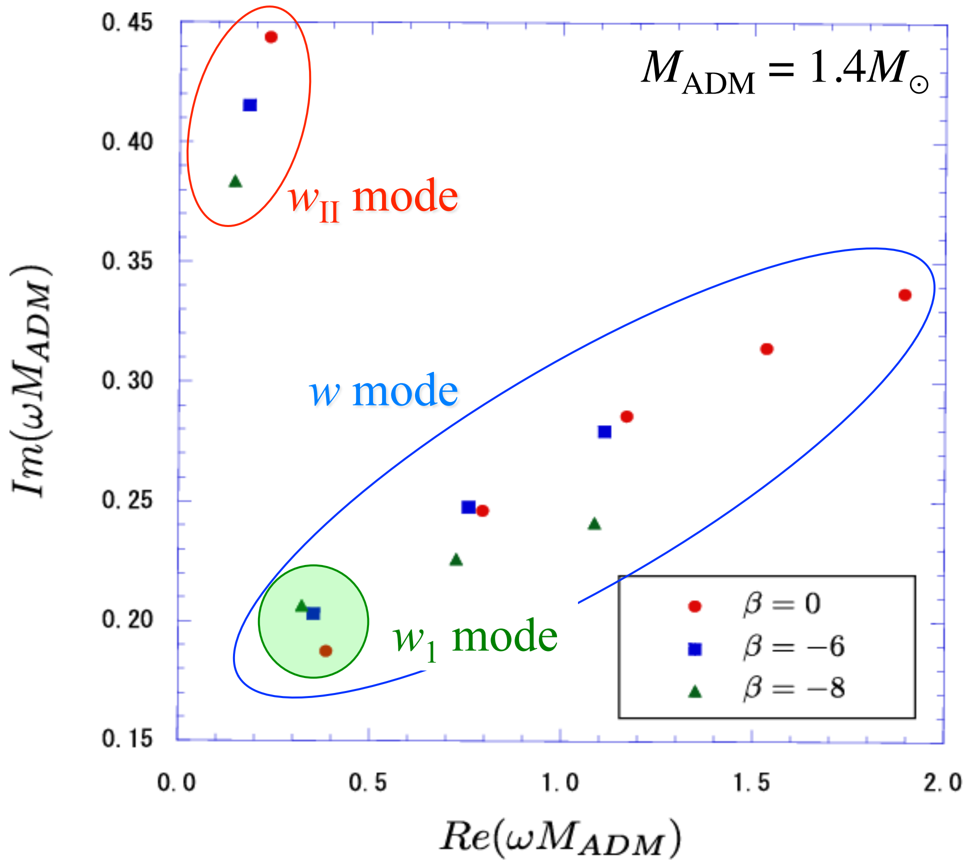
$$\tilde{h}^{(+)}_{\mu\nu} = \sum_{l=2}^{\infty}\sum_{m=-l}^{l}\begin{pmatrix} H_{0,lm}e^{2\Phi} & H_{1,lm} & 0 & 0 \\ * & H_{2,lm}e^{2\Lambda} & 0 & 0 \\ 0 & 0 & r^2K_{lm} & 0 \\ 0 & 0 & 0 & r^2K_{lm}\sin^2\theta \end{pmatrix} Y_{lm}$$

- Scalar field perturbation; $\phi = \phi^{(0)} + \delta\phi$
- perturbation of matter field; $P = P^{(0)} + \delta P\, Y_{lm}$, $\varrho = \varrho^{(0)} + \delta\varrho\, Y_{lm}$

$$\begin{aligned} \delta\tilde{U}^t &= \frac{1}{2A^3}e^{-\Phi}H_0Y_{lm}, \quad \delta\tilde{U}^r = \frac{1}{A}e^{\Phi-2\Lambda}WY_{lm}, \\ \delta\tilde{U}^\theta &= \frac{1}{Ar^2}e^{\Phi}\left(V\partial_\theta Y_{lm} - u\frac{1}{\sin\theta}\partial_\phi Y_{lm}\right), \quad \delta\tilde{U}^\phi = \frac{1}{Ar^2\sin^2\theta}e^{\Phi}\left(V\partial_\phi Y_{lm} + u\sin\theta\partial_\theta Y_{lm}\right). \end{aligned}$$

- *The metric perturbation in the Einstein frame is simplified and reduced to the "standard" Regge-Wheeler form of a perturbed spherical metric.*
- For axial perturbations, the equation does not coupled to $\delta\phi$.

spacetime oscillations in ST (2)



HS & Kokkotas (2005)

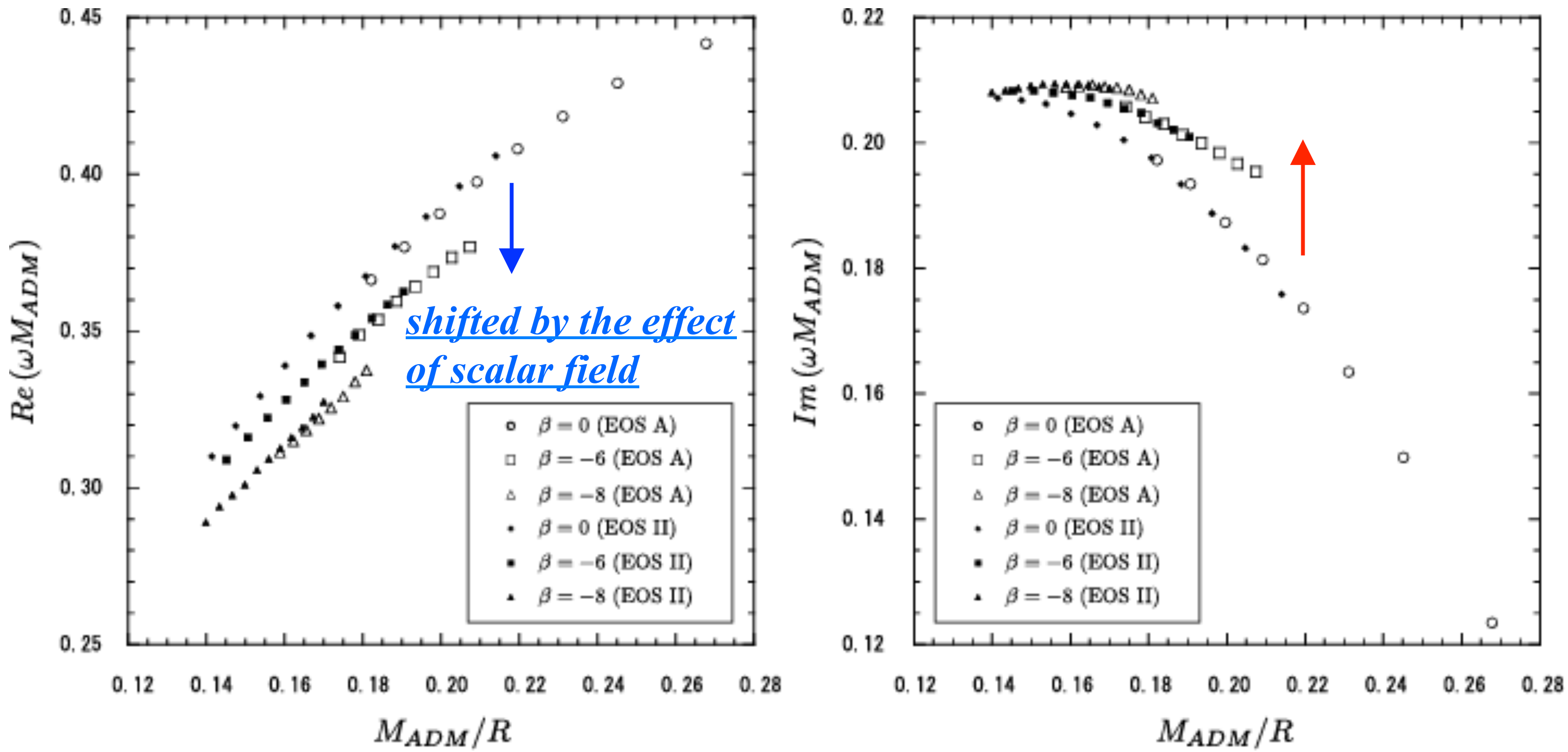
Axial GWs are coupled with only background scalar field.

The deviation of QNMs due to the scalar field arise for $\beta < -4.35$.

spacetime oscillations in ST (3)

- Dependence of w_1 mode on the stellar compactness

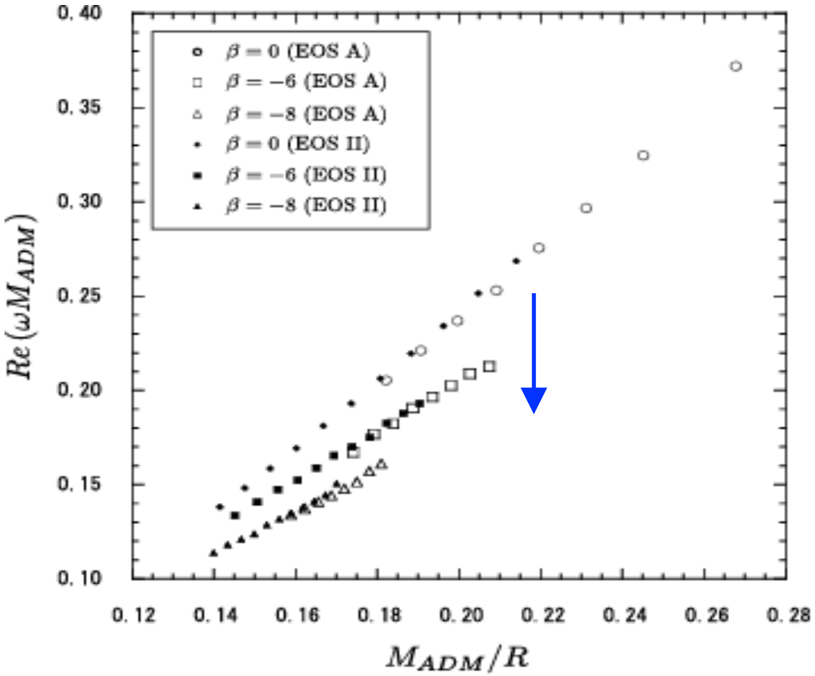
HS & Kokkotas (2005)

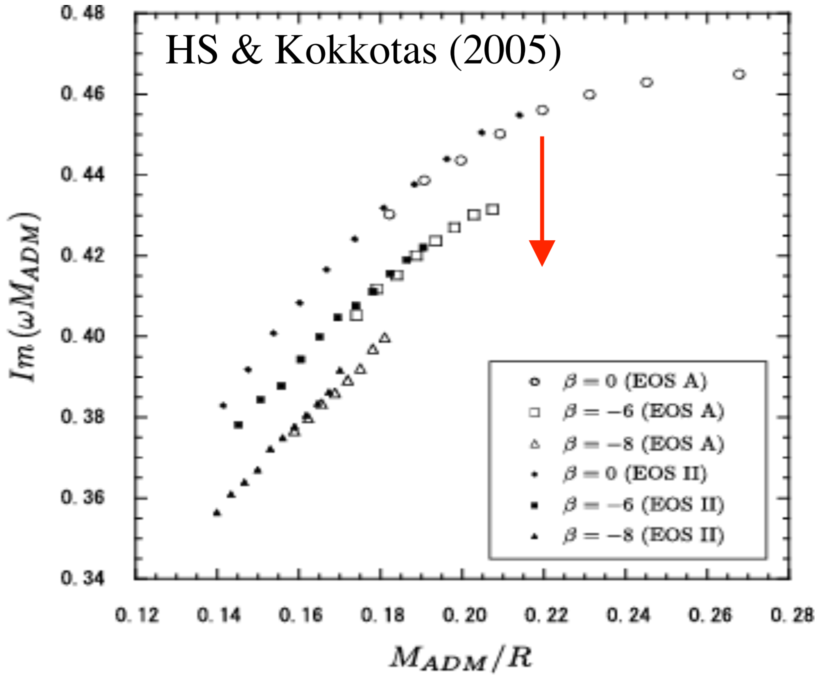


- *Independently of the EOS*, the frequency and damping rate depend on only β.

spacetime oscillations in ST (4)

- Dependence of (the lowest) w_{II} mode on the stellar compactness





- similarly to w_1 mode, *independently of the EOS*, the frequency and damping rate depend on only β**.**
- But, the dependence of damping rate is different.
- **the shift in GW spectrum could unambiguously signal the presence of a scalar field, if β < -4.35 in actuality.**

Tensor-Vector-Scalar theory (TeVeS) of gravity

- Modified Newtonian dynamics (MOND) was developed to describe the differences between the observed masses of galaxies and clusters of galaxies with the masses inferred from Newtonian dynamics.
- However, it is clear that MOND was never more than a toy model for gravity as it is not a covariant theory.
- A relativistic version has been put forward by Bekenstein (2004), where the standard Einstein tensor field of GR is coupled to a vector field as well as a scalar field.
- TeVeS has attracted considerable attention as an alternative gravitational theory.
  - It explains galaxy rotation curve and the Tully-Fisher law (roughly $L \propto v^4$ for spiral galaxy) without the existence of dark matter.
  - TeVeS has also successfully explained strong gravitational lensing, key features of the CMB, and galaxy distributions through an evolving Universe without CDM.

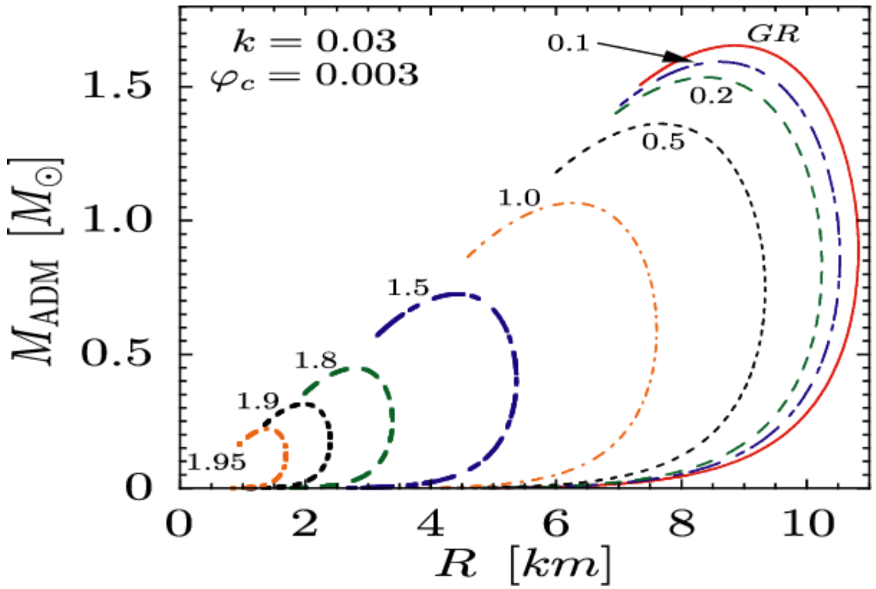
possibility to distinguish the TeVeS from the GR

- Laksy, HS, & Giannios (2008) : redshift of the atomic spectral lines radiated from the surface of NSs.
- Desai, Kahya, & Woodard (2008) : Shapiro delays of GWs and photons or ν.
- HS (2009a) : GWs associated with the fluid oscillations
- HS (2009b) : GWs associated with the spacetime oscillations
- HS (2010a) : rotational effects
- HS (2010b) : toroidal oscillations in NSs
- Lasky & Doneva (2011) : scalar perturbations of BH
- HS (2011) : torsional oscillations in NSs

stellar properties in TeVeS

- Mass-Radius relation

Laksy, HS, & Giannios (2008)

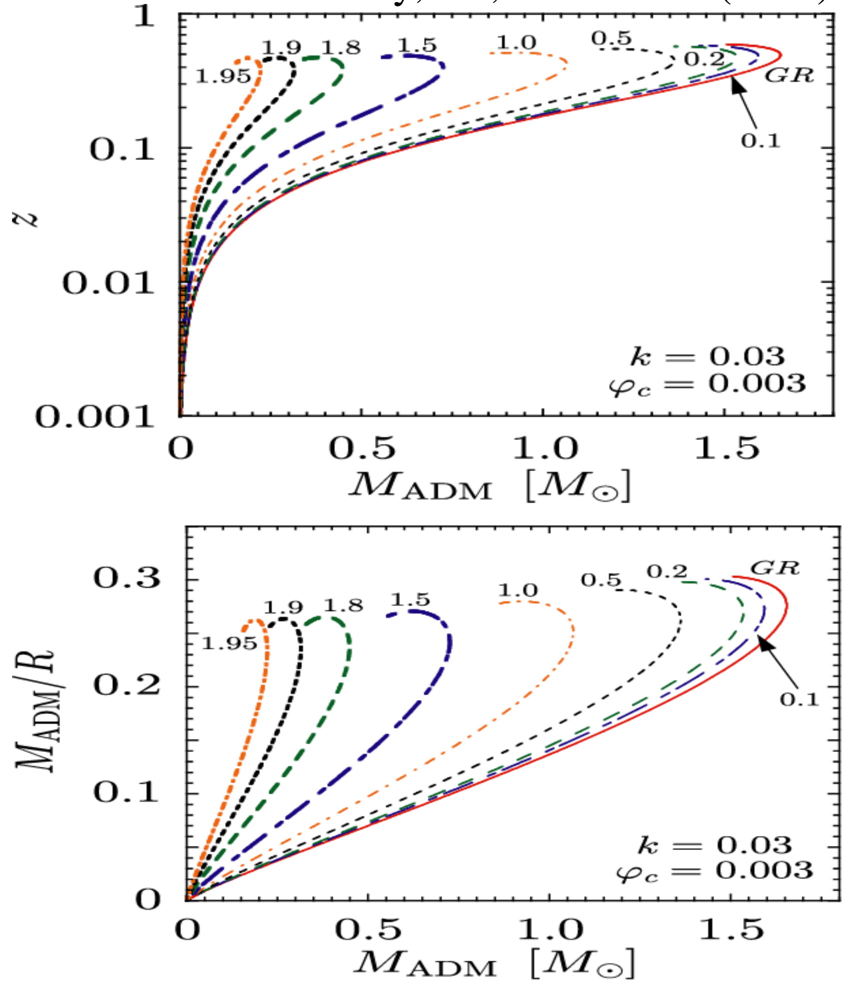


  – For small value of K, such as $K < 0.05$, the deviations from GR are small.
  – For larger value of K, the deviations from GR begin to become considerable.
  – The radius of NS in TeVeS is expected smaller than that in GR.

observation consequences

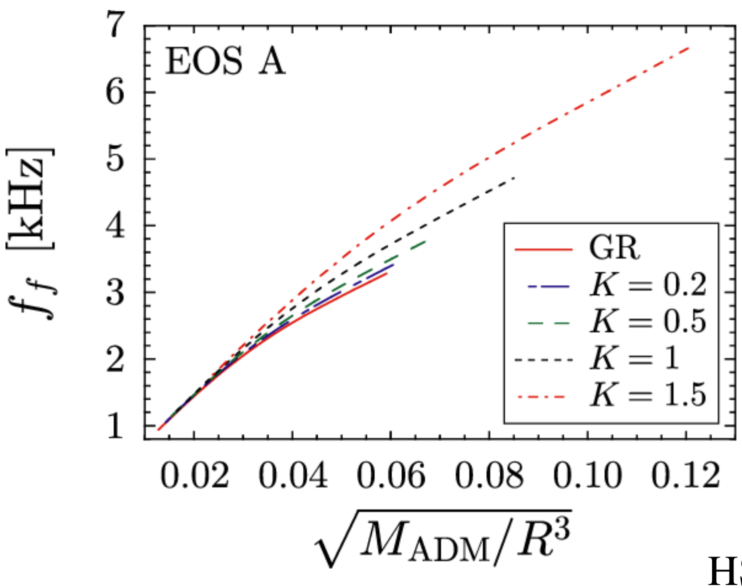
- Mass
- Radius
- Surface redshift factor
- Stellar compactness
- Stellar average density

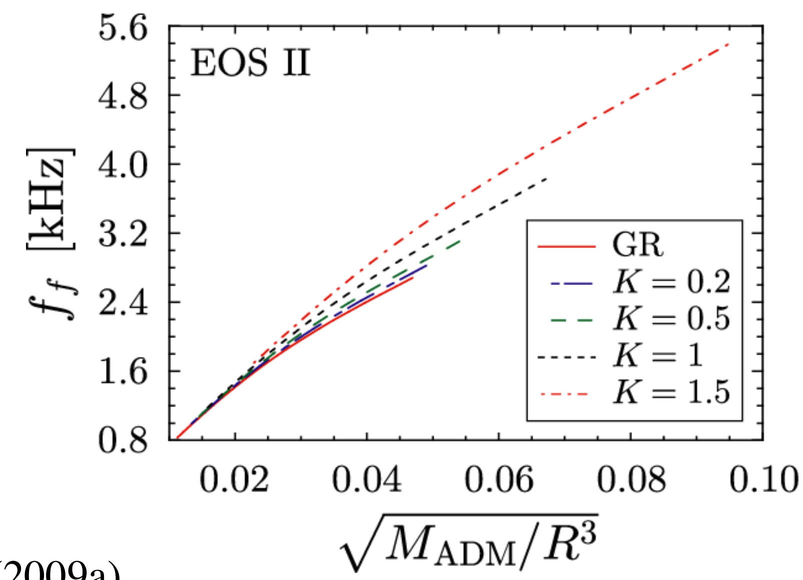
Laksy, HS, & Giannios (2008)



fluid oscillations (1)

- The deviation between the *f* mode frequencies expected in GR and in TeVeS is obviously.
  - Depending to K, the frequencies become around 20% larger than those in GR
  - This can be an observable effect and one might distinguish the gravitational theory in strong gravitational field by using the observations of GWs.



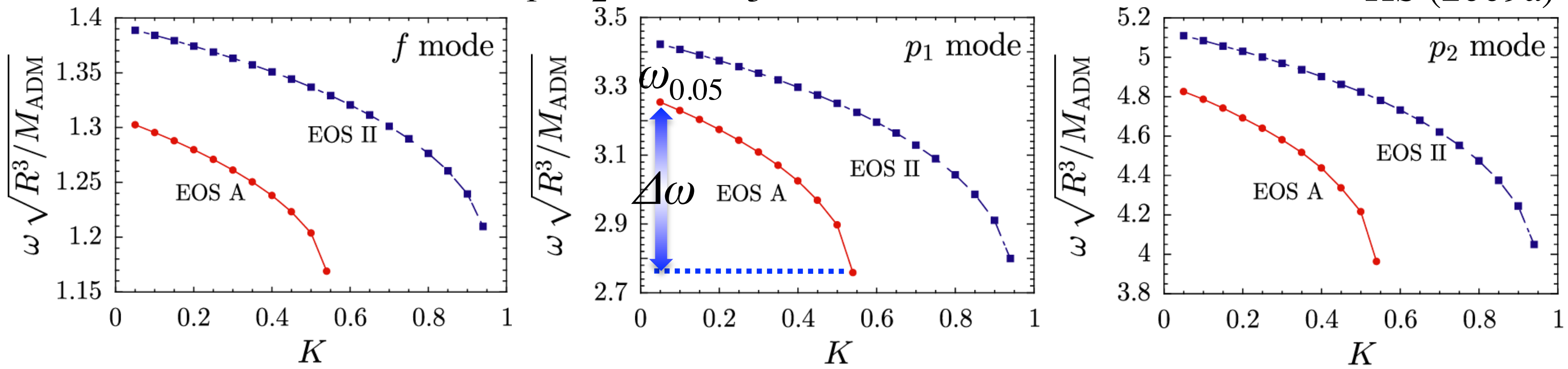


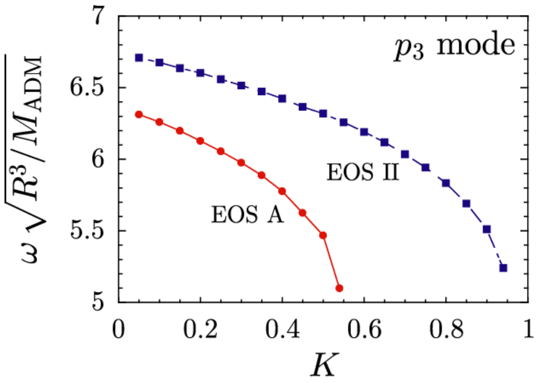
HS (2009a)

fluid oscillations (2)

- For the stellar models with $M_{\mathrm{ADM}} = 1.4M_{\odot}$, the normalized eigenvalues of the first few modes (f, p_1, p_2, and p_3).

HS (2009a)





Mode	EOS A	EOS II
f	10.28%	12.89%
p_1	15.21%	18.18%
p_2	17.88%	20.73%
p_3	19.23%	21.89%

relative frequency change of each eigenmode defined as $\Delta\omega/\omega_{0.05}$

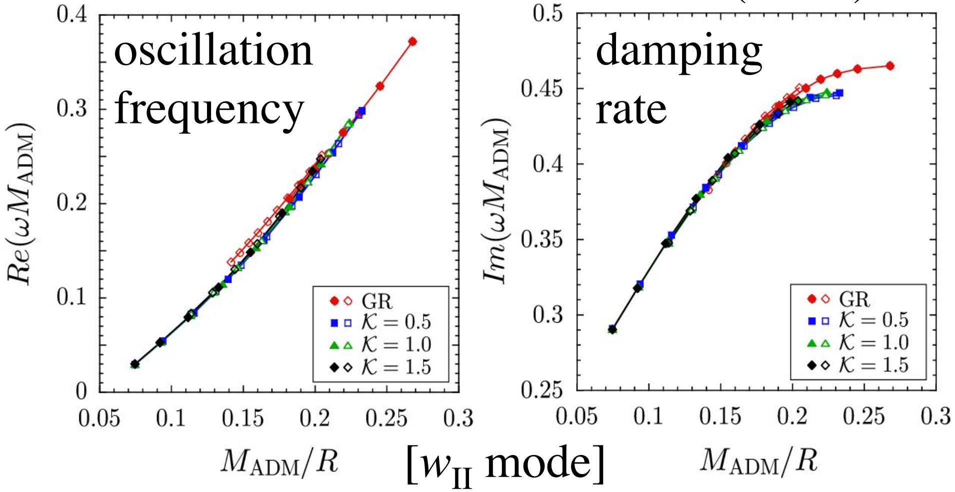
- We can find that with the help of observation of stellar mass, it might be possible to probe the gravitational theory in the strong-field regime by using observations of GWs.

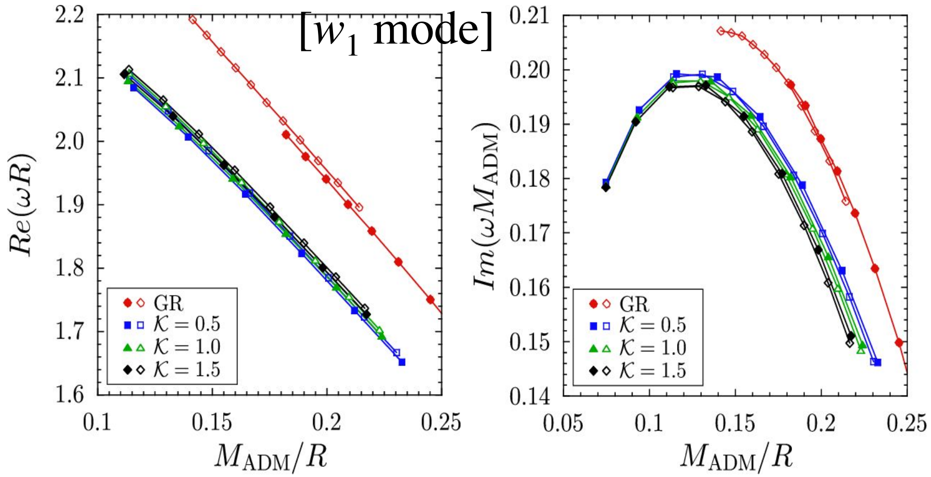
spacetime oscillations

- Dependence of w_{II} mode on M/R is almost *independent* of K and EOS.
- Frequencies of w_{II} mode is also almost independent of the adopted value K.
- We derive the empirical formula;

$$\mathrm{Re}(\omega M) = -0.041+0.71(M/R)+3.3(M/R)^2$$

HS (2009b)





- Frequency of w mode is also independent from K and EOS.
- However, those expected in TeVeS are *completely different* from those in GR.
- Via GW observations, one could distinguish the gravitational thoery in the strong-field regime.

$$\boxed{\mathrm{Re}(\omega M) = \alpha - \beta(M/R) \quad \begin{array}{l} (\alpha, \beta) = (2.80, 4.26) \text{ in GR} \\ (\alpha, \beta) = (2.53, 3.71) \text{ in TeVeS} \end{array}}$$

conclusion

- In the weak gravitational field, there are many experiments and the validity of GR has been shown, while the gravitational theory in the strong-field regime are still unconstrained by the observations.

- However, GWs observations could become the valuable tool to reveal the gravitational theory in the strong gravitational field.

- Via the projecting GW detectors, one can see the true gravitational theory and whether the scalar field exist or not…