Higher-dimensional Numerical Relativity

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0 Introduction

- Increasing roles in numerical relativity with development of observational and experimental technologies (Frans's talk)
- Numerical relativity plays a role in
  -- Gravitational-wave astrophysics/astronomy
  -- High-energy astrophysics; e.g., GRB
  -- Exploring nature of GR; e.g. critical behavior
  -- LHC, high-D gravity

This talk

1. Our current status & personal perspective in numerical relativity in
   GW physics & high-energy astrophysics
2. Higher-dimensional numerical relativity

Ingredients & Current status in 4D NR

- ○ 1. Einstein's evolution equations solver
- ○ 2. GR Hydrodynamic equations solver
- ○ 3. Gauge conditions (coordinate conditions)
- ○ 4. Realistic initial conditions
- ○ 5. Gravitational wave extraction techniques
- ○ 6. Apparent horizon (Event horizon) finder
- ○ 7. Special techniques for handling BHs
- △ 8. Physical modeling: ***EOS, neutrinos,*** B-field, **radiation transfer** ←------ **last frontier**
- ○ 9. AMR

Solving Boltzmann eq.
→ Another high-D NR

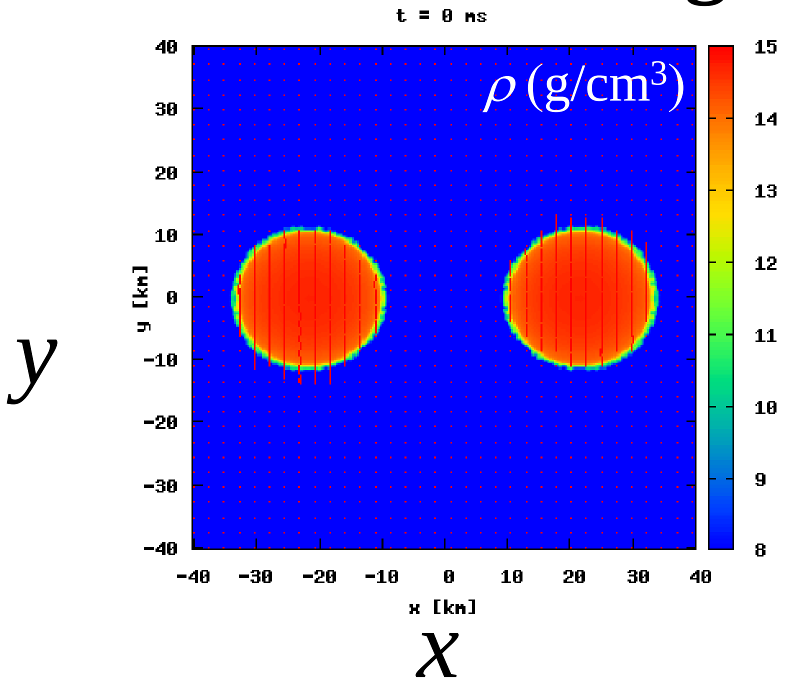
Our latest simulation

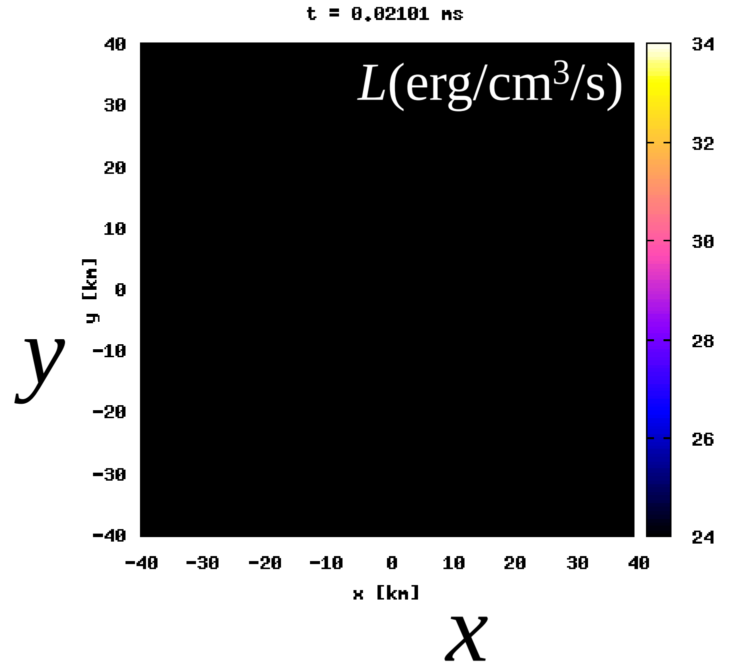
(Sekiguchi et al. 2011)

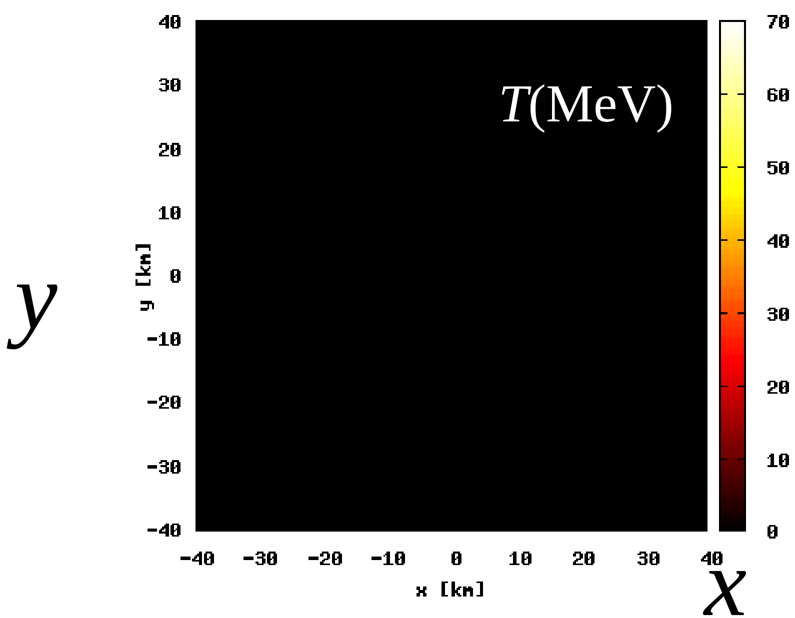
- Einstein's equation (BSSN)
- GR hydro (a shock capturing scheme)
- Physical EOS (finite-temperature EOS)
- Neutrino emission (simplified transfer)

- I will show a simulation for merger of binary neutron stars

NS-NS merger with microphysics







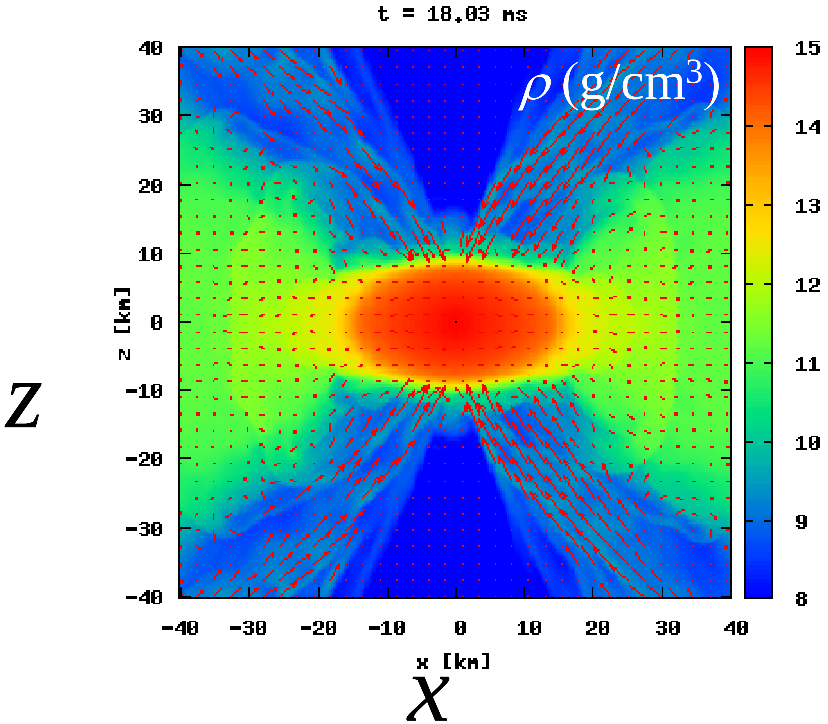
Shen's + hyperon EOS

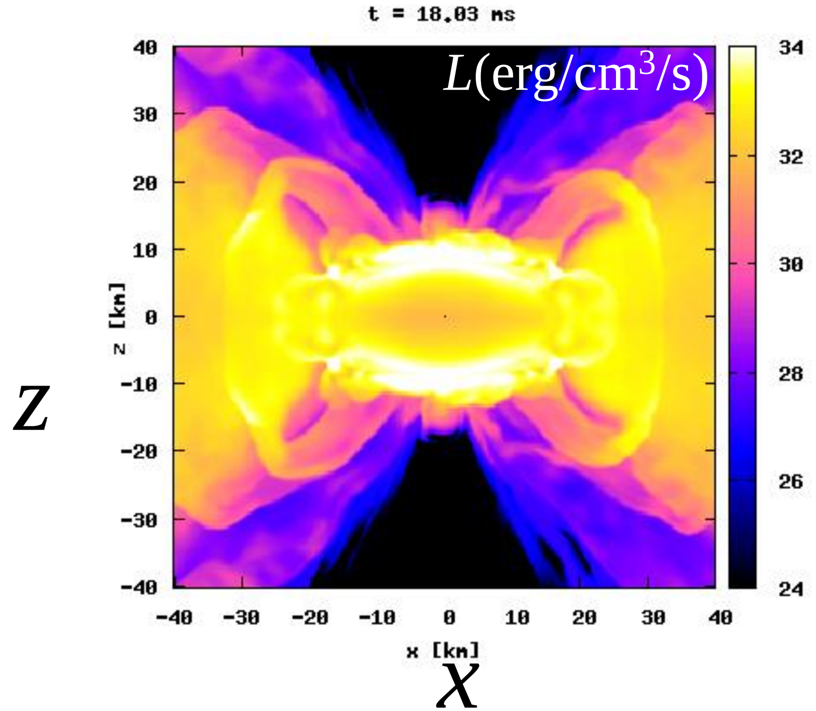
1.35—1.35 M_{sun}

Sekiguchi et al.

PRL & submission 2011

NS-NS merger with hyperon (x-z plane)

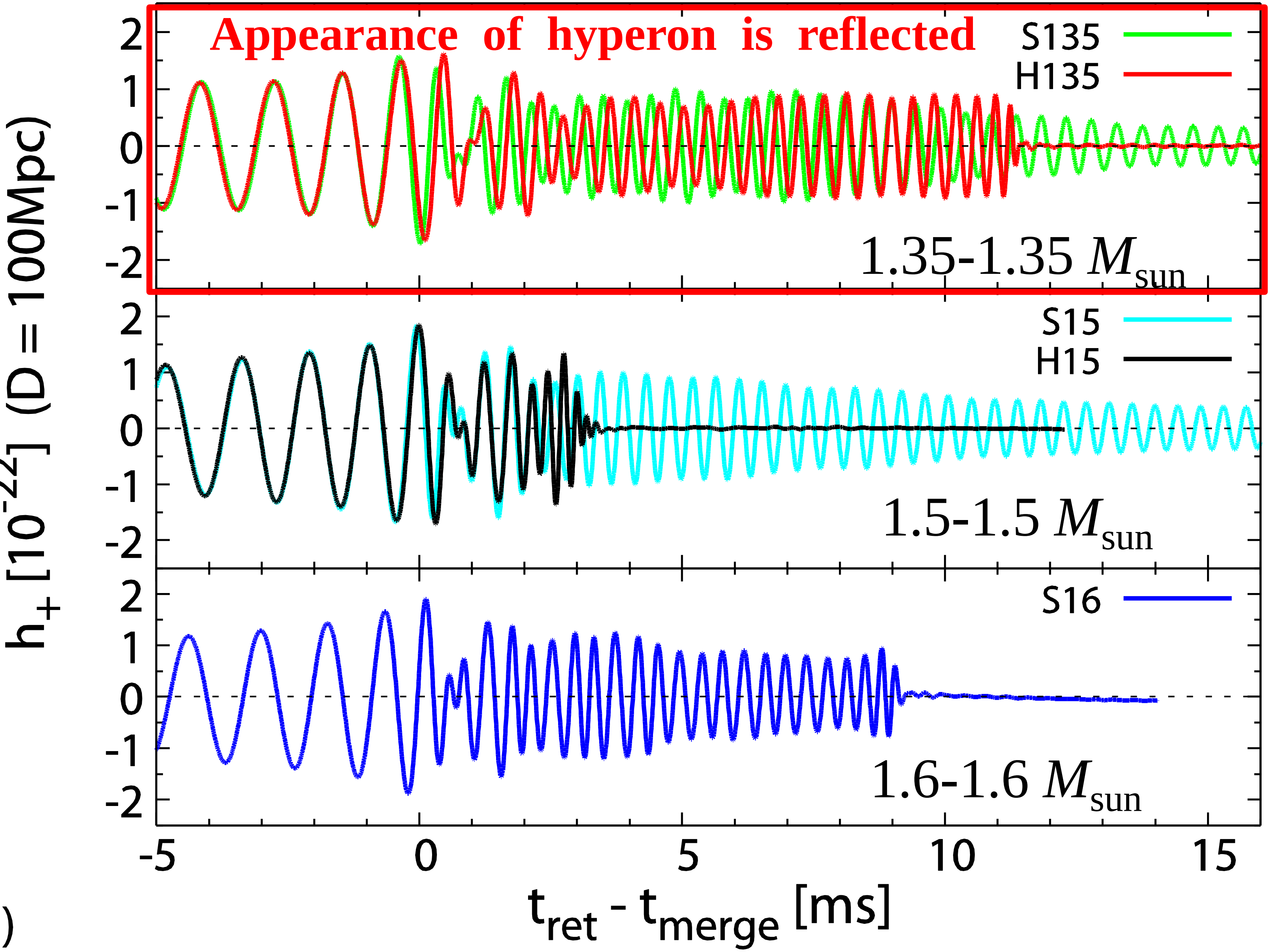




only during the BH formation is shown

- **Disk mass ~ 0.1 M_{sun}**
- **High mass & high luminosity disk**
- **Could be the engine of GRB**

Gravitational waveforms



(a)

I Motivations for high-D NR

- Exploring high-velocity collision of two particles/two black holes
  (Talks by Pretorius, Witek, Okawa)
- Exploring the stability of black objects, such as a Myers-Perry black hole, black string, etc
- Developing a new field in numerical rela.: E.g., Numerical relativity in AdS, AdS/CFT (Pretorius)
- Others ?

Variety of motivations

Several codes have to be developed

1. BH collision, stability of MP BH:
   Higher-dimensional code for asymptotically flat (AF) spacetime
   → Easily extended from 4D code
2. Stability of black string
   → Need simply to change boundary condition
3. Asymptotically AdS
   → Need a substantial change

In the following, I will talk on our effort in 1 & 3

II High-D numerical relativity: our approach to AF spacetime

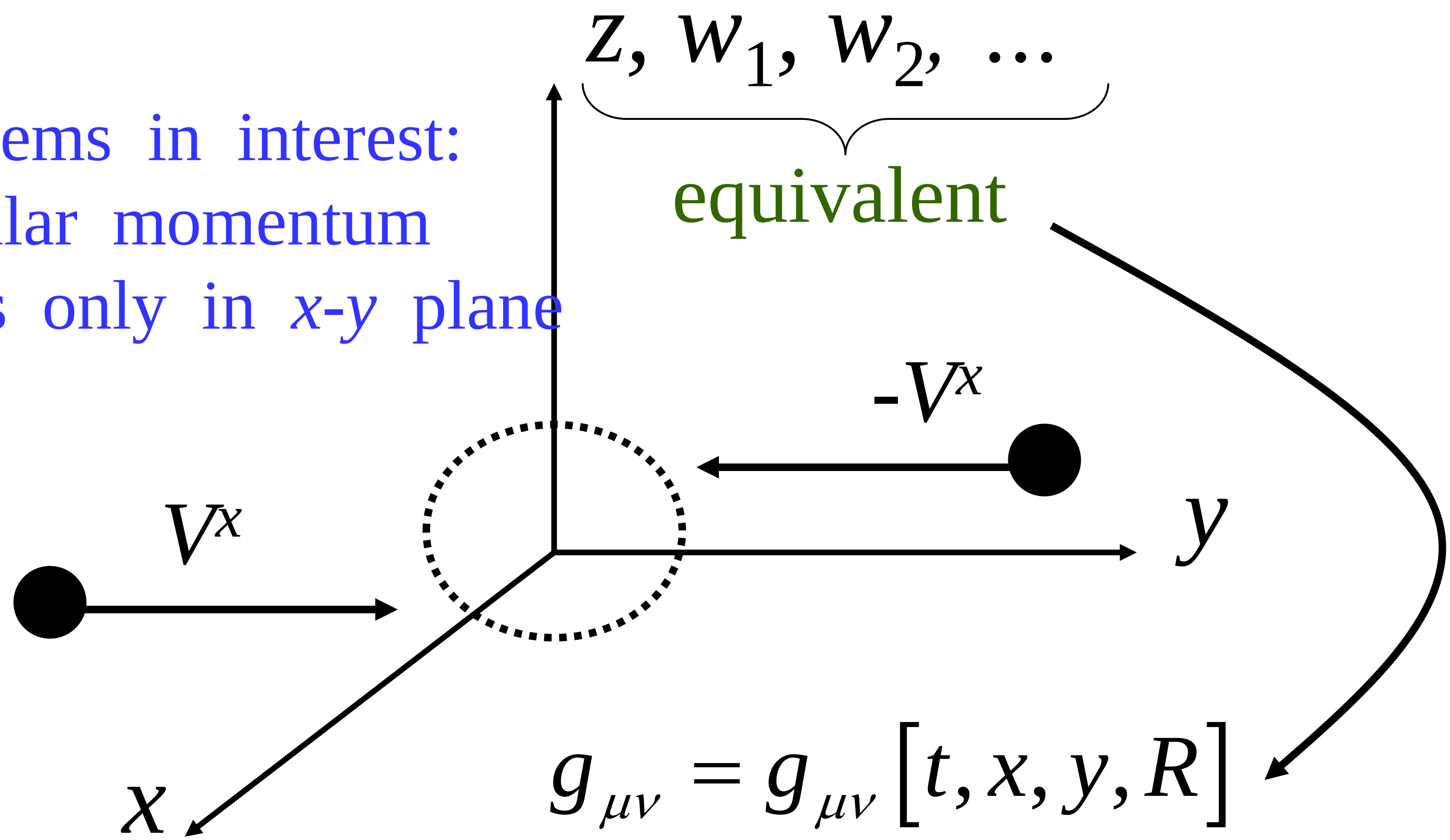
- Solve D-dim Einstein's equation $G_{\mu\nu}=0$ in (N+1) formalism (N=D–1)

$$ds^2 = -\left(\alpha^2 - \beta_k \beta^k\right) dt^2 + 2\beta_k dx^k dt + \chi^{-1} \tilde{\gamma}_{ij} dx^i dx^j$$

- Specifically, BSSN + puncture formalism is employed as in 3+1 case; works well
- Symmetry in the extra-dimensional directions is assumed; **SO(*D*–3) symmetry** → In computation, the number of dimension is "3" + 1 (time)

SO(D-3) Symmetry

Problems in interest:
Angular momentum
exists only in x-y plane



$$g_{\mu\nu} = g_{\mu\nu}\left[t, x, y, R\right]$$

$$R = \sqrt{z^2 + w_1^2 + w_2^2 + \cdots}$$

Cartoon method imposing symmetry

- Traditional method for symmetric space is to use curvilinear coordinates; e.g., 6D $z=R\cos\psi$, $w_1=R\sin\psi\cos\phi$, $w_2=R\sin\psi\sin\phi$
- In this method, coordinate singularities appear at $R=0$ and $\psi=0$
  $\rightarrow$ Special treatment is necessary & guaranteeing numerical stability is always messy problem in numerical relativity
- **Cartoon:** Solve equations in the Cartesian coordinates but only in the restricted space.

For simplicity, consider the 5D case with no rotation in subspace (*z*, *w*)

- Consider only (x, y, z) $(w=0)$ plane
- Symmetries give $\chi_{,w}=0$ and $\chi_{,ww}=\chi_{,z}/z$
- Vectors $\beta^x{}_{,w}=\beta^y{}_{,w}=0$, $\beta^z{}_{,w}=0$, $\beta^w{}_{,w}=\beta^z/z$
- Tensors $g_{ij,w}=0$ $(i, j = x, y, z)$,

  $g_{Aw,w}=g_{Az}/z$ $(A=x, y, w)$, $g_{zw,w}=(g_{zz}-g_{ww})/z$, etc.
- *Exception:* $z=0$ → Use finite difference

Every w derivatives can be replaced to (x, y, z) derivatives or simple relations using the symmetry relation !

→ **3D spatial grid is enough**

Second derivatives are also easily done

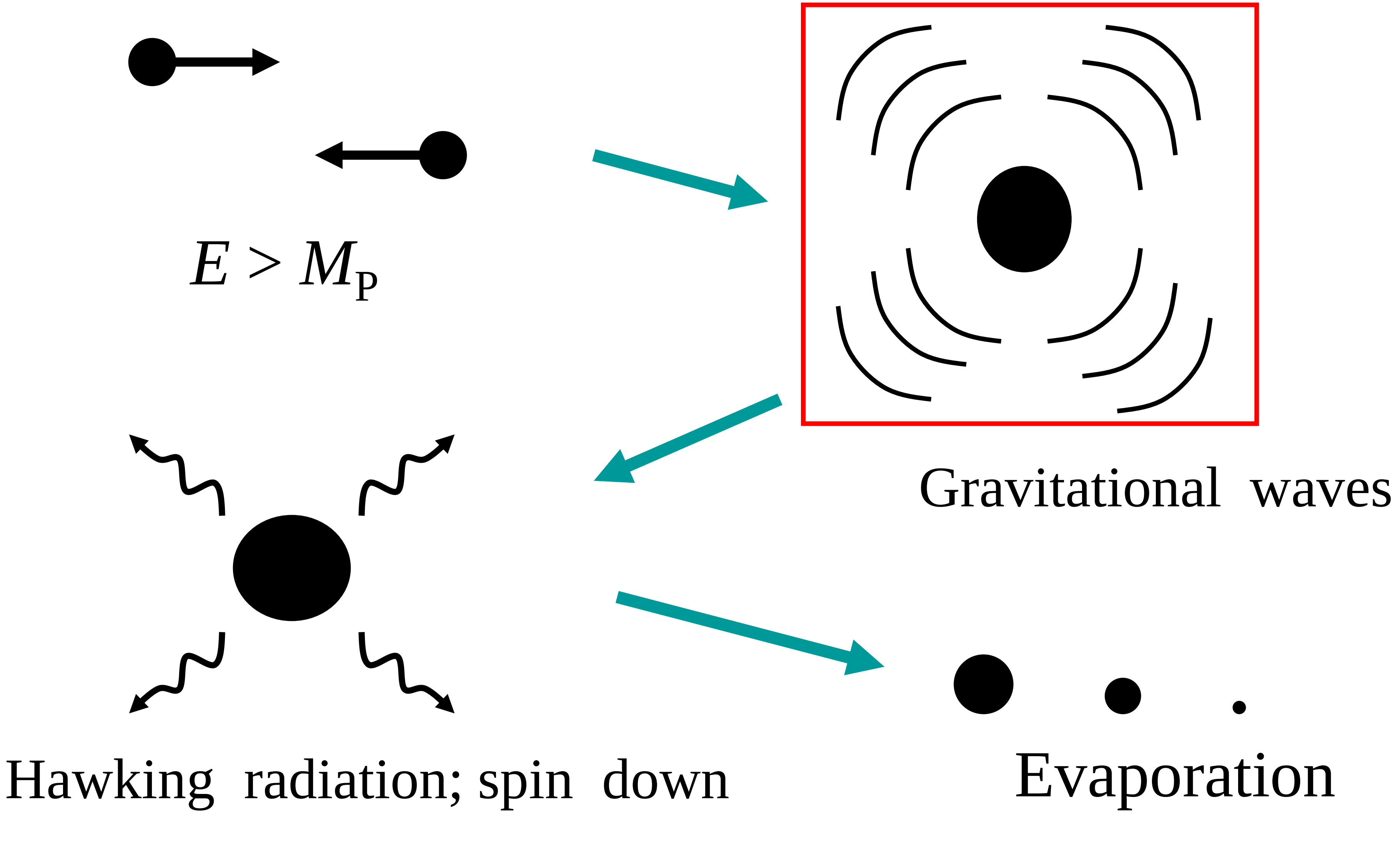
- Scalar $\chi_{,wz}=0$ (one $,w$ is always zero)
- Vectors $\beta^{w}{}_{,wk}$ is $(\beta^{z}/z)_{,k}$
- Tensors $\gamma_{ij,wk}=0$ $(i, j, k = x, y, z)$,
  $\gamma_{iw,wj}=(\gamma_{iz}/z)_{,j}$ is finite difference of γ_{Az}/z
- $\beta^{i}{}_{,ww}$, $\gamma_{ij,ww}$ are a little complicated to do, but straightforward

- For higher-dimensions, extension is easy: *e.g.*, $\beta^{i}{}_{,ww} \rightarrow (D\text{-}4)\,\beta^{i}{}_{,ww}$

Applications so far

- High-velocity collision of two BHs
  → Okawa's talk (5, 6D, *v* up to ~ 0.9c)
- Stability of MP BHs

Standard picture of particle collision
(Giddings, yesterday)

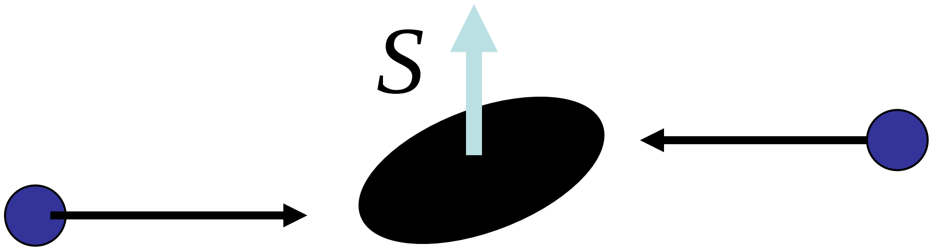


Is scenario really true ?

- If the formed BH is stable, it is OK
- For $D > 4$, no proof of stability for BH: likely, many instabilities (review later)
- **If the formed BH is unstable, it will not relax to a stable state soon → Different scenario could be the result**

III Stability of High-dim rotating black hole with single spin

- The formed BH in collision will have *one spin* parameter associated with orbital plane



→ MP BH with one spin

- Perturbation analysis for rotating BH is not easy

→ Robust method is **Numerical Relativity**

Setting

- Prepare Myers-Perry black hole (1986) with single rotation, and then perform simulations

$$ds^2 = -dt^2 + \frac{G_d \mu}{r^{D-5}\Sigma}\left(dt + a\sin^2\theta d\varphi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$
$$+\left(r^2 + a^2\right)\sin^2\theta d\varphi^2 + r^2\cos^2\theta d\Omega_{D-4}^2$$

$$\Sigma = r^2 + a^2\cos^2\theta; \quad \Delta = r^2 + a^2 - \frac{G_d \mu}{r^{D-5}}$$

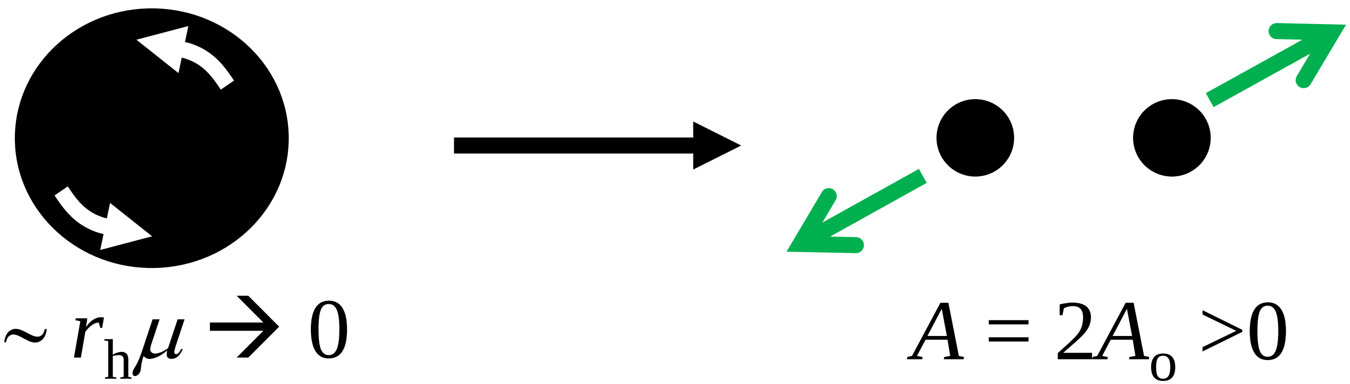
μ : mass parameter, a : spin parameter

$$\Rightarrow \quad M = \frac{(D-2)\Omega_{D-2}}{16\pi G_D}\mu, \quad J = \frac{2}{D-2}Ma$$

Length: $(G_D\mu)^{1/(D-3)}$, $\boxed{q = \frac{a}{(G_D\mu)^{1/(D-3)}} : \text{ nondim. spin}}$

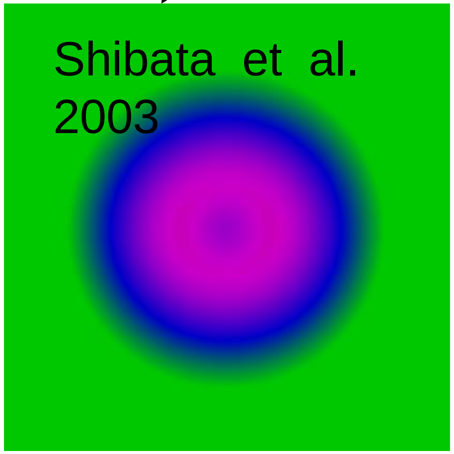
Brief review for previous studies

- **Axisymmetric instability sets in** for the ***ultra-spinning case***: $q=a/\mu^{1/(D-3)} > \sim 1.6$ *with* $D=6$—9
  (Dias et al. & Murata et al. 09)
- **Non-axisymmetric instability**:
  Emparan-Myers give a conjecture based on *Thermodynamical argument* (2003)
  → This suggests that **BHs are unstable for** $q=a/\mu^{1/(D-3)} > \sim 1$ **(smaller** q**)** **irrespective of** $D > 4$



Analogy: Rotating star in 4D

- Rapidly rotating stars are unstable against *nonaxisymmetric* deformation (many works done, e.g., by Eriguchi and collaborators since 1980)
- Often found criterion, $T/W \sim 0.27$;
  T = rotational kinetic energy
  W = gravitational potential energy;
  or of strongly differential rotation



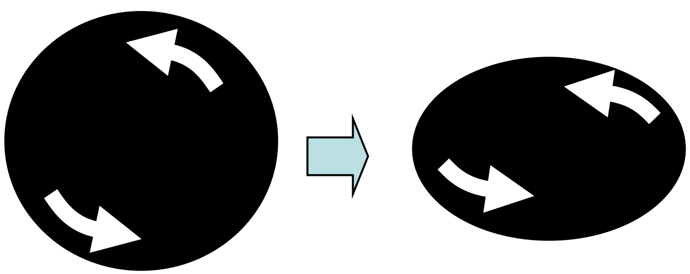
- ***not highly deformed; spheroid is unstable***
- By contrast, rotating stars (*like pancake*) only with $T/W > 0.4$ could be unstable against axisymmetric mode (ring formation)

Simulation, more specifically

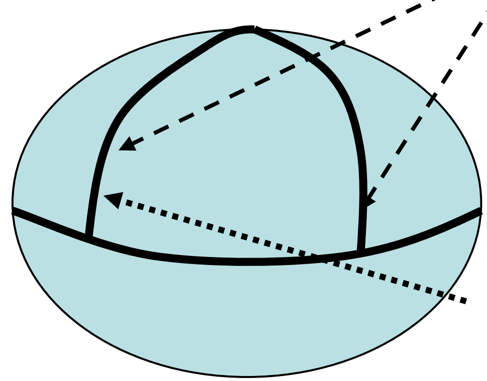
- Prepare a rotating Myers-Perry BH in the ***quasi-isotropic coordinates*** **(*good coordinates*)** and follow time evolution using puncture approach
- Initially, a small perturbation is given
- Method: 4th order finite difference in time and space & puncture-gauge with BSSN
- Fixed mesh refinement is used: High grid resolution is necessary for high spin case
- Perform simulations for various values of q

Method of analysis

- Analyze apparent horizon during simulation
- Calculate proper length of circumferential radius, and area
- Define deformation parameter



View from z-axis



Calculate the deformation, e.g.

$C_e(0)/\ C_e(\pi/2)$

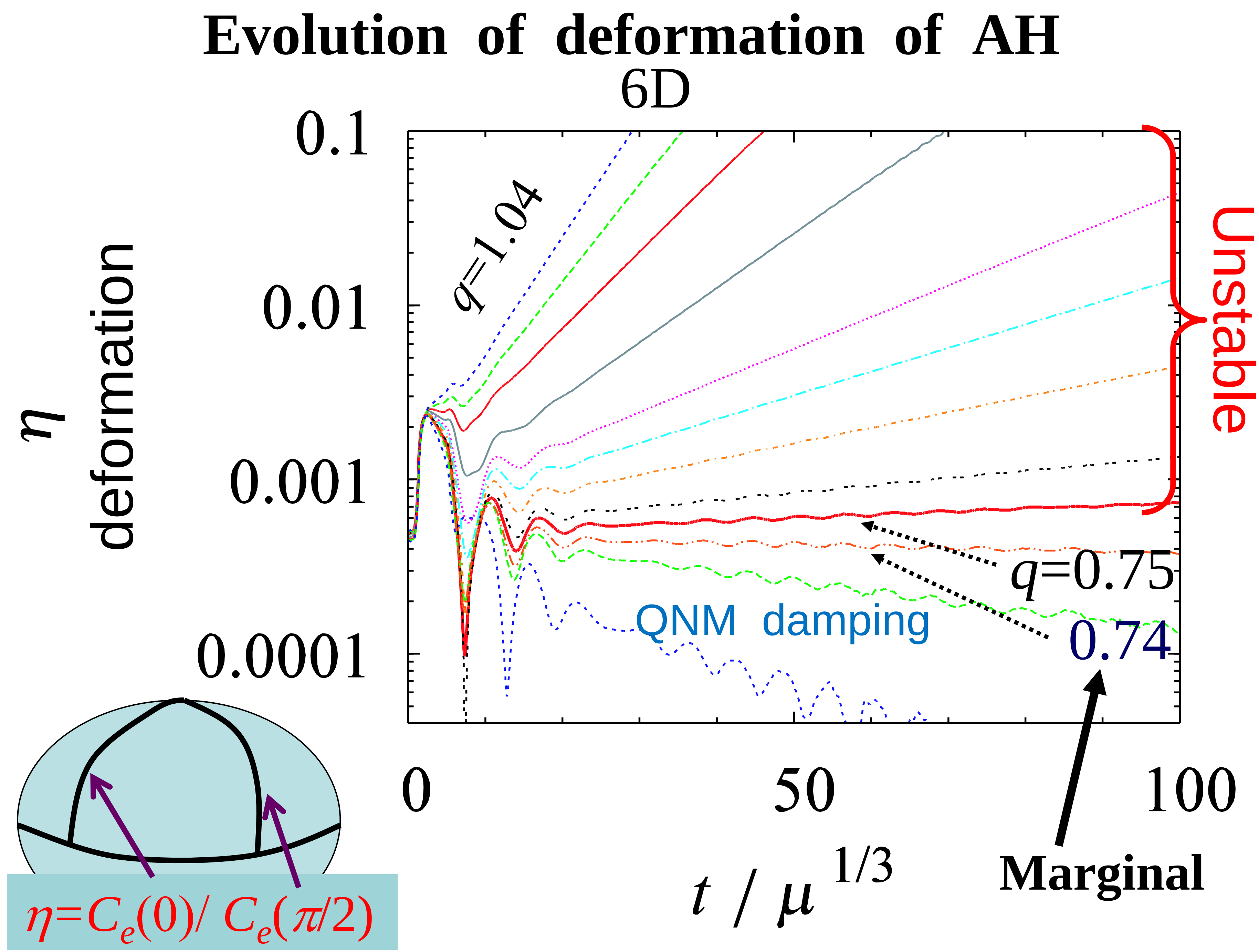
$C_e(\phi)$:
Meridian length

- Also, extract gravitational waves in the wave zone (along z axis)

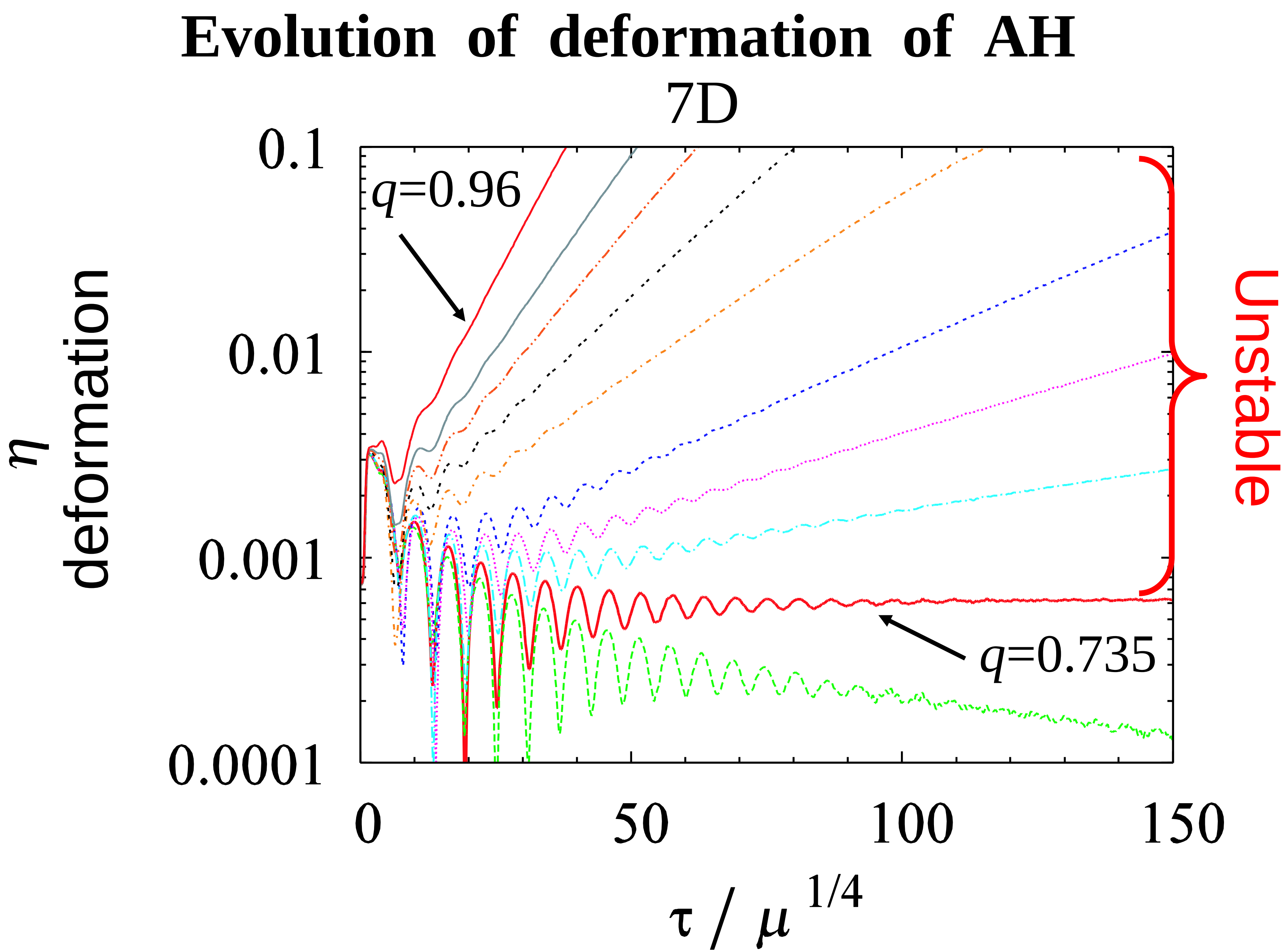
D-dim case ($D > 5$)

- Spin parameter $q = a/\mu^{1/(D-3)} = [0,\infty)$
- Ellipticity increases with q, but increases slowly with q
- BH with $q > \sim 1.6$ is unstable against axisymmetric deformation
  (Dias et al., Murata et al. 2009)
- Nonaxisymmetric instability sets in even for much smaller spin ! (for spheroidal BH)

(Shibata & Yoshino, PRD 81, 104035, 2010)



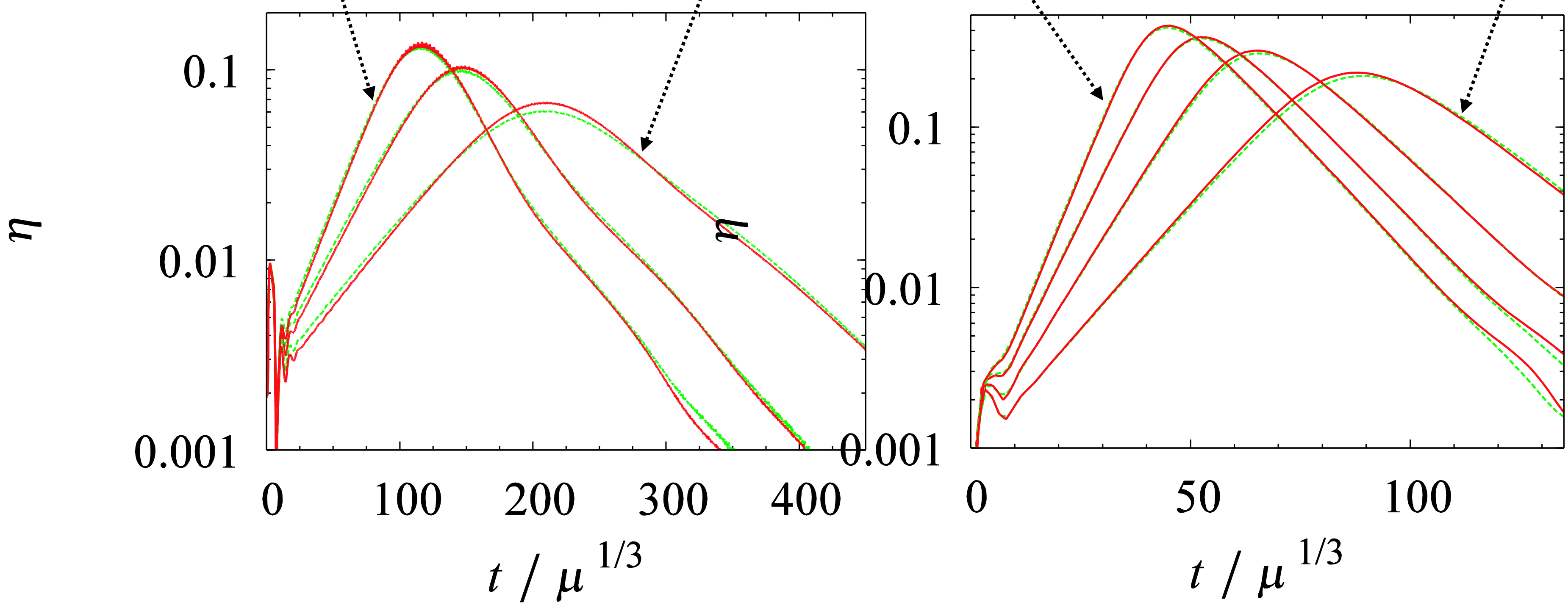
Evolution of deformation of AH
6D
0.1
0.01
0.001
0.0001
η
deformation
q=1.04
Unstable
q=0.75
0.74
QNM damping
Marginal
0
50
100
t / μ 1/3
η=Ce(0)/ Ce(π/2)



Evolution of deformation of AH
7D
0.1
0.01
0.001
0.0001
0
50
100
150
η deformation
τ / μ 1/4
q=0.96
q=0.735
Unstable

Longterm evolution of deformation of AH

D=6, q=0.82, 0.80, 0.78, 1.04, 0.99, 0.93, 0.88



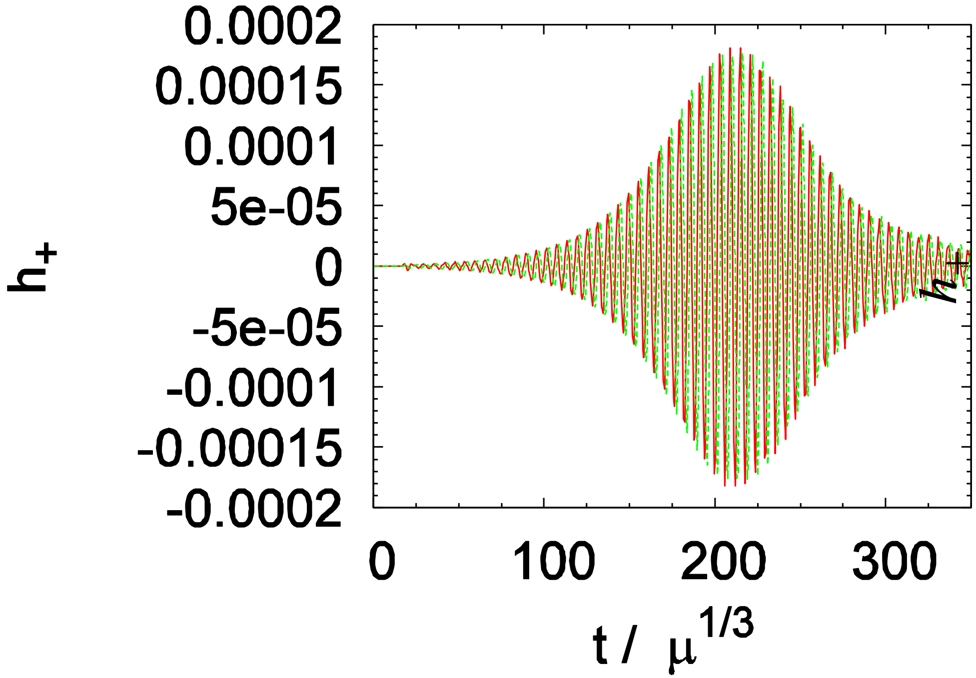
Growth → **Saturation by GW emission**

→ New stable BH of smaller spin

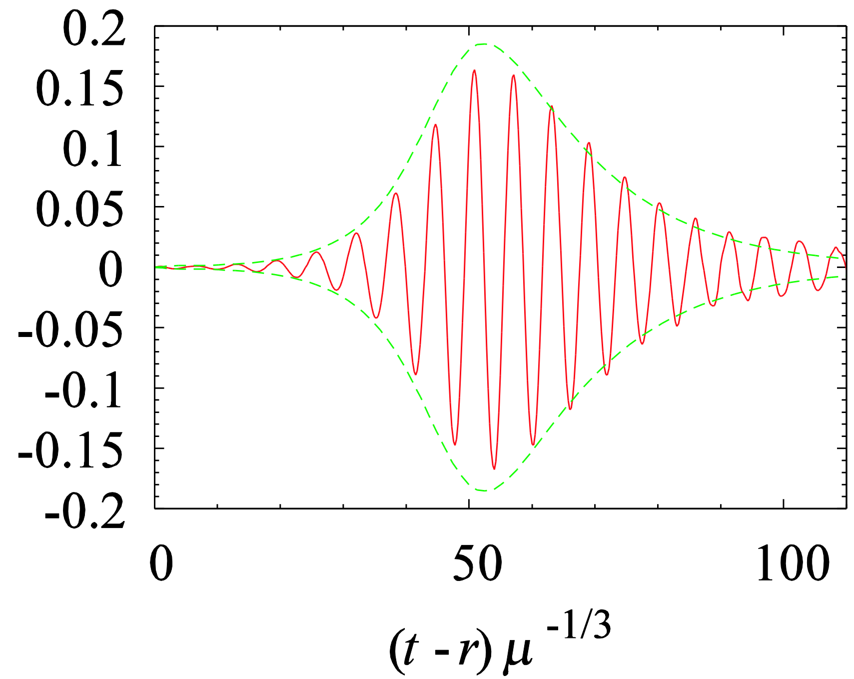
The same process of 4D fast rotating star

Gravitational waves

a=0.80$\mu^{1/3}$



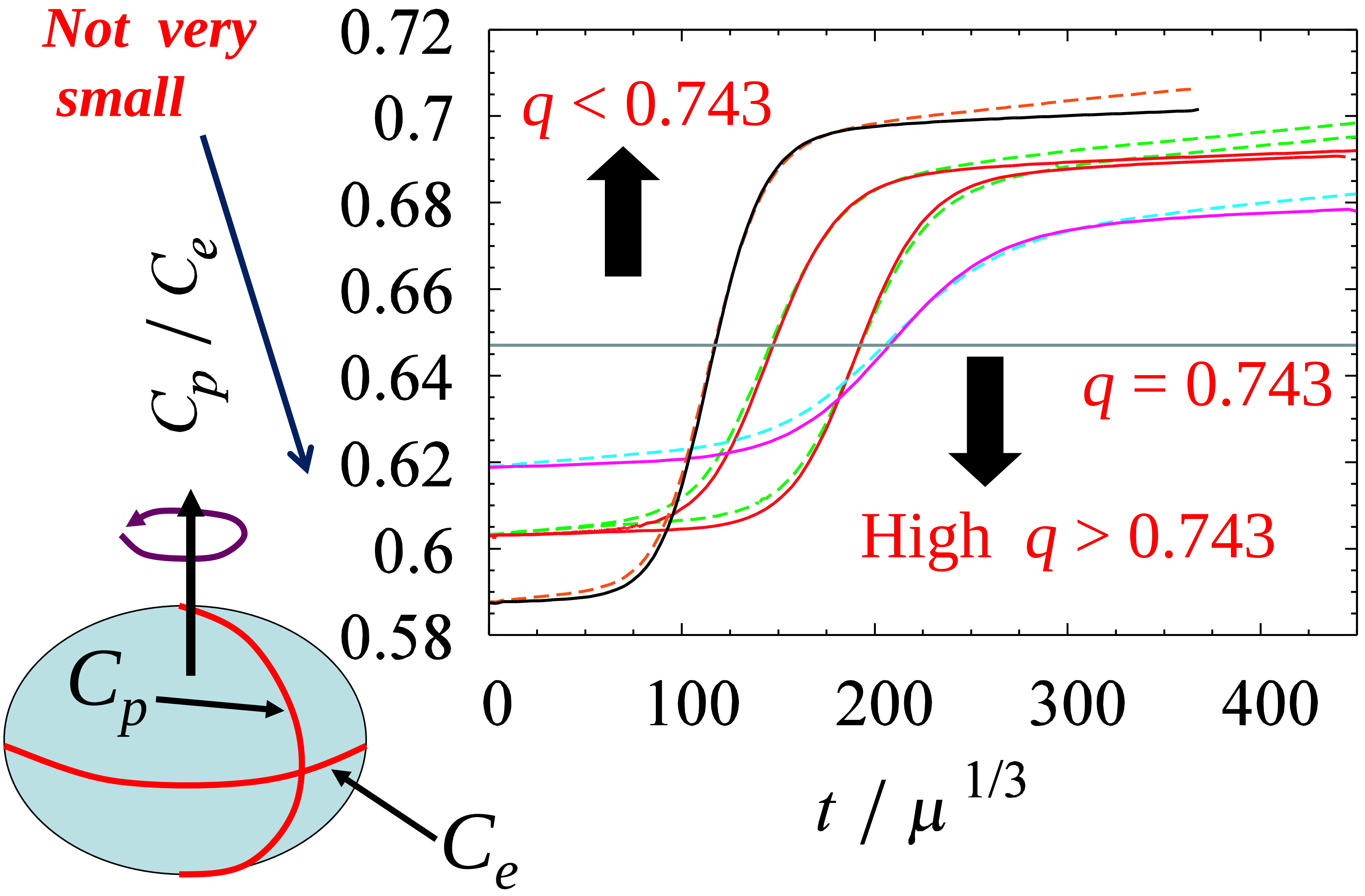
a=0.99$\mu^{1/3}$



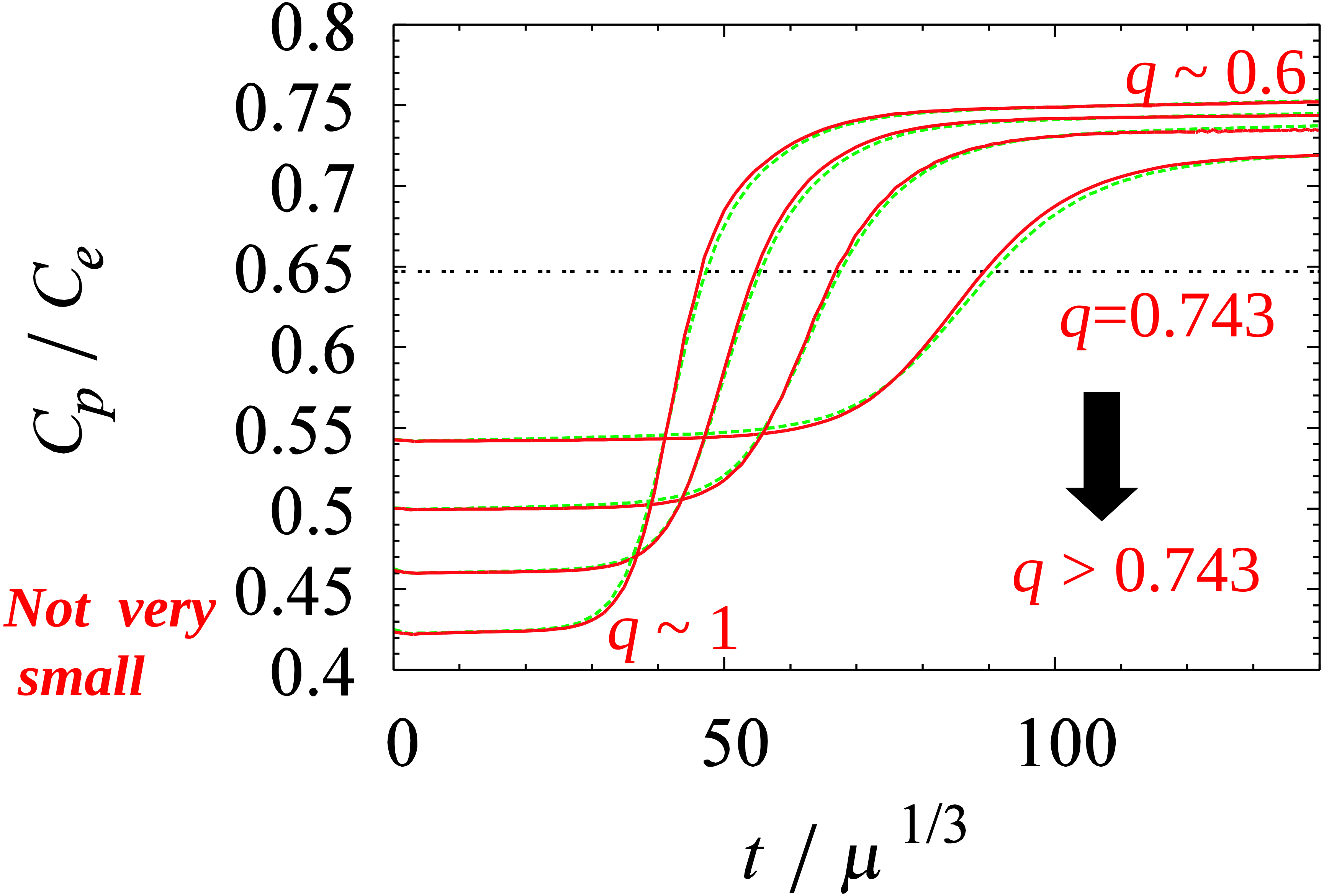
Slow growth
Slow damp

Rapid growth
Slow damp

Evolution of C_p / C_e: Spin down



Evolution of C_p / C_e for high spin



Summary for *D*-dim MP BH

- Rapidly rotating spheroidal BHs are unstable against bar-mode deformation
- The threshold value of spin is fairly small **q ~0.87 (D=5), q ~ 0.74 (D=6), 0.73 (D=7), and 0.77 (D=8)**
- We can follow BH for a very long time to determine the final fate for $D > 5$

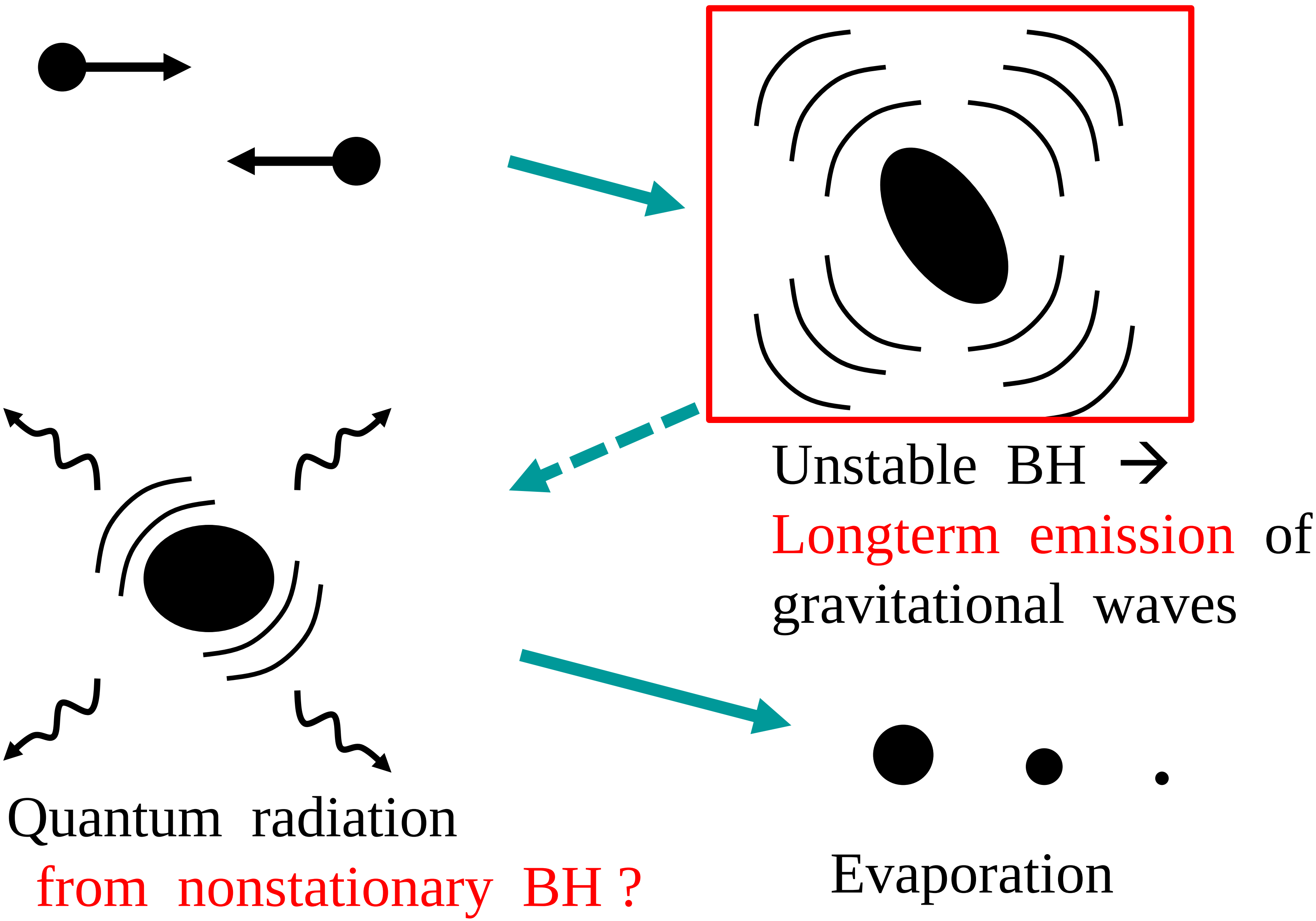
→ **Unstable BH radiates GWs and after the spin-down by sufficient radiation, the BH settles down to a new stable state**

Note: Ultra-spinning BH (pancake-shape) may have different fate; our study is only for spheroidal BH

q_{max}

- Analysis by Yoshino-Nambu (2002) indicates maximum impact parameter for formation of BH in ultra-rela. collision
- This gives the maximum spin of BH formed in the collision as 0.93 (D=5), 1.47 (D=6), 1.98 (D=7), 2.50 (D=8): much larger than q_{crit} found in our work
- Formation of larger impact parameter seems to be more frequent
  → Many of formed BHs are unstable ?

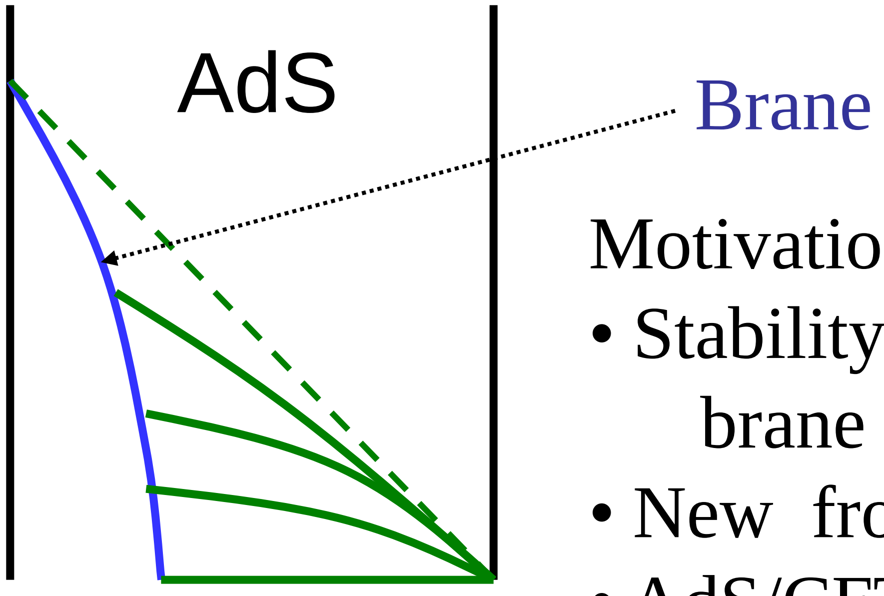
Corrected picture ?



IV Latest effort for numerical relativity of Asymptotic AdS

(Takahashi, Okawa, Shibata)

- We want to consider Randall-Sundrum II type spacetime with domain wall



Motivation:

- Stability of a BH on the brane (by T. Tanaka)
- New frontier in NR
- AdS/CFT ?

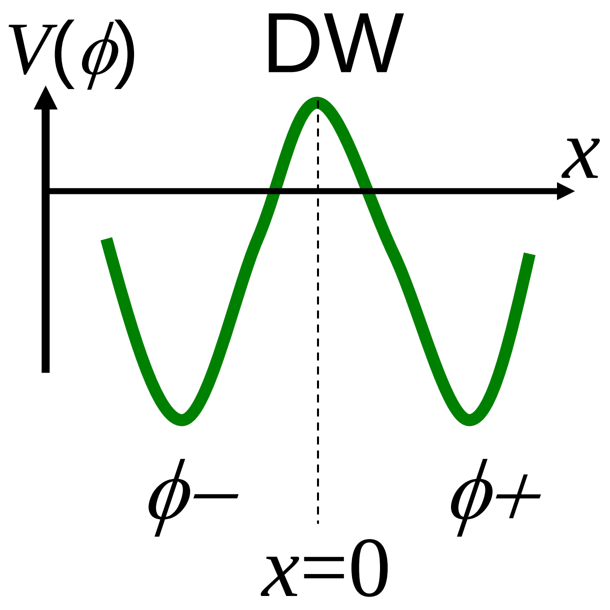
Setting

- ***Handling singular surface is not easy in numerical relativity***: Regularity is not easily guaranteed in numerical simulation
- Consider a thick wall by sin-Goldon type scalar-field potential (Giovannini '01)

$$V(\phi) = \frac{3b^2}{16\pi}\left[1 - 5\sin^2\left(\sqrt{\frac{8\pi}{3}}\phi(x)\right)\right]$$

$$\phi(x) = \sqrt{\frac{3}{8\pi}}\tan^{-1}(bx)$$

$$ds^2 = \frac{1}{b^2x^2+1}ds^2_{\text{flat}} : \quad \text{AdS for } x \gg b^{-1}$$



Formulation

$$ds^2 = \frac{1}{a^2}\left[-\alpha^2 dt^2 + \chi^{-1}\tilde{\gamma}_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)\right]$$

Original BSSN variables: $\tilde{\gamma}_{ij},\ \tilde{A}^0_{ij},\ \chi^0,\ K^0,\ \tilde{\Gamma}^i$

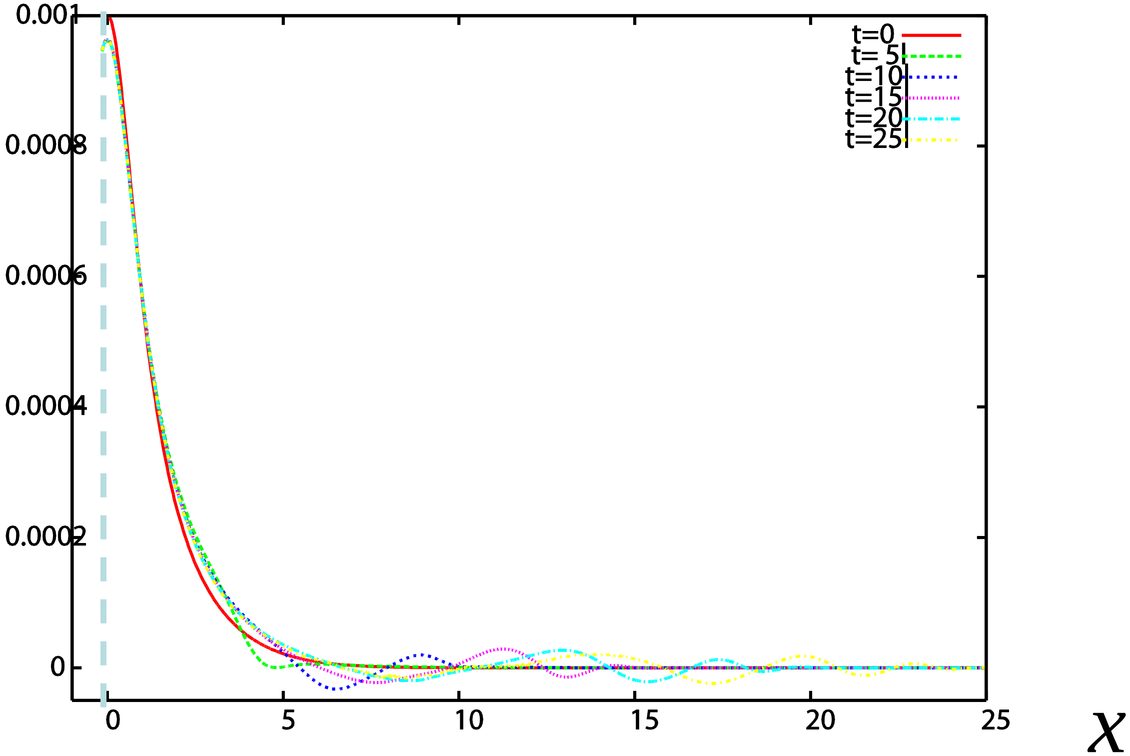
$$\Downarrow \quad a^2 = 1 + b^2 x^2$$

Variables in AdS: $\tilde{\gamma}_{ij},\ \tilde{A}_{ij} = \tilde{A}^0_{ij}/a,\ \chi = \chi^0/a^2,\ K = K^0/a,\ \tilde{\Gamma}^i$

- Modified BSSN with a few additional factors associated with a, a', a''
- Boundary conditions, similar to asymptotic flat case, work for new variables.
- At x=0, reflection symmetric (or asymmetric) BCs are simply imposed.

Status (just started)

- Test 1: Put the exact solution & evolve
  → Static solution remains static; OK
- Test 2: Put a perturbation & evolve
  → looks OK



Next step

- Prepare BH at the center & evolve; prepare BH as Frans does ?
- Consider applications seriously; suggestions are welcome

Thanks

Evolution of deformation of AH: *D*=6