Latest News from Numerical Relativity

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Numerical Relativity and High Energy Physics Conference

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Outline

- Motivation : Gravity in the coming decade
  - where does numerical relativity fit in?
- A one-sided and partial overview of recent results of possible interest to high energy physics
  - black hole formation in super-Planck scale "particle" collisions
  - high-speed black hole scattering
  - the Gregory-Laflamme instability of black strings
  - black holes and hydrodynamics via gauge/gravity dualities

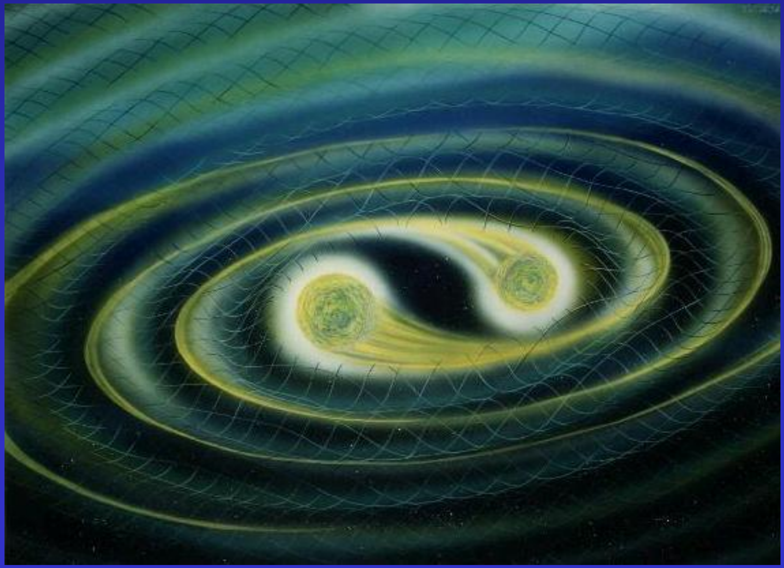
Gravity in the coming decade – GW astrophysics

- A concerted effort is underway to observe gravitational waves from a variety of sources in the universe, with plausible first detection within ~5 years
- **Ground based interferometers**
  targeting sources in the 10's of Hz to KHz range
- **Pulsar timing**
  sensitive to waves in the 10's of nanoHz to 10's of microHz range
- **B-modes in the CMB**
  frequencies ~ 10^{-15} - 10^{-18} Hz at decoupling
- More distant future : space-based detectors such as LISA, 10's milliHz – 1/10 Hz

LIGO Livingston

Gravity in the coming decade – GW astrophysics

- GW's hold promise to be a driving force in learning more about the universe
  - provide overwhelming evidence for the existence of black holes
  - explore the properties, populations and interactions of compact objects (black holes, neutron stars) and the consequences (e.g. sources of short gamma ray bursts?)
  - test the nature of strong-field, dynamical gravity; constrain (discover!) alternative theories
  - clues to the early universe in a stochastic background, CMB polarization
  - "exotica" like cosmic strings, etc.
  - discover the unknown?



An artist's impression of the merger of two neutron stars

Gravity in the coming decade – understanding fundamental physics

- Still major challenges for fundamental physics
  - Theory : a framework to resolve the incompatibility between a classical (GR) description of spacetime and the quantum world, that is experimentally or observationally verifiable
  - Observational puzzles : dark energy and dark matter
- Gravity could be expected to play an imported role in these endeavors
  - higher dimensional Einstein gravity?
  - modifications to general relativity, in 4 or higher dimensions?
  - clues from GW astrophysics?

Gravity in the coming decade – gauge/gravity dualities

- AdS/CFT and related correspondences showing remarkable connections between seemingly disparate physics
  - quark-gluon plasma formation in heavy ion collisions and black hole collisions
  - "hairy" black holes and superfluids, superconductors and other condensed matter systems
  - black hole dynamics and fluid mechanics
- Even if string theory is not *the* theory of everything, that such a mapping exists is astonishing, and provides an alternative route to understanding gravity and strongly coupled gauge theories
- Since the dualities are holographic, understanding "ordinary" 4D physics in terms of geometry requires study of higher dimensional gravity, in particular spacetimes with event horizons
  - much richer set of solutions & dynamics compared to 4D

Where does numerical relativity fit in?

- In many of these problems, *understanding* classical general relativity is required
- Often, understanding require *solutions* to the underlying equations
- In the 21st century, numerical methods should be considered one of the standard tools that can be brought to bare to find solutions to equations

Black hole formation in super-Planck scale particle collisions

- The presence of extra dimensions could allow for a very different Planck scale than what would then be the effective 4D one of *10^{19} GeV [Arkani-Hamed , S. Dimopoulos & G.R. Dvali PLB 429 (1998) ; Randall & R. Sundrum PRL 83 (1999)]*
  - current lower limits on the Planck scale are several TeV; if slightly above this, super-Planck scale collisions could be occurring in cosmic ray collisions with the atmosphere, and the LHC at maximum energies
  - it is *generically* expected that the consequence of this would be black hole formation, regardless of any non-gravitational interactions between particles *[Banks & Fishler hep-th/9906038, Dimopoulos & Landsberg PRL 87 161602 (2001), Giddings & Thomas PRD 65 (2002) , Feng & Shapere, PRL 88 021303 (2002)]*
- *see Talks by Giddings, Parker, Landsberg, Moeller*

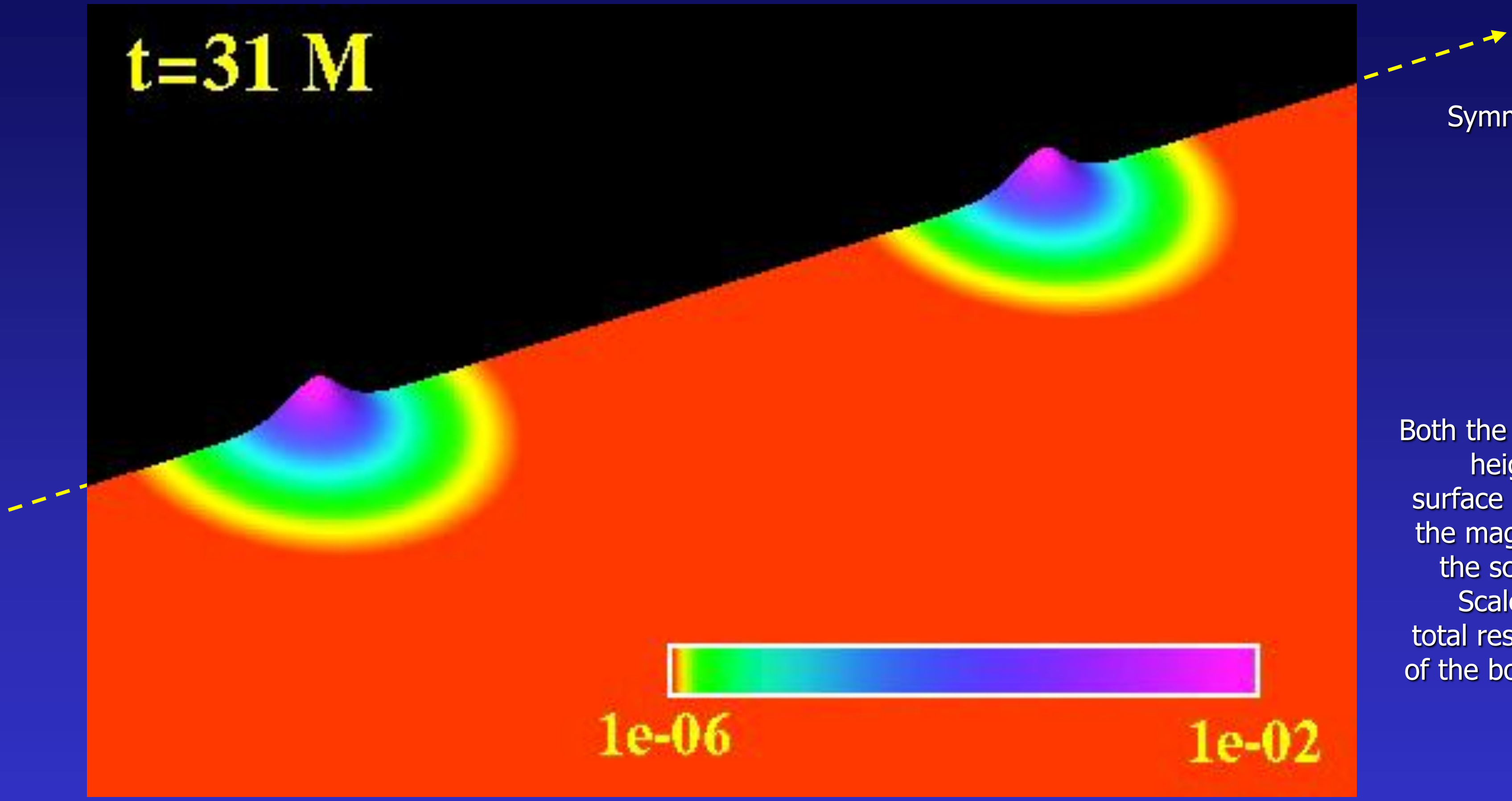
Black hole formation in super-Planck scale particle collisions

- The expected genericity is essentially for two reasons
  - at such energies gravity dominates the interaction, and sufficiently above the Planck scale *classical GR* should be a good description of it
  - classical GR, by *Thorne's hoop conjecture*, implies a black hole will form, hiding all details of non-gravitational interactions inside it
    - squeeze matter/energy into a "ball" with radius smaller than the corresponding Schwarzschild radius, and a black hole forms
    - Planck's constant enters the picture for quantum particle interactions in defining the size of the fundamental particle through de Broglie's relation
- however, sparse evidence in the form of solutions to the GR field equations to support this conjecture
    - colliding plane-fronted gravitations waves (Penrose) and generalizations there of in the *infinite* boost limit

High speed soliton collision simulations

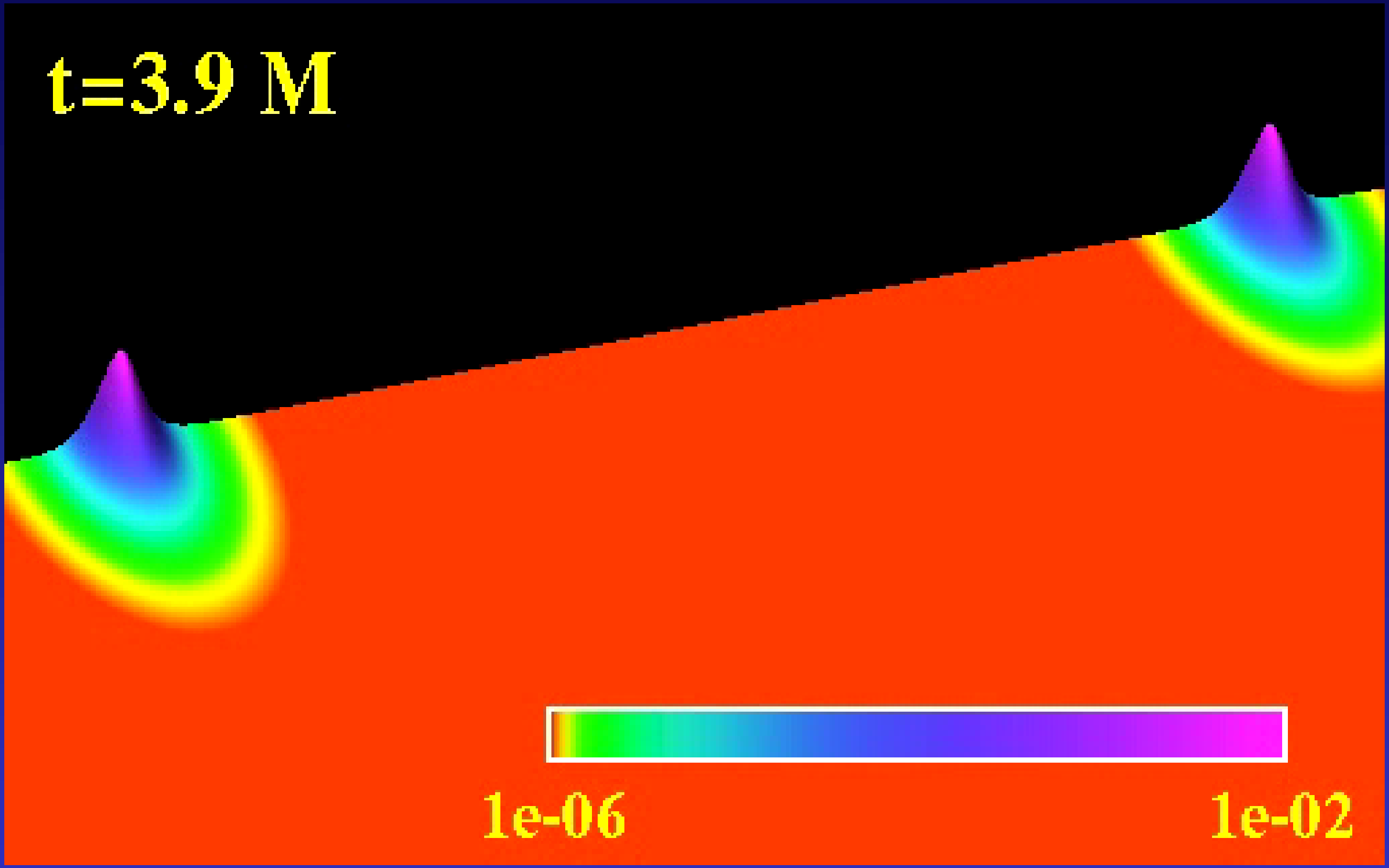
- Test this hypothesis by colliding self-gravitating solitons, boson stars in this case *[M.W. Choptuik & FP, PRL 104, (2010)]*
- Very computationally expensive to run high-γ simulations, so need to start with a relatively compact boson star that will reach hoop-conjecture limits with reasonable γ 's.
  - LHC-type scenario is essentially all kinetic energy; here, a sizeable amount of rest-mass energy as well
- Choose parameters to give a boson star with $R/2M \sim 22$
  - thus, hoop-conjecture suggests a collision of two of these with $\gamma=11$ in the center of mass frame will be the marginal case

Case 1: free-fall collision from rest



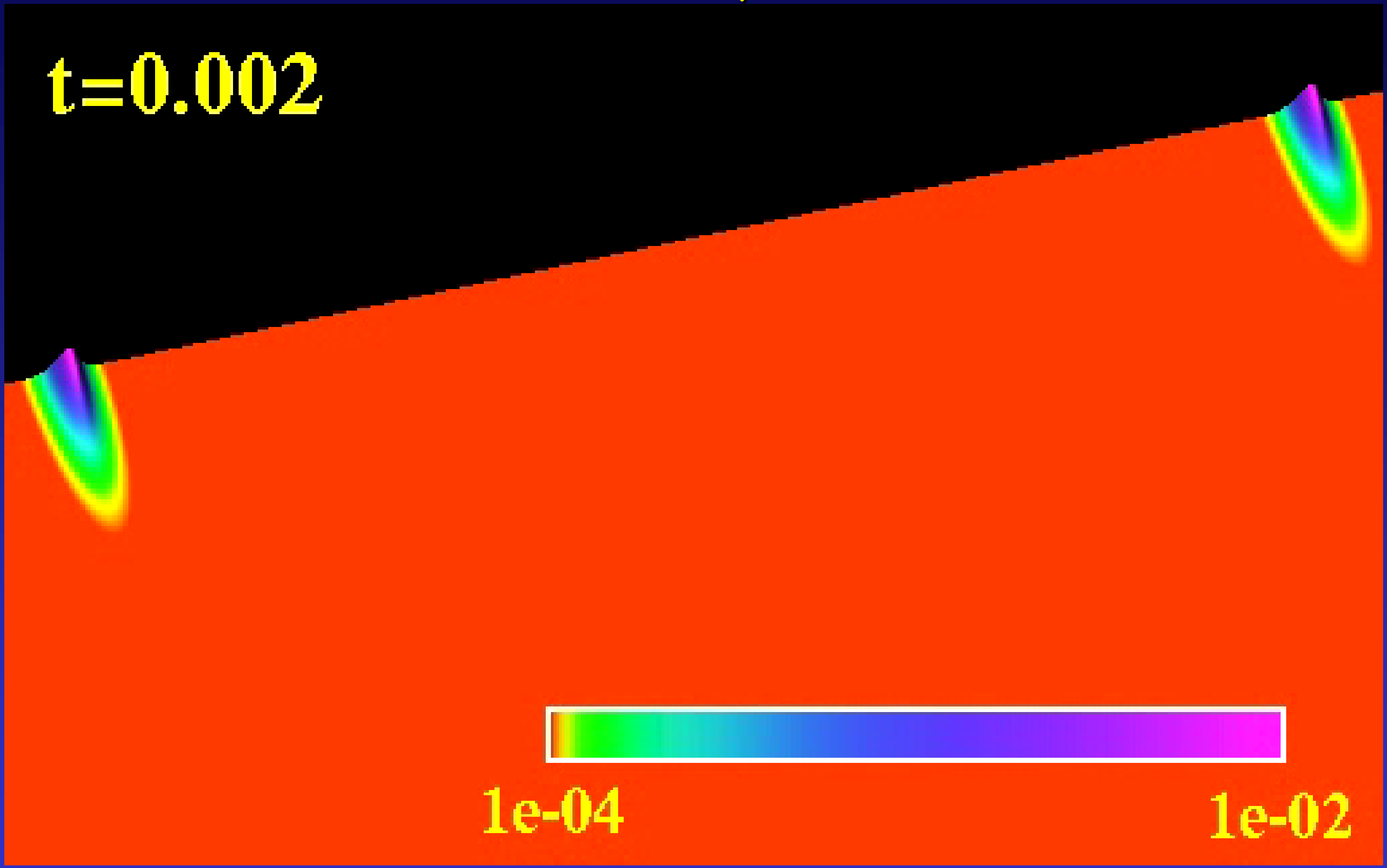
- Here, gravity dominates the interaction, causing the boson stars to coalesce into a single, highly perturbed boson star (this case eventually collapses to form a black hole)

Case 2 : $\gamma = 2$



- Here, though gravity strongly perturbs the boson stars, kinetic energy “wins” and causes them to pass through each other
  - soliton-like interference pattern can be seen as the boson star matter interacts
  - superposition of initial data, and subsequent truncation, cause some component of the field to move in the wrong direction; the truncation error part converges away with resolution, the initial data part lessens the further the initial separation

Case 3 : $\gamma = 4$



- Here, the early matter interaction looks similar, but now the gravitational interaction of the kinetic energy of the solitons causes gravitational collapse and black hole formation
  - NOTE: gauge than previous case: the coordinate spreading of the solitons before collision, and shrinking of the horizon afterwards, are just coordinate effects; also, different color scale

High speed collisions and black hole formation

- All this implies one can use *any* model of a particle to study the nature of sufficiently super-Planck scale scattering, including *black holes!*

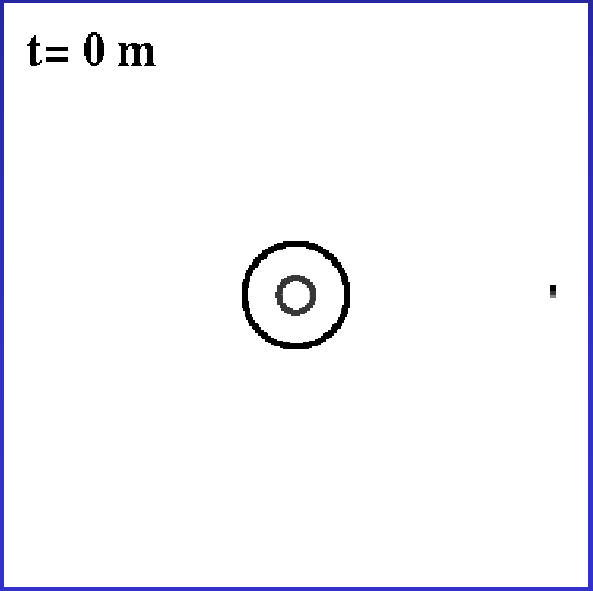
- *Several groups working on this; see talks later today by Witek, Zilhao, Okawa, and Shibata on Friday*

- Here will discuss one example of interesting "zoom-whirl" behavior seen in grazing collisions from *Sperhake, Cardoso, P, Berti, Hinderer ,Yunes PRL 103 (2009)*
  - results obtained with U. Sperhake's Lean code
  - "just" 4D, so make extrapolations to LHC-type scenarios with caution
  - again, see talks later today for the latest, and in higher dimensions

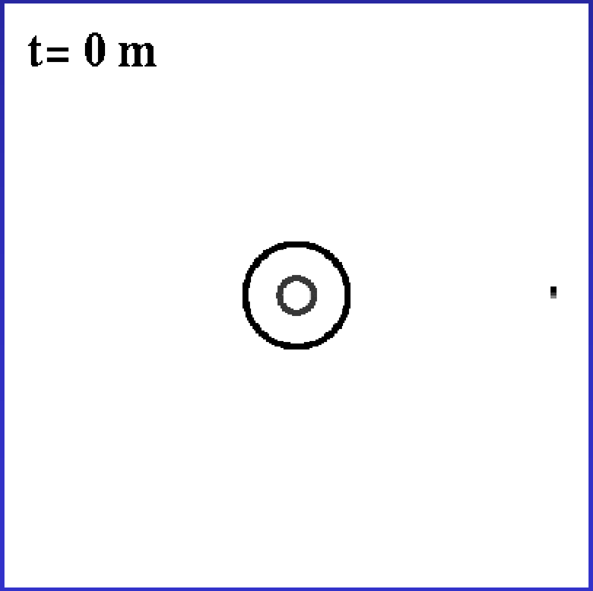
Zoom-whirl behavior

- Zoom-whirl behavior originally noted in geodesics about black holes
- At first glance, seems like "extreme" pericenter precession, but in fact zoom-whirl orbits are perturbations of *unstable circular orbits* that exists within the inner-most stable spherical orbit (ISCO)
  - In Schwarzschild, radial perturbations of circular orbits in the range 4M to 6M lead to elliptic zoom-whirl orbits, 3M (the "light ring") to 4M lead to a hyperbolic orbit with one whirl episode

Schwarzschild geodesics; inner circle is event horizon, outer one is ISCO



usual (but large) pericenter precession

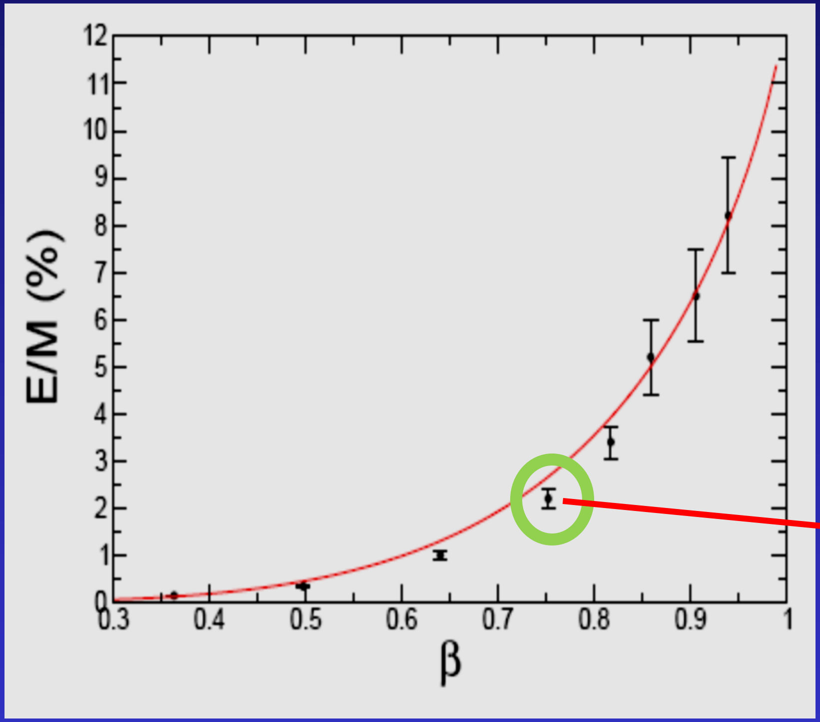


zoom-whirl orbit

Zoom-whirl behavior

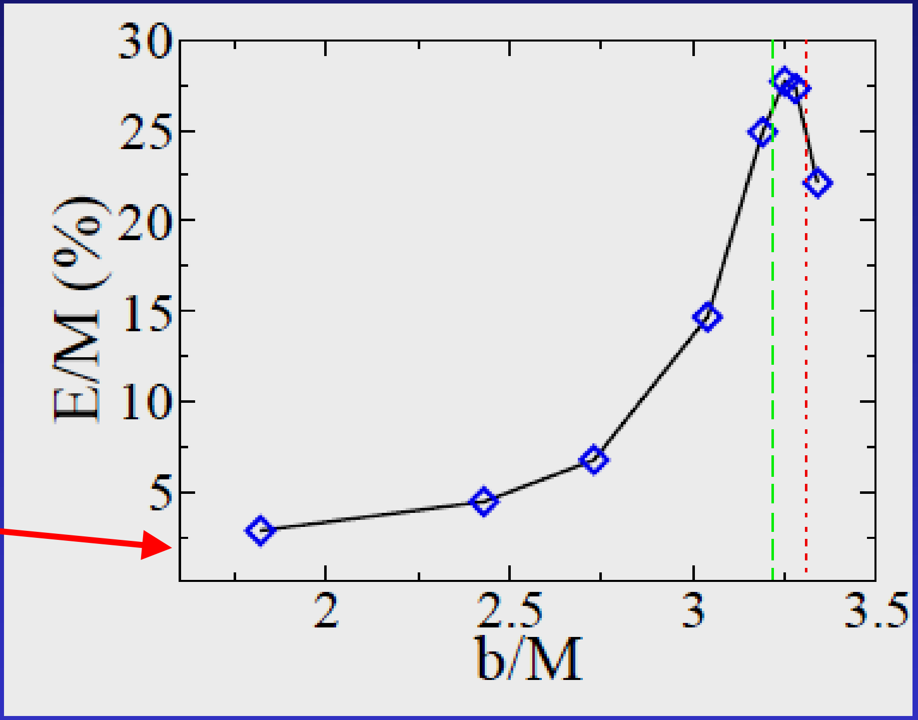
- Might imagine ZW orbits are particular to geodesic motion, however one can also understand ZW behavior as arising due to their being (at least) *two distinct end-states* in the BH scattering problem : one BH or two BH's
  - thus, *qualitative* behavior should arise generically in all BH scattering problems
- Consequences near the critical impact parameter
  - strong sensitivity to initial conditions
  - enhanced total gravitational wave emission, even in scattering cases
    - In 4D also get a huge enhancement in gravitational wave luminosity, though may be peculiarity of 4D in that in an effect geodesic model the light ring frequency is commensurate with the least-damped quasinormal mode frequency of the black hole
  - at threshold, expect *all* kinetic energy to be converted to GW energy

Enhanced energy emission in grazing collisions



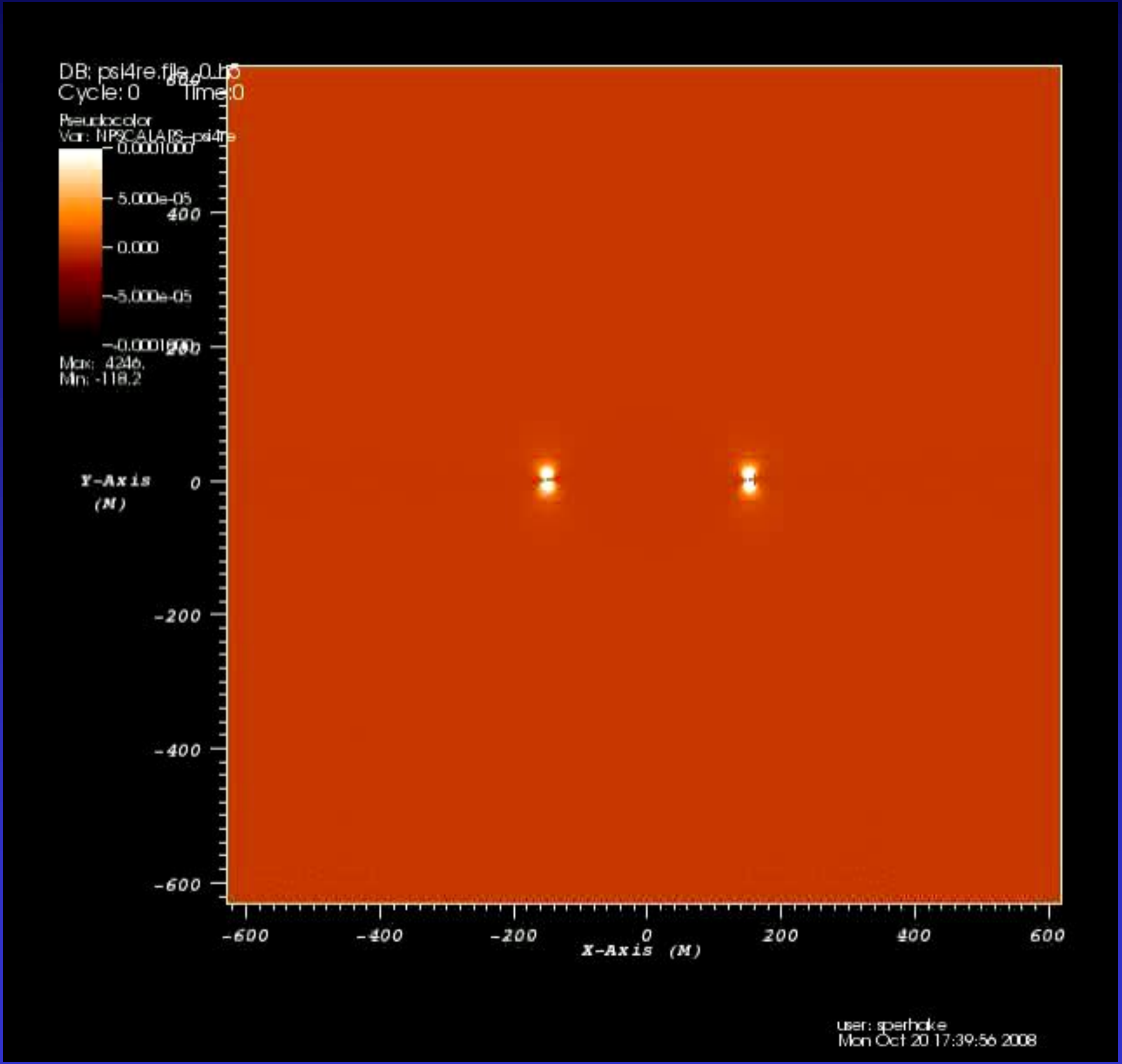
GW energy emitted in head on collisions vs center of mass velocity

(from Sperhake, Cardoso, P, Berti, Gonzalez PRL 101 (2008))



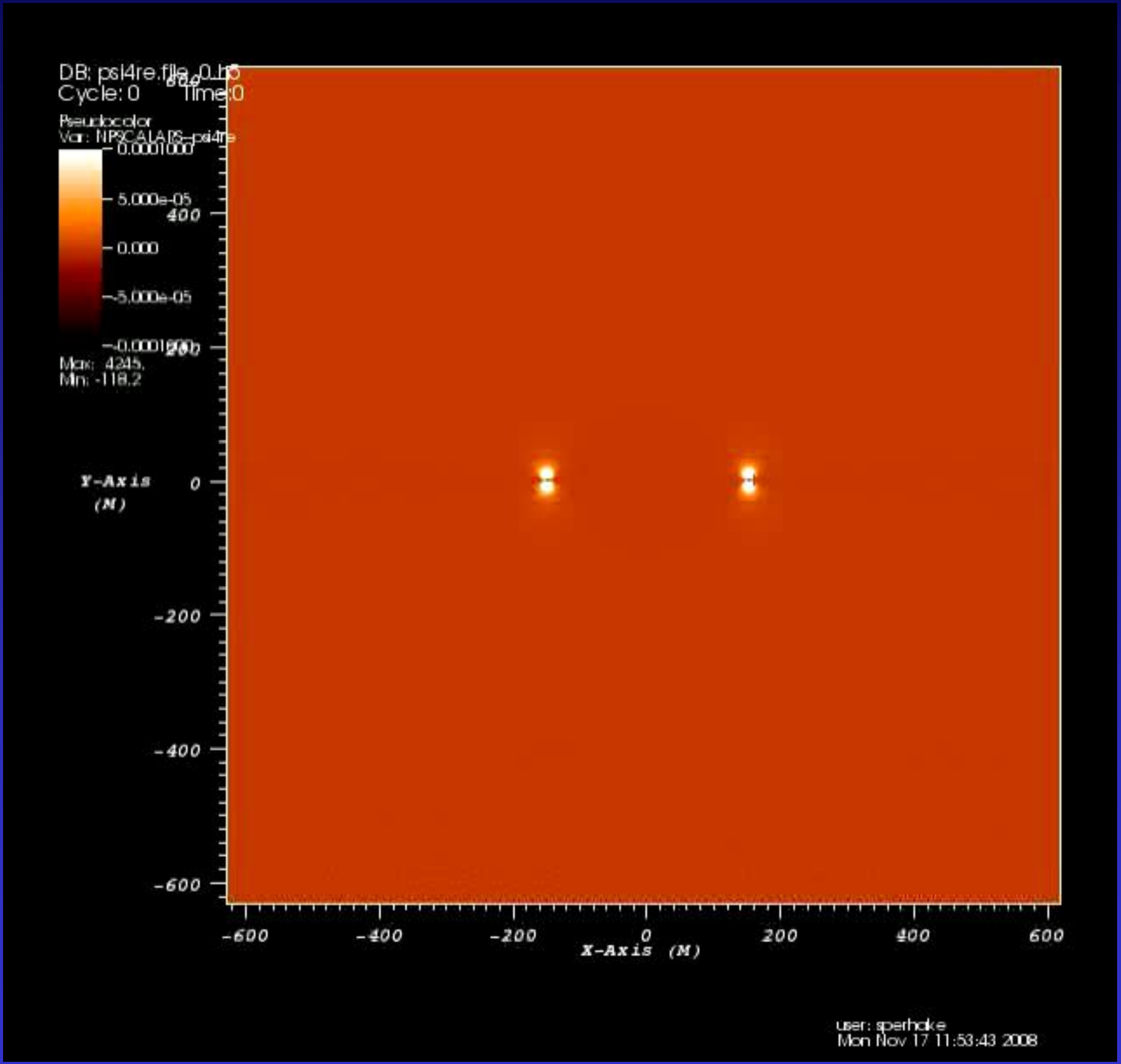
GW energy for v=0.75 case vs impact parameter (green line is merger threshold for initial interaction)

Scatter example, γ=1.5



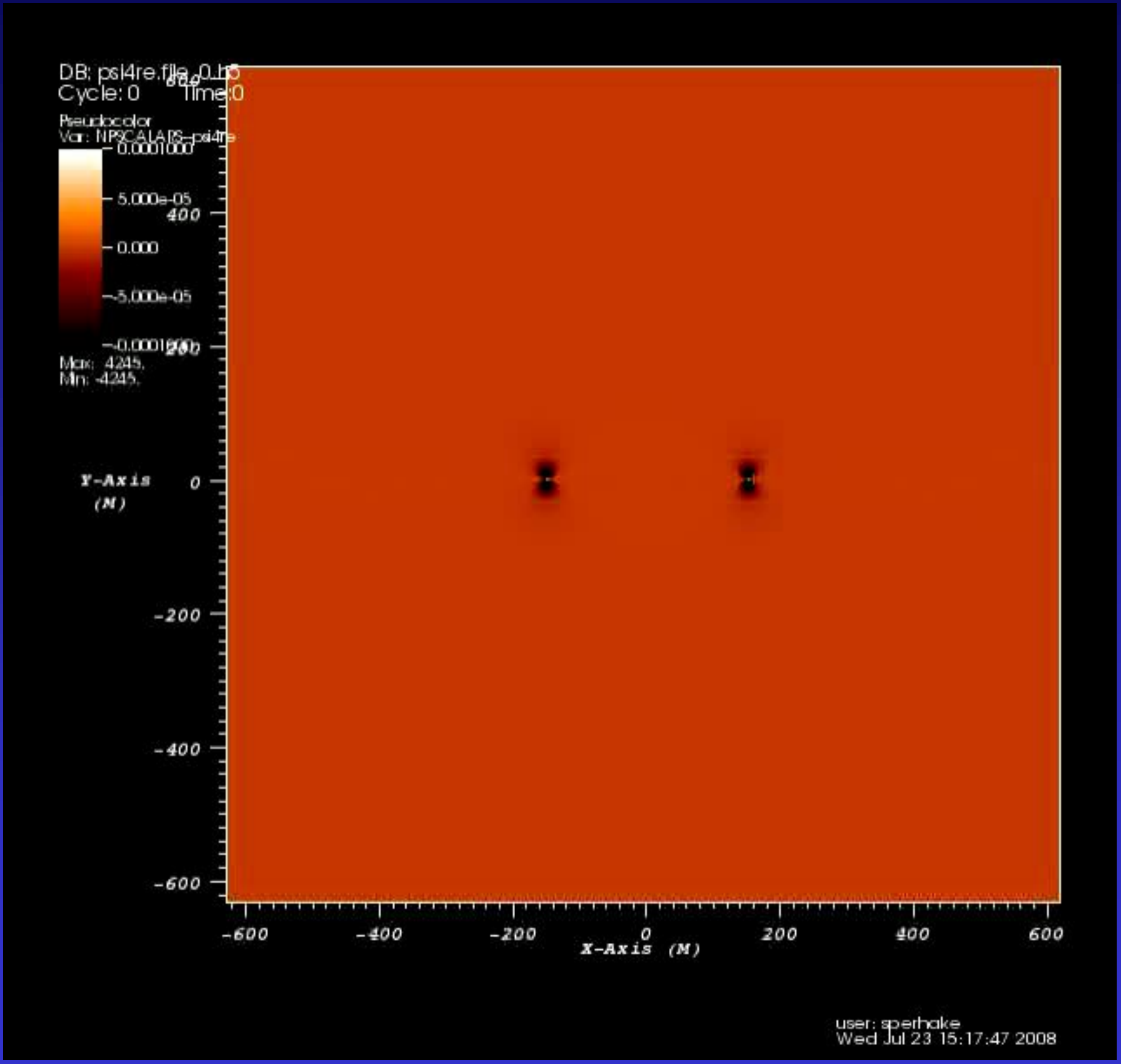
$Re[\Psi_4]$

Whirl, then scatter , $\gamma=1.5$



Re[Ψ_4]

Whirl, then merger, $\gamma=1.5$



Re[Ψ$_4$]

Higher dimensional black holes

- Higher dimensional black holes have many properties in common with their 4D counterparts, e.g.
  - can be defined using global (event horizons) or local (isolated horizons) constructs, contain geometric singularities, quasi-stationary processes are governed by the usual laws of black hole mechanics, end-points of gravitational collapse, Hawking radiate at the semi-classical level, etc.
- However, no strict uniqueness as in 4D, and many black objects are *unstable* to perturbations
- Within the context of gauge-gravity dualities, black holes play a prominent role
  - associated with states where a thermal/hydrodynamic description is valid
- Will show two examples of numerical solutions of higher dimensional BHs:
  - Gregory-Laflamme instability of the 5D black string
  - quasinormal ringdown of 5D S^3 black holes in asymptotically AdS spacetime
- *see Talks by Hubeny, Emparan, Gregory, Wiseman, Mateos, Dias, Ishibashi*

Black Strings

- Black strings are a particularly simple class of higher dimensional black hole solutions
  - in N spacetime dimensions, the metric is *4D Schwarzschild X (N-4)D Euclidean flatspace*; e.g. for N=5, in Schwarzschild coordinates

$$ds^2 = -\left(1 - 2m/r\right)dt^2 + \frac{1}{\left(1 - 2m/r\right)}dr^2 + r^2 d\Omega^2 + dw^2$$

  - here m is interpreted as mass per unit length; a segment of length $\Delta\omega = L$ of the spacetime has asymptotic mass $M = mL$

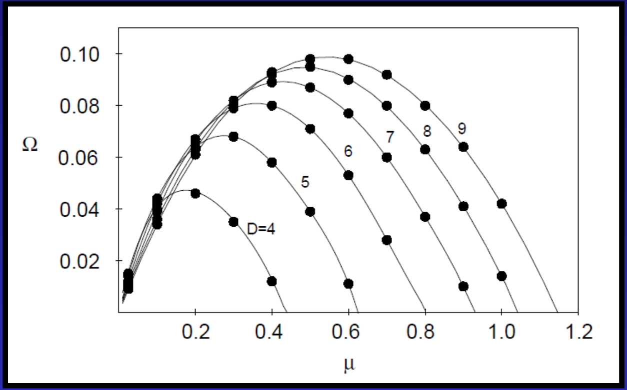
Gregory-Laflamme instability

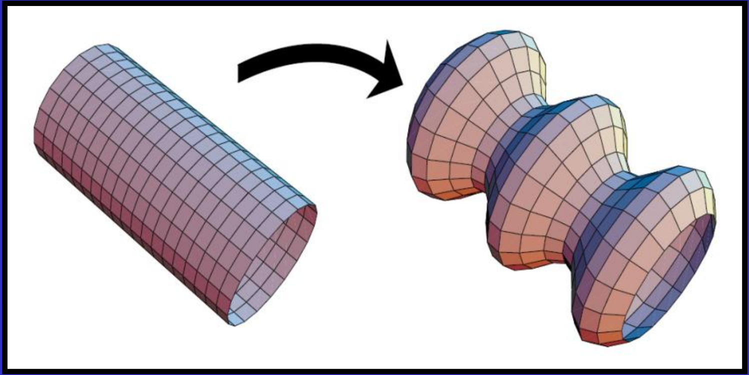
- Gregory and Laflamme *[PRL 70 (1993)]* first showed that black strings are linearly unstable to long-wavelength perturbations

$$g = g_0 + \delta g \cdot e^{\Omega t/m + i\mu w/m}$$

  - Images from R. Gregory and R. Laflamme, Nucl.Phys.B428 (1994)
  - the D=4 curve corresponds to the 5D black string, and the critical wavelength above which modes are unstable is

$$\lambda_c \approx 14.3m$$



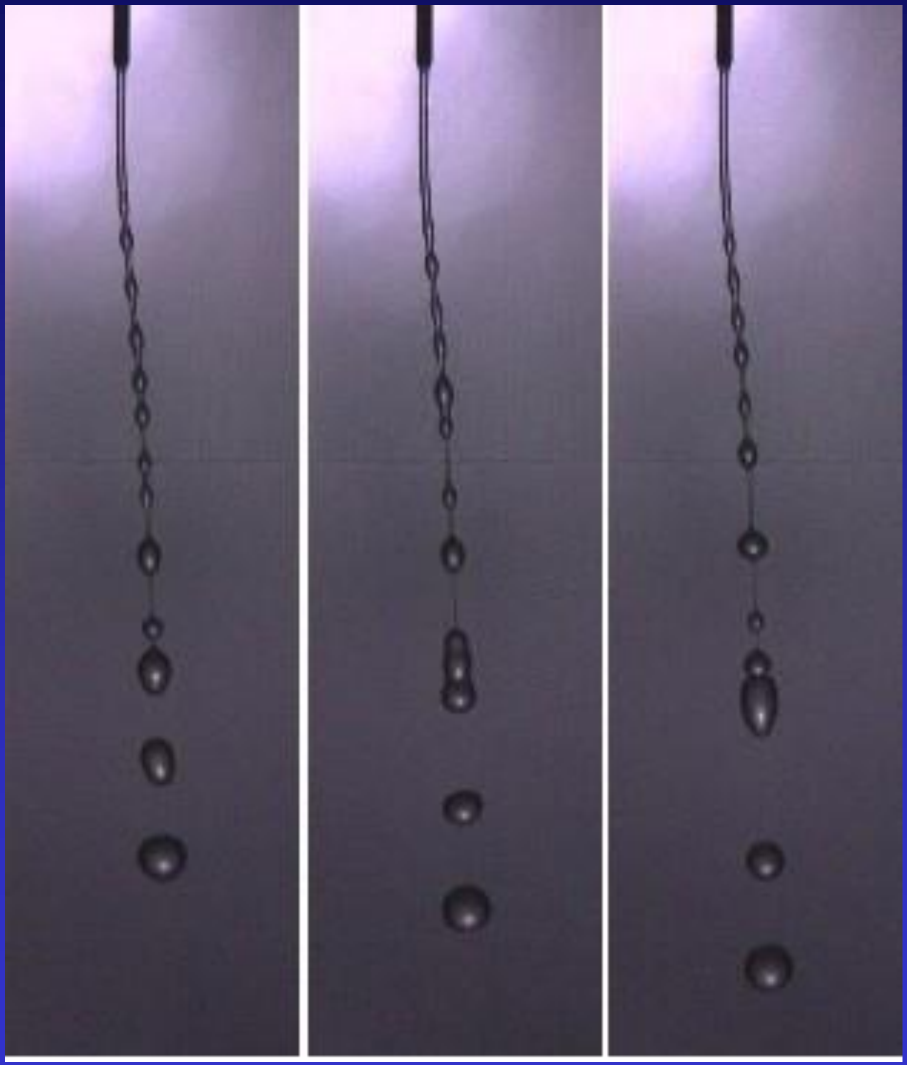


End-state of the instability?

- much speculation about the end-state
  - from entropic considerations, GL argued that black string would "pinch-off" into a sequence of spherical black holes (in the process violating cosmic censorship)
    - earlier numerical simulations seemed to indicate in favor of this, though "crashed" before a conclusive statement could be made, *Choptuik et al. [PRD 68, 044001 (2003)]*
  - Horowitz and Maeda *[PRL 87, 131301 (2001)]* argued the end-state would be a new, static, non-uniform solution with the same topology as the black string, based on a proof that any cross-sectional radius of the horizon cannot shrink to zero in *finite* affine time of the horizon generators
    - non-uniform black hole solutions were found, though they had too high entropy to be the end-state of the GL instability *[S. S. Gubser, CQG. 19, 4825 (2002), T. Wiseman, CQG. 20, 1137 (2003), E. Sorkin, PRD74:104027 (2006)]*;
  - Cardoso and Dias *[PRL 96 (2006)]* showed that the spectrum of unstable modes of a Rayleigh-Plateau-unstable fluid stream was *qualitatively* similar to GL; Camps, Emparan & Haddad *[JHEP 1005 (2010)]* derived the GL dispersion relationship using the blackholds GR-hydronamics duality

End-state of the instability?

- The hydrodynamic analogies, in particular GL-RP, give weight to the pinch-off scenario
  - of course, the caveat is that at present the GL-RP relation is just an analog, and the blackfolds approach only describes the linear regime of the instability
  - need solutions in the non-linear regime
    - will show results from a simulation with L. Lehner *[PRL 105 2010]* that shows the "answer" is pinch-off in a manner qualitatively consistent with RP.



Embedding Diagram of Apparent Horizon Unstable 5D Black String

- map the geometric 1D shape of each *t=x=y=constant* slice of the apparent horizon to a flat *(R,Z)* Euclidean space; i.e. in parametric form

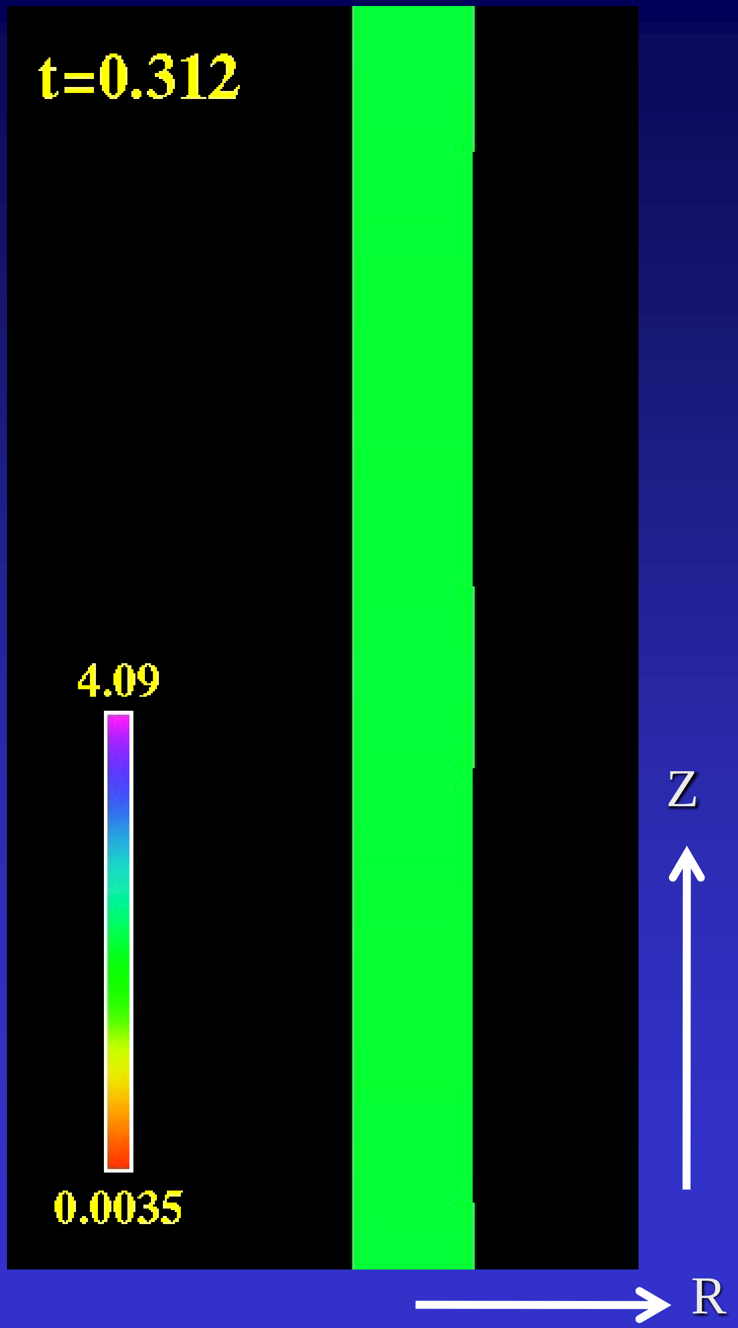
$$(R,Z)=\left(R(\xi),Z(\xi)\right)$$

- *R(ξ)* is the areal radius of that point on the horizon, and *Z(ξ)* is defined so that the proper length of the curve in the flat space is identical to that of the corresponding curve in the physical geometry

- the movie shows this curve spun around *R=0* to form a surface for visual aid

- color is mapped to *R*

- note that time is "slowing down"



Embedding Diagram of Apparent Horizon Unstable 5D Black String, close-up and in “real time”

- map the geometric 1D shape of each *t=x=y=constant* slice of the apparent horizon to a flat *(R,Z)* Euclidean space; i.e. in parametric form

$$(R,Z) = \left(R(\xi), Z(\xi)\right)$$

- $R(\xi)$ is the areal radius of that point on the horizon, and $Z(\xi)$ is defined so that the proper length of the curve in the flat space is identical to that of the corresponding curve in the physical geometry
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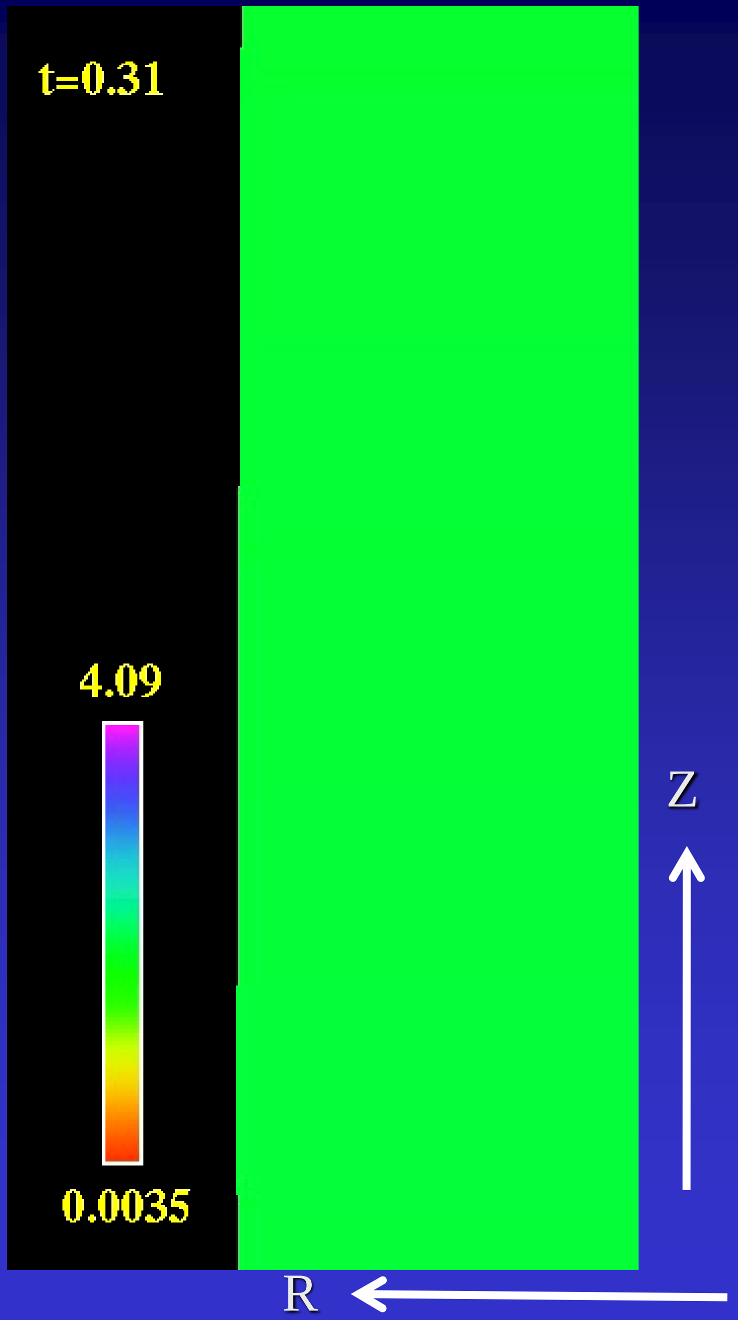


Gregory-LaFlamme and Raleigh-Plateau

- In the Rayleigh-Plateau hydrodynamic analogue, a self-similar cascade can also occur
  - the lower the viscosity of the fluid, the more generations of self-similar behavior are seen before break-up
  - the membrane paradigm and gravity/gauge dualities suggests black holes have much lower shear viscosity to entropy ratio that any "real-world" fluid
- An exact scaling solution of the Navier Stokes equation *[Eggers, PRL 71 (1993); Miyamoto, JHEP 1010 (2010)]* near pinch-off is known, giving

$$r \propto (t_0 - t)$$

and this is consistent with the numerical results of the Gregory-Laflamme instability

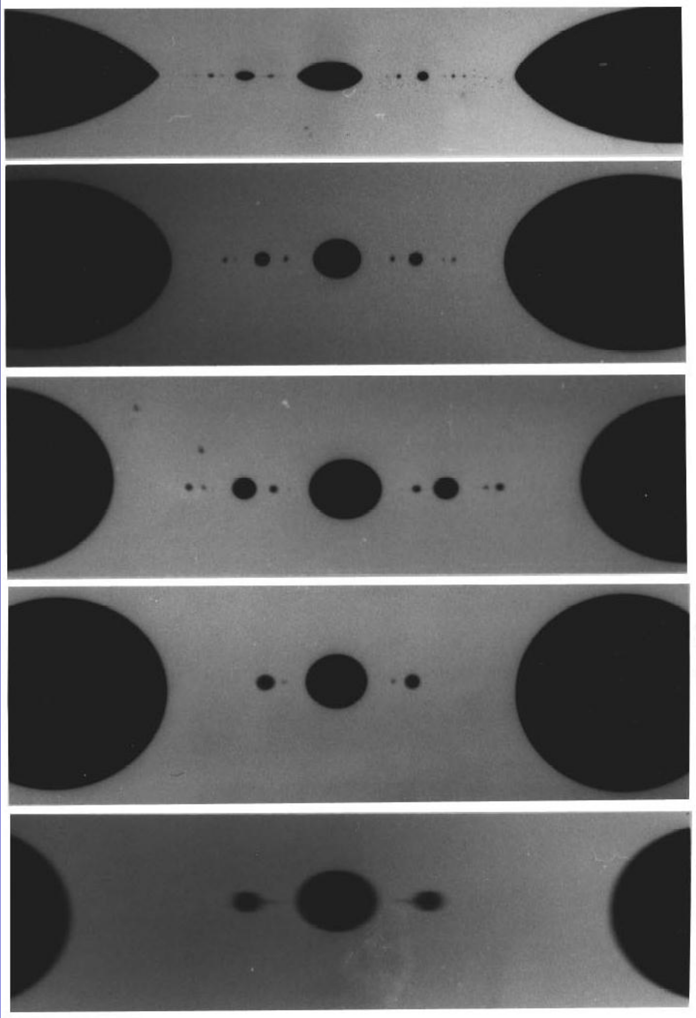


Image from Review article by Eggers *[Rev.Mod.Phys 59 (1997)]*, from work of Tjahjadi, Stone, and Ottino, *[Fluid Mech. 243 (1992)]*

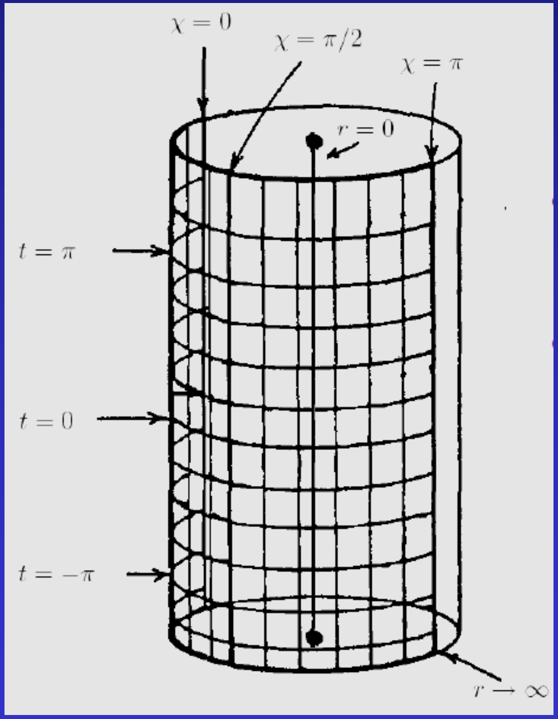
Quasinormal ringdown of highly distorted black holes in 5D Asymptotically AdS spacetime

- Will show preliminary results from a new code designed to study 5D AAdS spacetimes (with Hans Bantilan)
  - eventual goal is to study gravity dual of quark-gluon plasma formation in heavy ion collisions, conjectured to be black hole collisions *[Nastase (2005)]*
    - first numerical approach to this problem by Chesler & Jaffe *[PRL 106 (2011)]*
  - here we impose SO(3) symmetry in global AdS, hence we can study axisymmetric bulk geometries, dual to spherically symmetric states on the boundary with topology $R \times S^3$
  - immediate goals to see if any non-linear phase of ringdown apparent (corresponding to thermalization time?), and when the boundary dynamics becomes consistent with that of a thermal, conformal fluid

5D AdS spacetime

- Global AdS in spherical-polar type coordinates

$$ds^2 = -\left(1 + r^2/L^2\right)dt^2 + \left(1 + r^2/L^2\right)^{-1} dr^2 + r^2\left(d\chi^2 + \sin^2\chi d\Omega_2^2\right)$$



- spacetime of constant negative curvature $R=-20/L^2$
- the boundary metric ($r\to\infty$) is the 4D Einstein static universe ($R \times S^3$)
- Poincare coordinates cover a conformally flat piece of global AdS (the Poincare patch)

$$ds^2 = -W^2\left(-dt^2 + d\vec{x}_4^2\right)$$
$$W^2 = \sqrt{1+r^2/L^2}\cos(t/L) + r/L\cos(\chi/L);\ W > 0$$

this segment of AdS is usually used for applications with a CFT on $R^{3,1}$; we will use global coordinates, and can transform a patch as needed

- The boundary is timelike, and though an *infinite proper distance* from any point in the interior to a point on the boundary on a $t=const.$ slice, null signals will propagate back and forth in *finite proper time,* experiencing *infinite red/blue* shift in the process

AAdS Black Holes

- The 5D AdS-Schwarzschild black hole has metric is:

$$ds^2 = -\left(1 + {r^2}/{L^2} - {r_0^2}/{r^2}\right)dt^2 + \left(1 + {r^2}/{L^2} - {r_0^2}/{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2$$

the horizon is at $r = r_H$, where

$$r_0 = r_H \sqrt{1 + r_H^2 / L^2}$$

and it has mass, entropy and temperature

$$M = \frac{3\pi}{8} r_0^2;\ \ S = \frac{\pi^2 r_H^3}{2};\ \ T = \frac{r_H}{\pi L^2}\left(1 + \frac{L^2}{2r_H^2}\right) \approx \frac{r_H}{\pi L^2}$$

Quasi-normal modes of AAdS Black Holes

- Gravitational and scalar field perturbations of 5D AdS-Schwarzschild black holes exhibit quasi-normal (QN) decay *[Horowitz & Hubeny PRD 62 (2000); Review: Berti, Cardoso & Starinets CQG 26 (2009)]*
  - in general for the metric there are scalar, vector & tensor modes; here due to axisymmetry only scalar modes can be excited
  - decompose scalar perturbation into scalar spherical harmonics on S^3, $S_{klm}(\chi,\theta,\varphi)$; again due to symmetry only $k \neq 0;\ l=m=0$.
  - A given QN mode can then schematically be written as

$$f_{klm}(t,\rho,\chi,\theta,\varphi) = A_{klm}(\rho) S_{klm}(\chi,\theta,\varphi) e^{-i\omega t}$$
$$\omega = \omega_r + i\omega_i$$

  - the decay times (imaginary modes) of most interest to heavy ion collisions $\leftrightarrow$ equilibration/thermalization time scale of boundary state

Quasi-normal modes of AAdS Black Holes

- For large BHs relative to L ($r_H > L$), there are *fast*

$$\omega \approx (3.0 - 2.7i)\frac{r_H}{L^2} \qquad (k = 2, l = m = 0; \text{fund.mode})$$

and *slow*

$$\omega \approx 1.6\frac{1}{L} - 0.8i\frac{1}{r_H} \qquad (k = 2, l = m = 0; \text{fund.mode})$$

gravitational QNMs; it has been suggested the former can be thought of as related to "microscopic" perturbations of the boundary state, the latter "hydrodynamic" *[Friess et al. JHEP 0704 (2008)].*

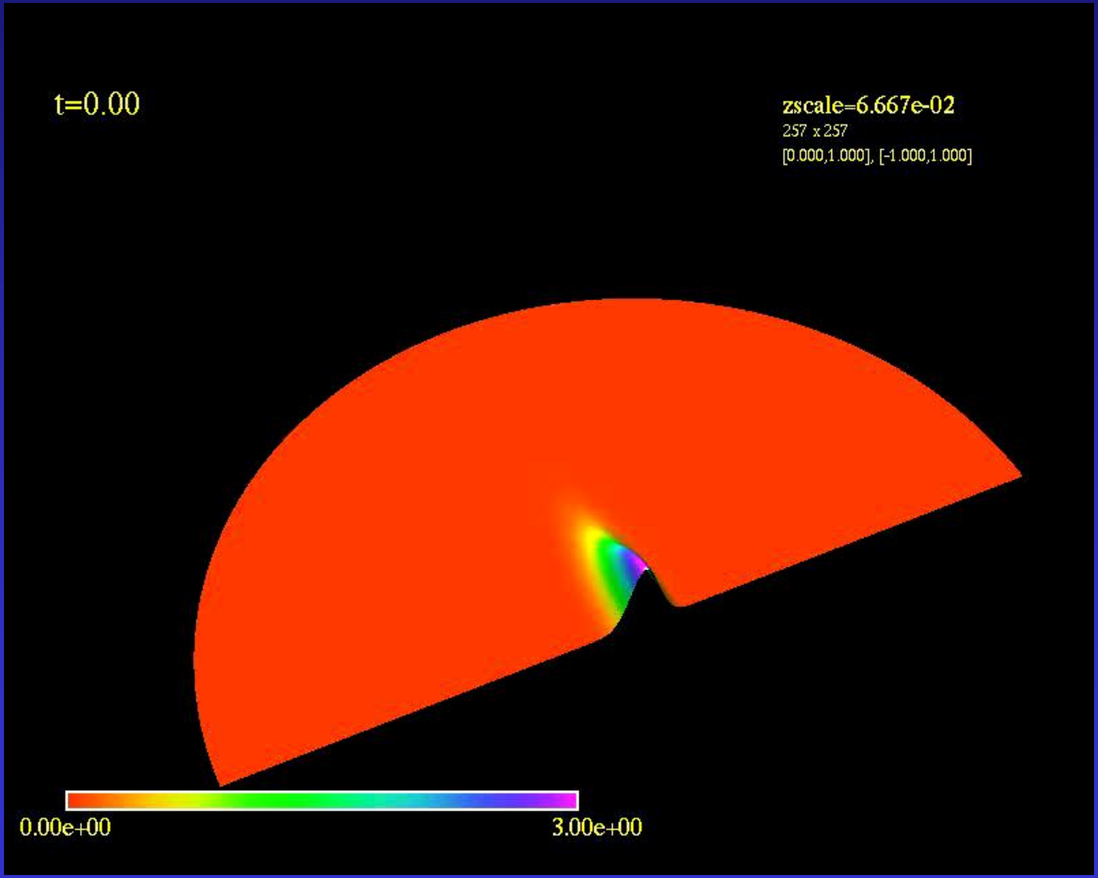
- The scalar field only has fast modes

$$\omega \approx (3.0 - 2.7i)\frac{r_H}{L^2} \qquad (k = 0, l = m = 0; \text{fund.mode})$$

Quasi-normal modes of AAdS Black Holes

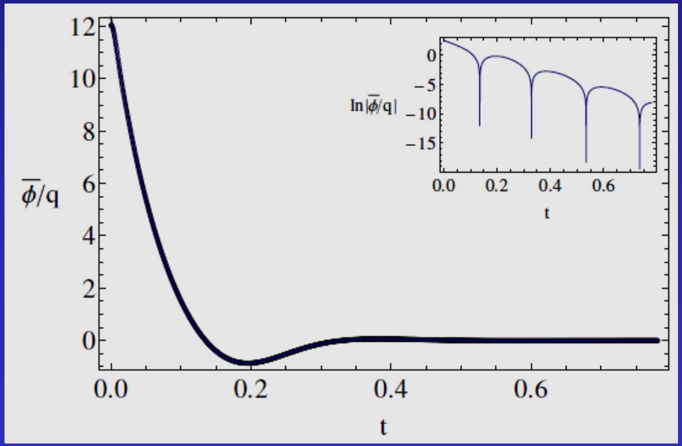
- form a distorted BH via asymmetric scalar field collapse

$$\overline{\Phi}(\rho,\chi,t=0) = Ae^{-\frac{\rho^2\cos^2\chi}{w_x^2}-\frac{\rho^2\sin^2\chi}{w_y^2}}$$



$\overline{\Phi}(\rho,\chi,t)$

r_H=12.2

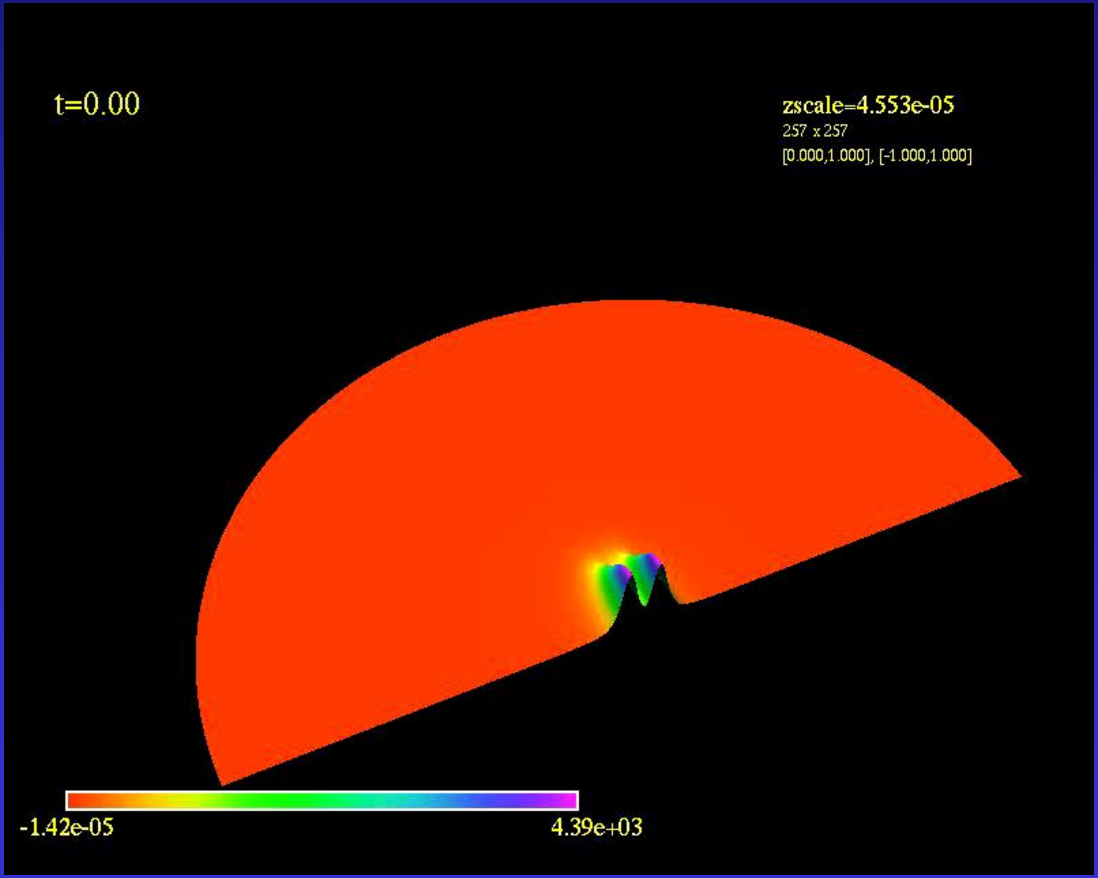


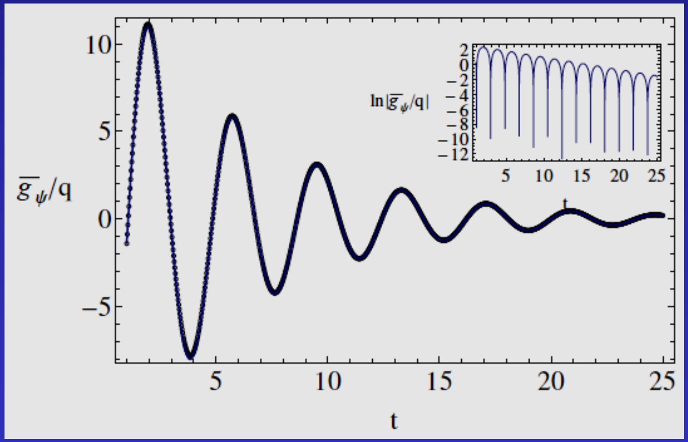
r_H=5.0; k=0

Quasi-normal modes of AAdS Black Holes

- form a distorted BH via asymmetric scalar field collapse

$$\overline{\Phi}(\rho,\chi,t=0) = Ae^{-\frac{\rho^2\cos^2\chi}{w_x^2}-\frac{\rho^2\sin^2\chi}{w_y^2}}$$





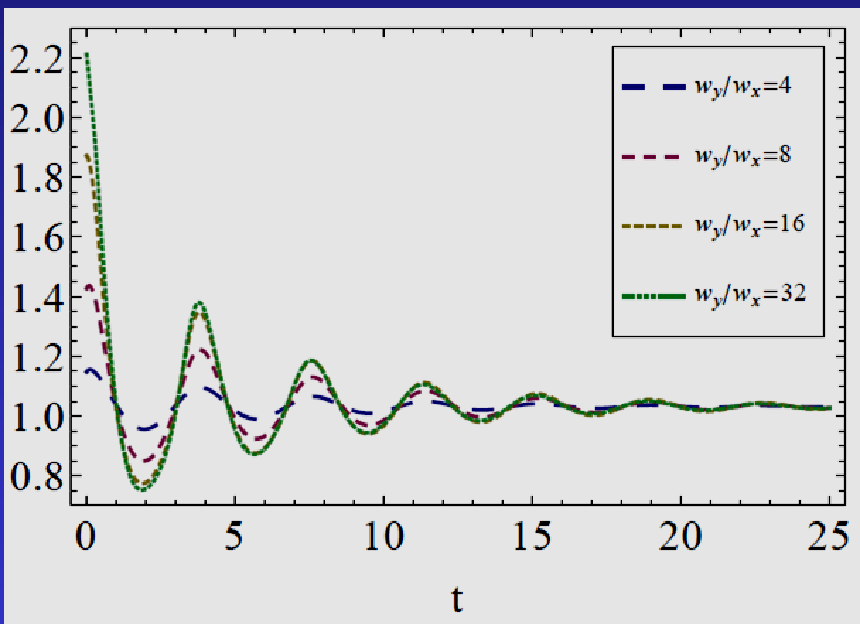
r_H=5.0; k=2

$$\overline{\Phi}(\rho,\chi,t)$$

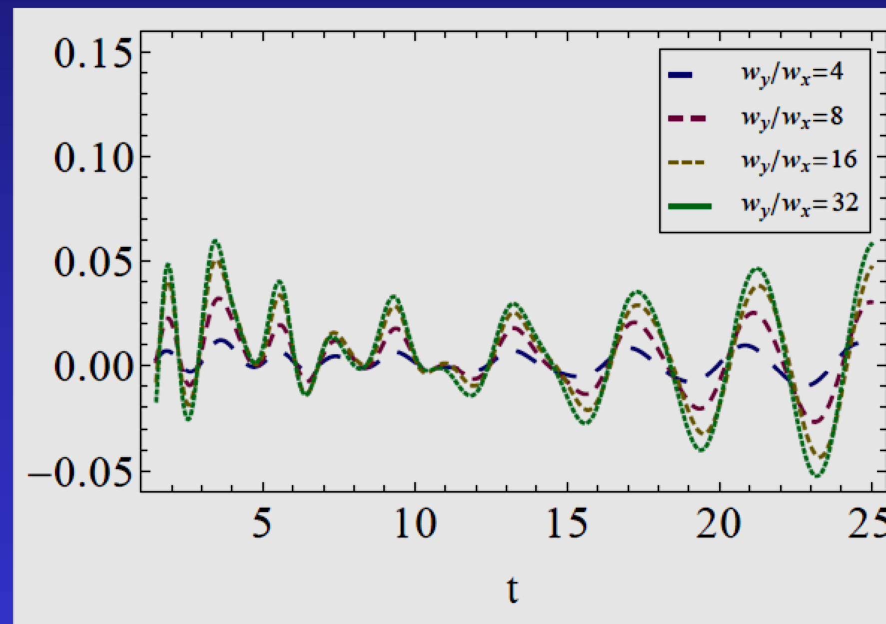
r_H=12.2

Quasi-normal modes of AAdS Black Holes

- Can quite accurately describe the metric perturbation purely in terms of the dominant QN mode, despite starting with initial data representing highly distorted BH's.



ratio of equatorial to polar circumference of AH for increasingly asymmetric ID



"residual" of QN mode; i.e. normalized difference between the actual metric behavior and extracted QN mode

Boundary stress energy

- The AdS/CFT dictionary says

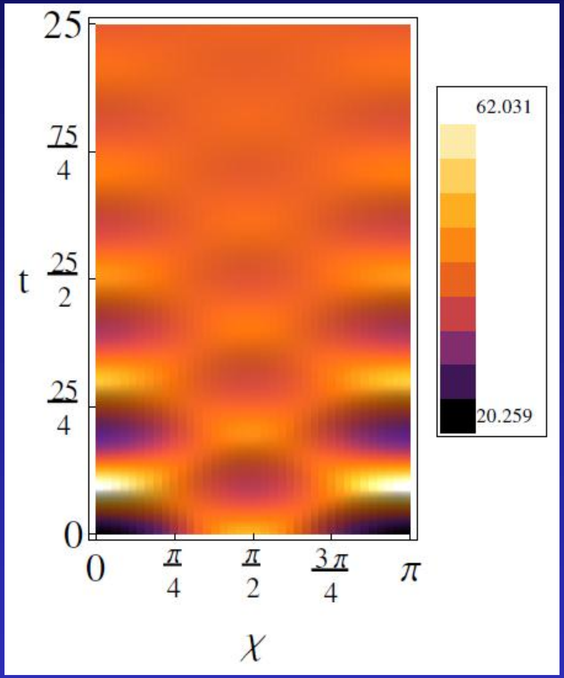
$$\langle T_{\mu\nu} \rangle_{CFT} = \lim_{q \to 0} \frac{1}{q^2} \left({}^{(q)}T_{\mu\nu} - {}^{(q)}T^{ADS}_{\mu\nu} \right)$$

where ${}^{(q)}T_{uv}$ is the Brown-York quasi-local stress energy tensor associated with a q=const. surface (with intrinsic metric Σ_{uv}, extrinsic curvature K_{uv}, and intrinsic Einstein tensor G_{uv})

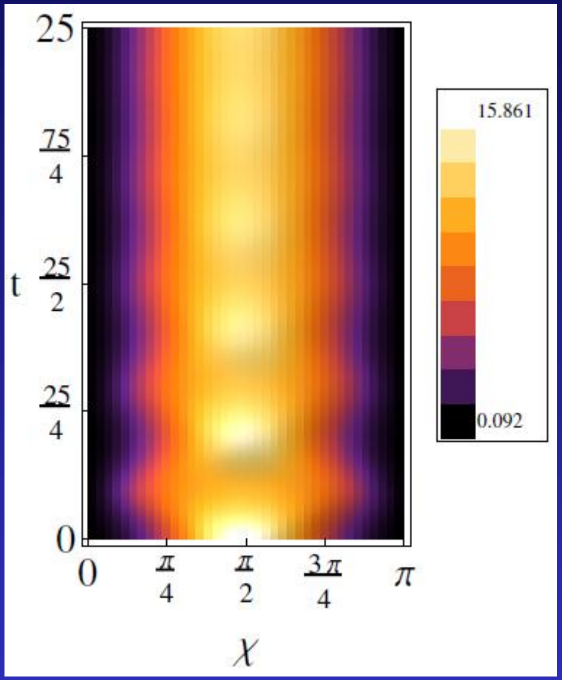
$${}^{(q)}T_{\mu\nu} = \frac{1}{8\pi} \left({}^{(q)}K_{\mu\nu} - \left({}^{(q)}K - \frac{3}{L} \right) \Sigma_{\mu\nu} + {}^{(q)}G_{\mu\nu} \frac{L}{2} \right)$$

and we have subtracted off the AdS Casimir term (arising due to the chosen S^3 topology)

Boundary stress energy



$<T_{tt}>, \ r_H=5.0$



$<T_{xx}>, \ r_H=5.0$

- For reference, the AdS-Schwarzschild solution describes a thermal state on S^3 with ($L=1$):

$$T_{ab} \approx \frac{r_H^{\ 4}}{16\pi} \cdot \mathrm{diag}\left[3,1,\sin^2\chi, \sin^2\chi \sin^2\theta\right]$$

Is the Stress Tensor on the Boundary that of a Conformal Fluid?

- To answer this question, will attempt to map the extracted boundary stress tenser to that of a conformal fluid, which in a derivative expansion is

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu} - 2\eta\sigma_{\mu\nu} + \Pi_{\mu\nu}$$

u_v is the fluid 4-velocity

ρ, P and η are the rest-frame energy density, pressure and shear viscosity respectively

σ_{uv} is the shear tensor of the flow

Π_{uv} denotes all higher derivative terms of the velocity field.

Hydrodynamics on the boundary

- With SO(3) symmetry

  $u^{\nu}=\gamma(1,v,0,0)$

  in (t,χ,θ,φ) coordinates, and $\sigma_{\mu\nu}$ has only one independent component

$$\sigma_{\chi\chi} \equiv \sigma$$
$$\sigma_{tt} = v^2\sigma$$
$$\sigma_{t\chi} = -v\sigma$$
$$\sigma_{\theta\theta} = \frac{\sigma_{\phi\phi}}{\sin^2\theta} = -\frac{\sin^2\chi}{2\gamma^2}\sigma$$

- Likewise, with SO(3), there are only 4 independent components of the stress tensor extracted on the boundary of the AAdS spacetime

$$\varepsilon_t \equiv {}^{(q)}T_{tt}$$
$$P_\chi \equiv {}^{(q)}T_{\chi\chi}$$
$$P_\Omega \equiv \frac{{}^{(q)}T_{\theta\theta}}{g_{\theta\theta}} = \frac{{}^{(q)}T_{\varphi\varphi}}{g_{\varphi\varphi}}$$
$$q_\chi \equiv -{}^{(q)}T_{t\chi}$$

Hydrodynamics on the boundary

- Thus, ignoring the higher order terms Π_{uv} , there is a one-to-one mapping between "hydrodynamic" and extracted stress tensor variables

$$\left(\varepsilon_t, P_\chi, P_\Omega, q_\chi\right) \rightarrow \left(\rho, P, v, [\eta\sigma]\right)$$

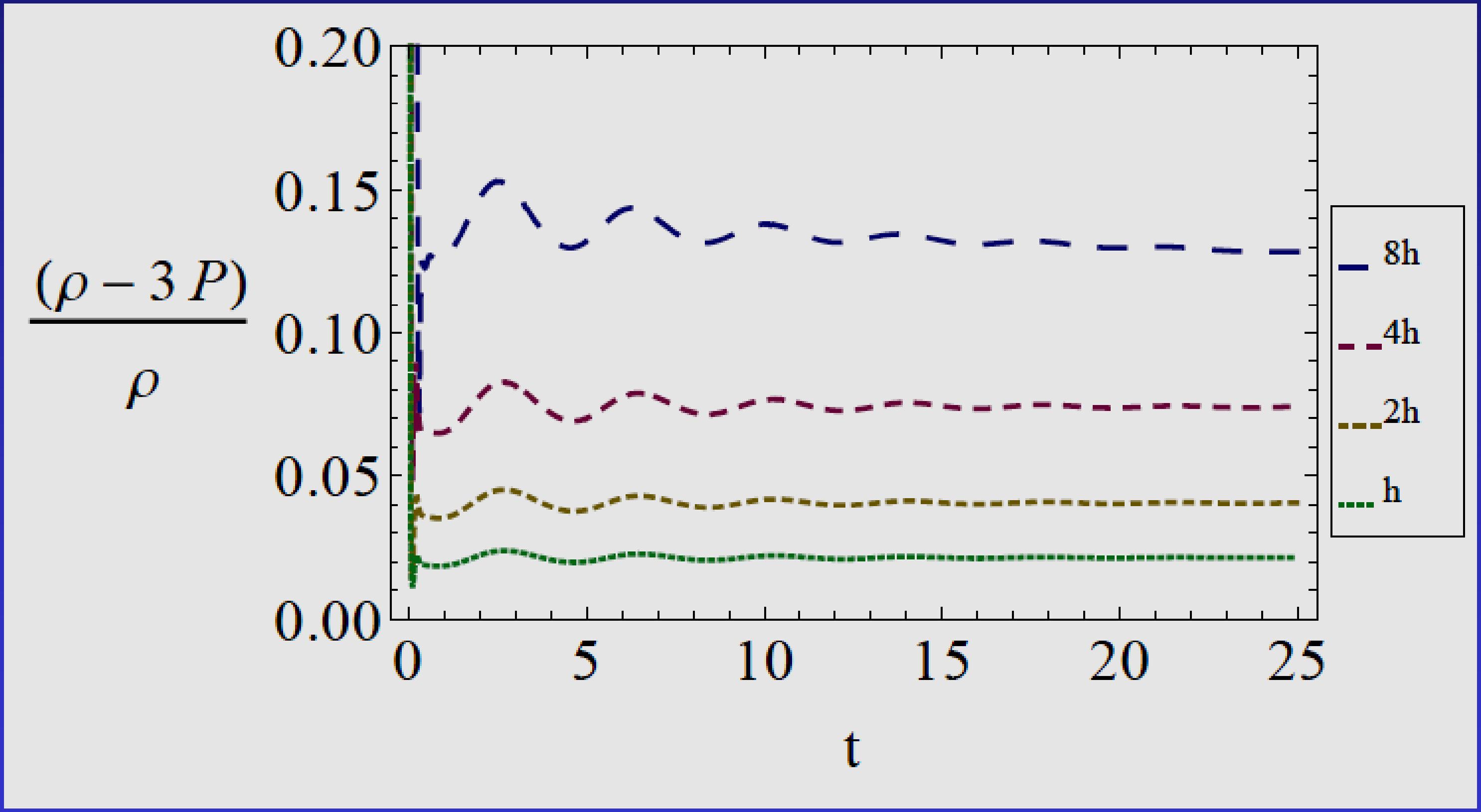
  though it's a different question whether the mapping gives "sensible" hydrodynamics

- For the QN spacetimes we have looked at so far, we find
  - $\rho > 0$
  - $P > 0$
  - $v \in [-1,1]$
  - $T^u{}_u = 0$ and $T^{uv}{}_{;v} = 0$ to within truncation error

- i.e., consistent with some conformal fluid satisfying the Navier-Stokes equations

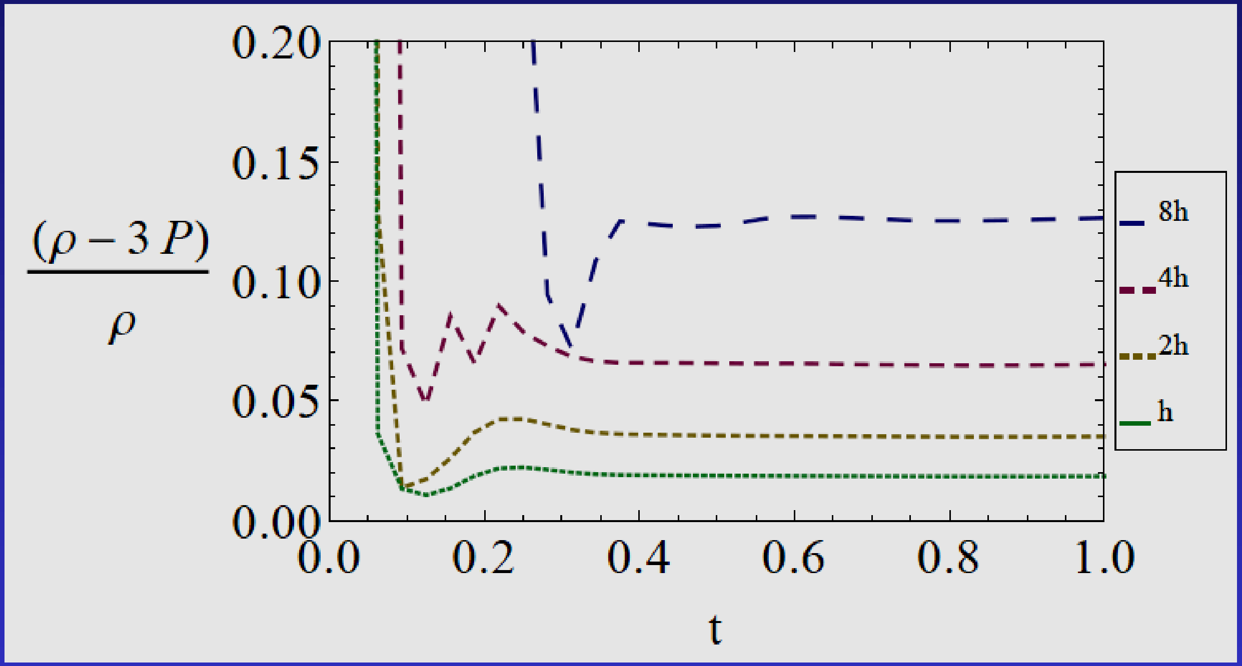
Boundary Fluid Equation of State

- Moreover, modulo a possible early time "transient", the equation of state of the fluid is converging to that of an isotropic conformal fluid, $\rho=3P$:



Boundary Fluid Equation of State

- Close-up of transient:



- For this case (r_h=*5.0*), the e-folding time of the "fast, microscopic" QNM is ~ 0.075; could be that this is causing the non-conformal fluid behavior, though early time transients from rapid gauge dynamics at the boundary make a clean interpretation of this challenging

Boundary Fluid Shear Viscosity to Entropy Ratio

- To check whether the shear viscosity to entropy ratio is $1/4\pi$ as expected for a conformal gauge theory dual to Einstein gravity *[Polcastro, Son and Starinets, PRL 87(2001)]*, we need to make a couple of extra assumptions
  - that the extracted u^α *is* the velocity field of the fluid, and hence we can plug it into the definition of the shear tensor to independently calculate σ:

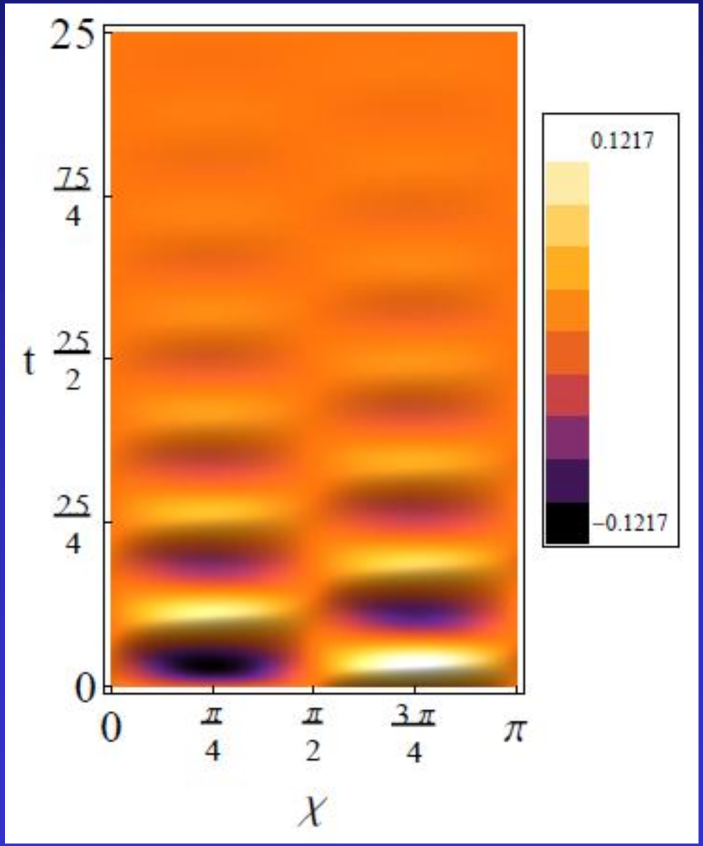
$$\sigma^{\mu\nu} = \perp^{\mu\alpha}\perp^{\nu\beta}\nabla_{(\alpha}u_{\beta)} - \frac{1}{4}\nabla_\alpha u^\alpha \perp^{\alpha\beta}$$

$$\perp^{\alpha\beta} = g^{\alpha\beta} + u^\alpha u^\beta$$

Define $\sigma_v = \sigma$ to computed using the extracted u^α in the above; then

$$\bar{\eta} \equiv \frac{^{(q)}[\eta\sigma]}{\sigma_v}$$

This quantity will only be close η where σ_{uv} is large compared to the higher order terms Π_{uv} ignored in the mapping



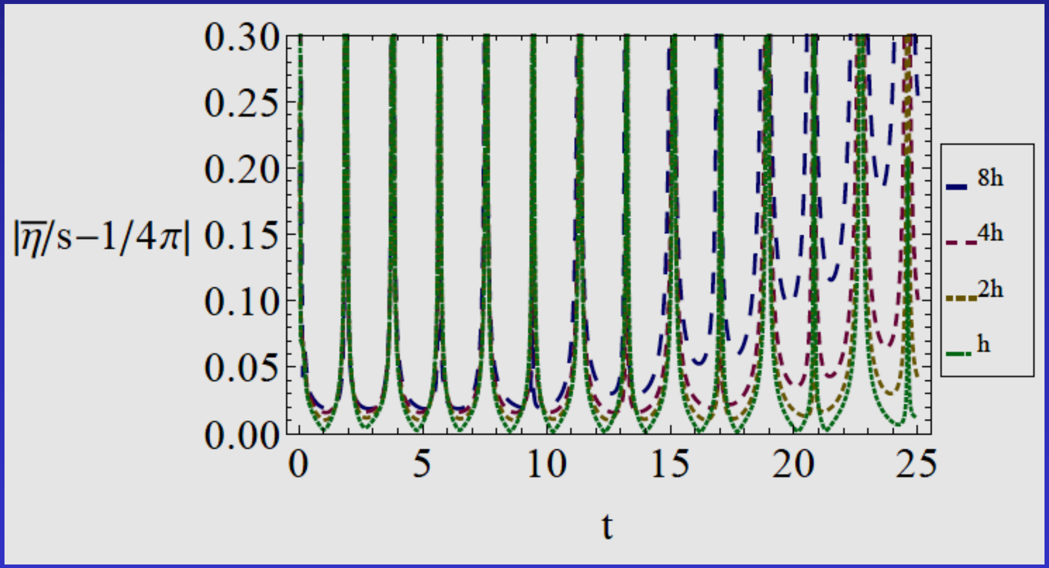
v, r_H=5.0

Boundary Fluid Shear Viscosity to Entropy Ratio

- The second assumption is that the constituent relationship for a thermal fluid holds, to relate ρ to the entropy density s

$$\rho = \frac{3\pi^3}{16} T^4; \; s = \frac{4\pi^3}{16} T^3$$

- Then



$\nu r_H = 5.0$

i.e., when σ_v is large, we appear to be converging to the expected value