

Full Numerical Simulation of Black Holes

Carlos Lousto

In collaboration with M.Campanelli, H.Nakano, M.Ponce, and Y.Zlochower.

Viz: H.-P.Bischof

Center for Computational Relativity and Gravitation

<http://ccrg.rit.edu>

Rochester Institute of Technology



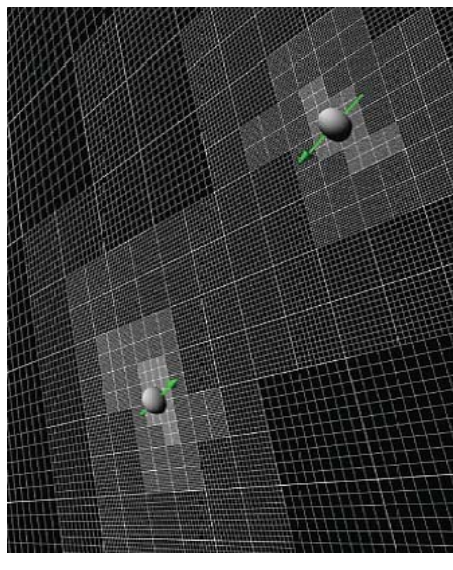
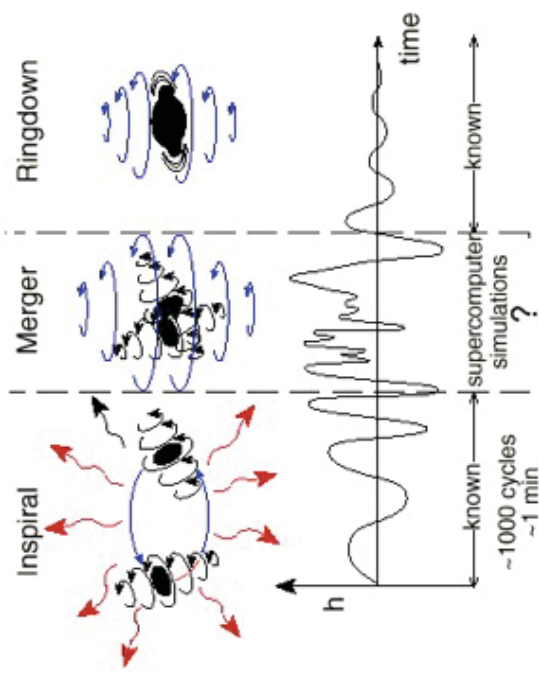
Madeira, September 2nd, 2011

Overview

- Brief History of NR breakthroughs
- BBH dynamics and astrophysical recoils
- Extensions of BBHs techniques
 - Exploring the extremes of BBH parameters
 - Beyond BBH problem
- Discussion of HE $\leftrightarrow$ NR

Simulation of Black-Hole Binaries with Full Numerical General Relativity

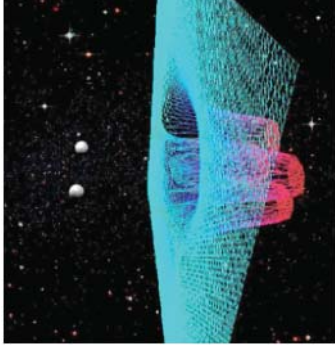
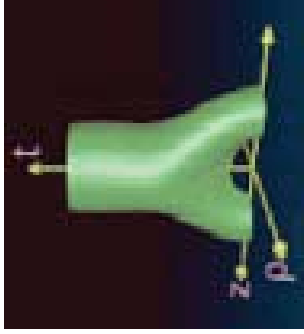
- Numerical Relativity:
 - Solve numerically the full GR eqs for dynamical spacetime, in strong gravity where no approximations hold.
 - Very difficult problem ...
- Goals:
 - Understand gravity at its strongest manifestation
 - Inform gravitational wave detection
 - Determine characteristics of compact objects (final black hole)



- Challenges:
 - Several scales required for physics
 - Mass of the smallest BH, BH spins, etc
 - Wavelength of the waveform in the wave zone
 - Long waveforms matching early PN inspiral
 - Large parameter space: mass ratio, spins, eccentricity ...

Credits: NASA GSFC group, 2007

A Brief Historical Overview



40+ years of hard labor:

1964 First Simulation (Hahn & Lindquist)

... then **LIGO** ...

1990s Grand Challenge

BSSN-NOK evolution system

Puncture Initial Data (Brandt-Bruegman)

Gauge: Fixed Punctures (Alcubierre et al.)

Lazarus (Campanelli et al)

2004 One Orbit (Buegman et al.)

Breakthrough:

2005 Binary Inspiral and Merger

Pretorius, PRL 95 (2005)

2006 Moving Punctures (RIT & NASA)

Campanelli et al PRL 96 (2006)

Baker et al PRL 96, (2006)

Numerical Relativity today:

2006+ GW Waveforms & Orbits,

Spin dynamics, Mass ratios,

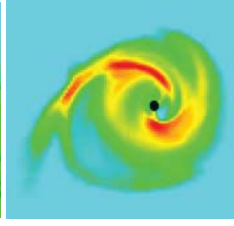
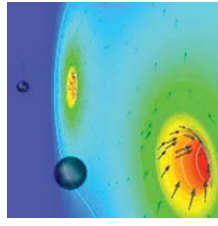
GW Recoils, BH remnants,

BHs multiplets

2009+ Community Collaborations

2010+ Extreme BH Binaries

BH Binaries in a gaseous environment



Spectral Einstein Code (SpEC):

Generalized Harmonic, but 1st order

Physical BCs

Highly-accurate, but less flexible (care

needed to get BH-BH merger)

Extended to GRMHD (BH-NS, Duez)

Moving Puncture Codes:

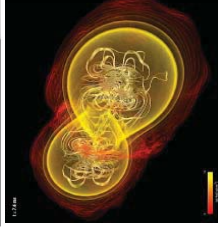
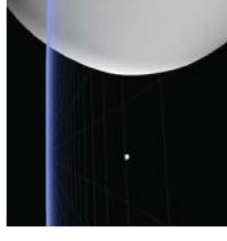
BSSN + Punctures, AMR

Less-accurate, but more flexible and

robust (NBBH -BH/NS mergers)

Community Codes, including GRMHD

(<http://einstein toolkit.org>)



F1

Rally

The conformal BSSN system with moving punctures

Modified BSSN system:

$$\partial_0 \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}, \quad \partial_0 = \partial_t - \mathcal{L}_\beta,$$

$$\partial_t \chi = \frac{2}{3} \chi (\alpha K - \partial_a \beta^a) + \beta^i \partial_i \chi,$$

$$\partial_0 \tilde{A}_{ij} = \chi (-D_i D_j \alpha + \alpha R_{ij})^{TF} + \alpha (K \tilde{A}_{ij} - 2 \tilde{A}_{ik} \tilde{A}_j^k),$$

$$\partial_0 K = -D^i D_i \alpha + \alpha \left(\tilde{A}_{ij} \tilde{A}^{ij} + \frac{1}{3} K^2 \right),$$

$$\partial_t \tilde{\Gamma}^i = \tilde{\gamma}^{jk} \partial_j \partial_k \beta^i + \frac{1}{3} \tilde{\gamma}^{ij} \partial_j \partial_k \beta^k + \beta^j \partial_j \tilde{\Gamma}^i -$$

$$\tilde{\Gamma}^j \partial_j \beta^i + \frac{2}{3} \tilde{\Gamma}^i \partial_j \beta^j - 2 \tilde{A}^{ij} \partial_j \alpha +$$

$$2\alpha \left(\tilde{\Gamma}^i{}_{jk} \tilde{A}^{jk} + 6 \tilde{A}^{ij} \partial_j \phi - \frac{2}{3} \tilde{\gamma}^{ij} \partial_j K \right),$$

$$\tilde{\Gamma}^i = -\partial_j \tilde{\gamma}^{ij}.$$

Replace $\phi (O(\log r))$ with $\chi = e^{-4\phi} (O(r^4))$

$$\partial_0 \alpha = -2\alpha K$$

$$\partial_t \beta^a = B^a, \quad \partial_t B^a = 3/4 \partial_t \tilde{\Gamma}^a - \eta B^a$$

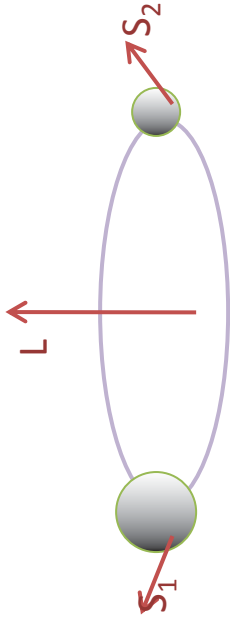
$$\alpha(t=0) = \psi_{BL}^{-2} \quad \beta^i = B^i = 0.$$

Numerical Code: LazEv

- Modular
 - Cactus-based framework
- Flexible
 - *Mathematica* scripts used to generate C routines (257108 lines)
- Use 8th order finite differencing in space with MoL integration
 - standard 8th order centered stencils for all spatial derivatives
 - upwinded 4th order stencils for the advection (shift) terms
 - standard 4th order RK for time evolution

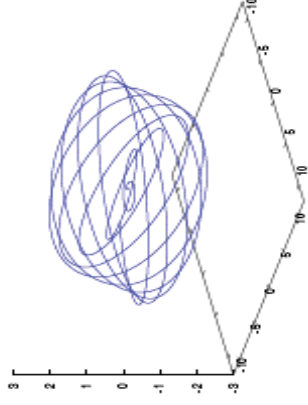
Exploring BH Merger Spin Dynamics

- Merger of a generic, precessing BH binary [Campanelli, Lousto, Nakano and Zlochower, PRD 2009]

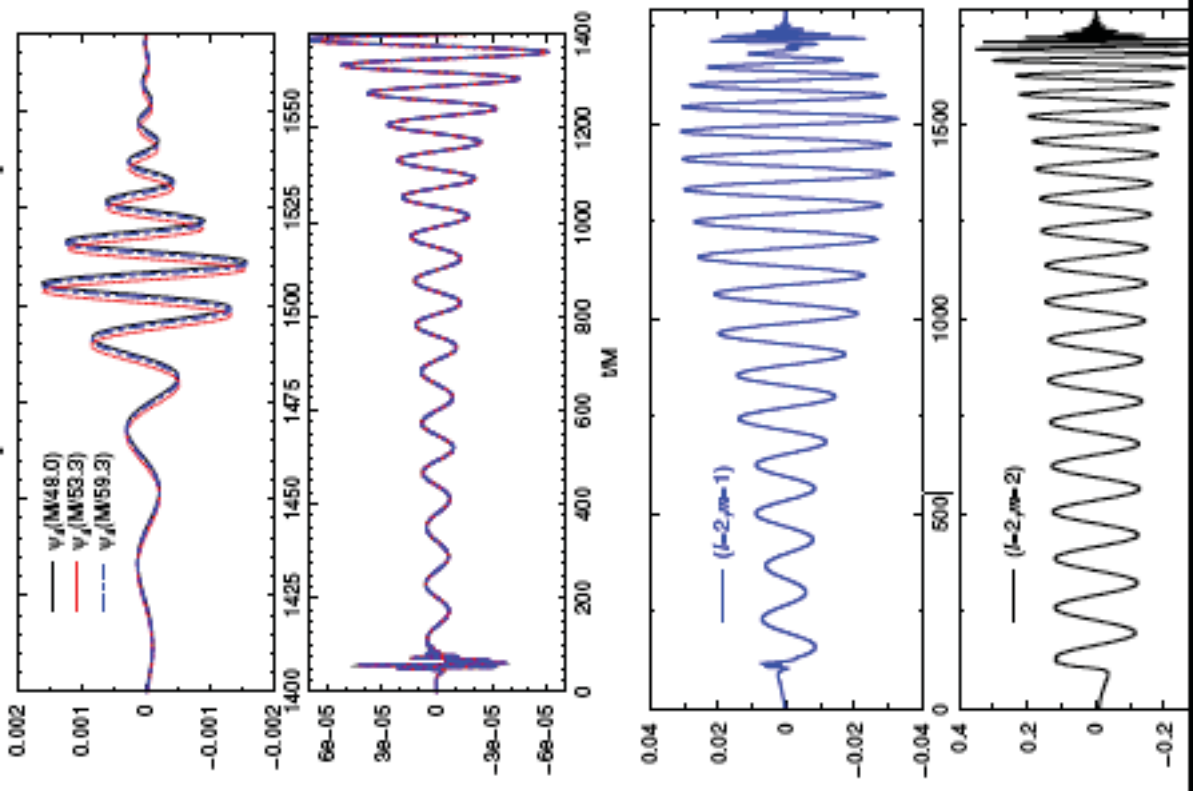


- Random Spin, non-equal mass, small eccentricity

- $m_1/m_2 \sim 0.8$, $a_1/m_1 \sim 0.6$, $a_2/m_2 \sim 0.4$
- spins initially at arbitrary orientations
- complete 9 orbits prior merger
- initial parameters from 3.5PN inspiral evolution (> 300 orbits)



Trajectory difference $\underline{r} = \underline{x}_1 - \underline{x}_2$



- These Simulations are Computationally Challenging!
- Large simulations .. 1000s of CPUs, many TBs of data
- Need highly-efficient AMR (very different length scales)
- Need high accuracy to get small BH dynamics
- Spinning data requires much higher resolutions

Merger of Spinning Black Holes: Hang-Up Orbits

-- Track

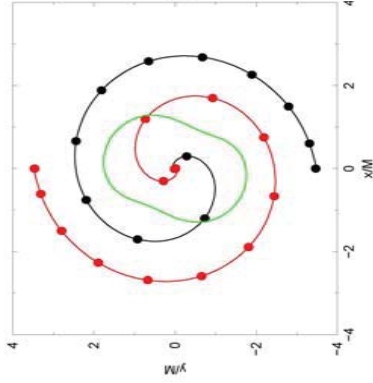


Figure 4: Puncture tracks for the -- configuration.

00 Track

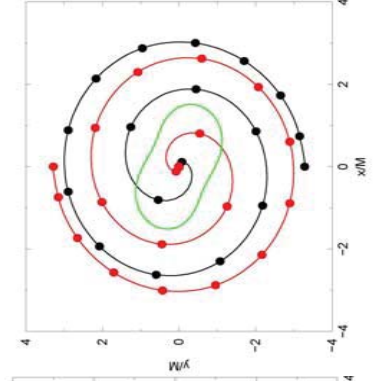


Figure 5: Puncture tracks for the 00 configuration.

++ Track

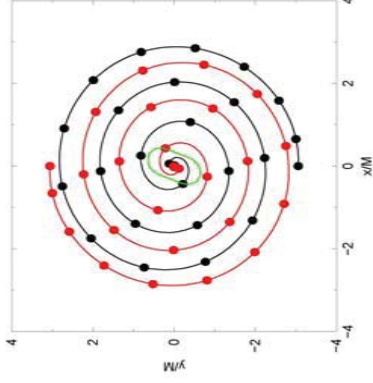
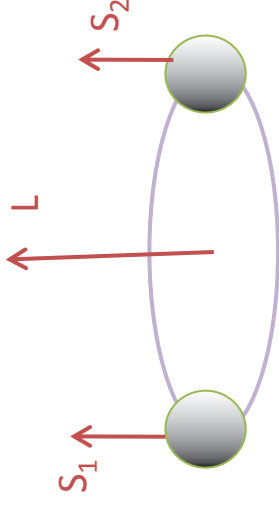
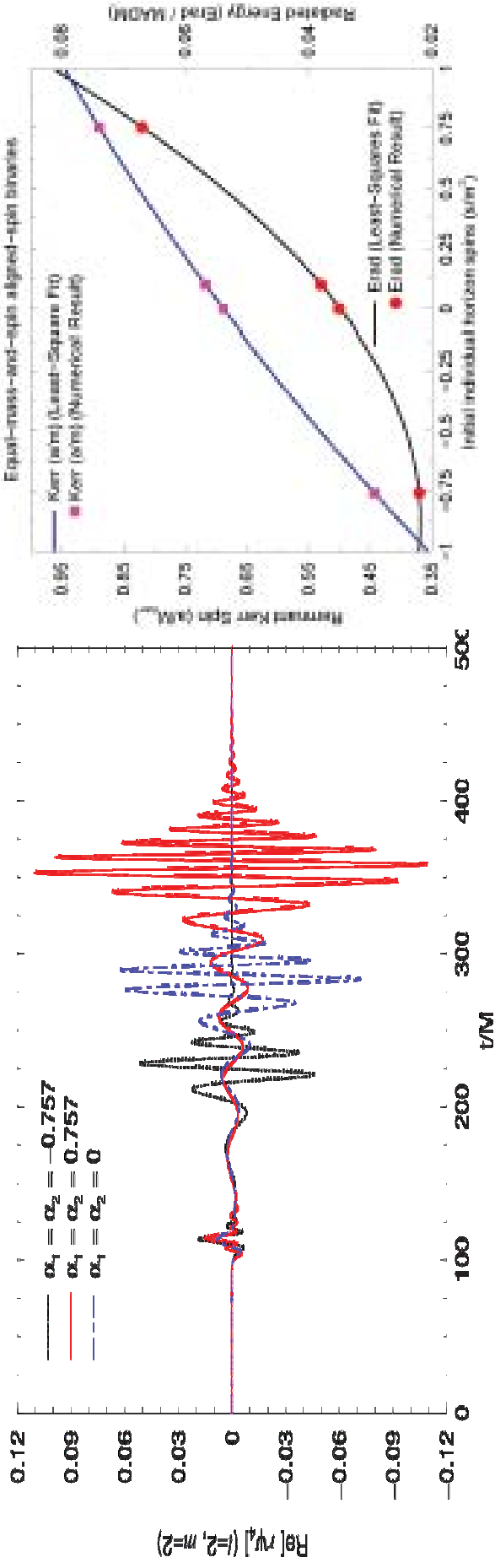


Figure 6: Puncture tracks for the ++ configuration.



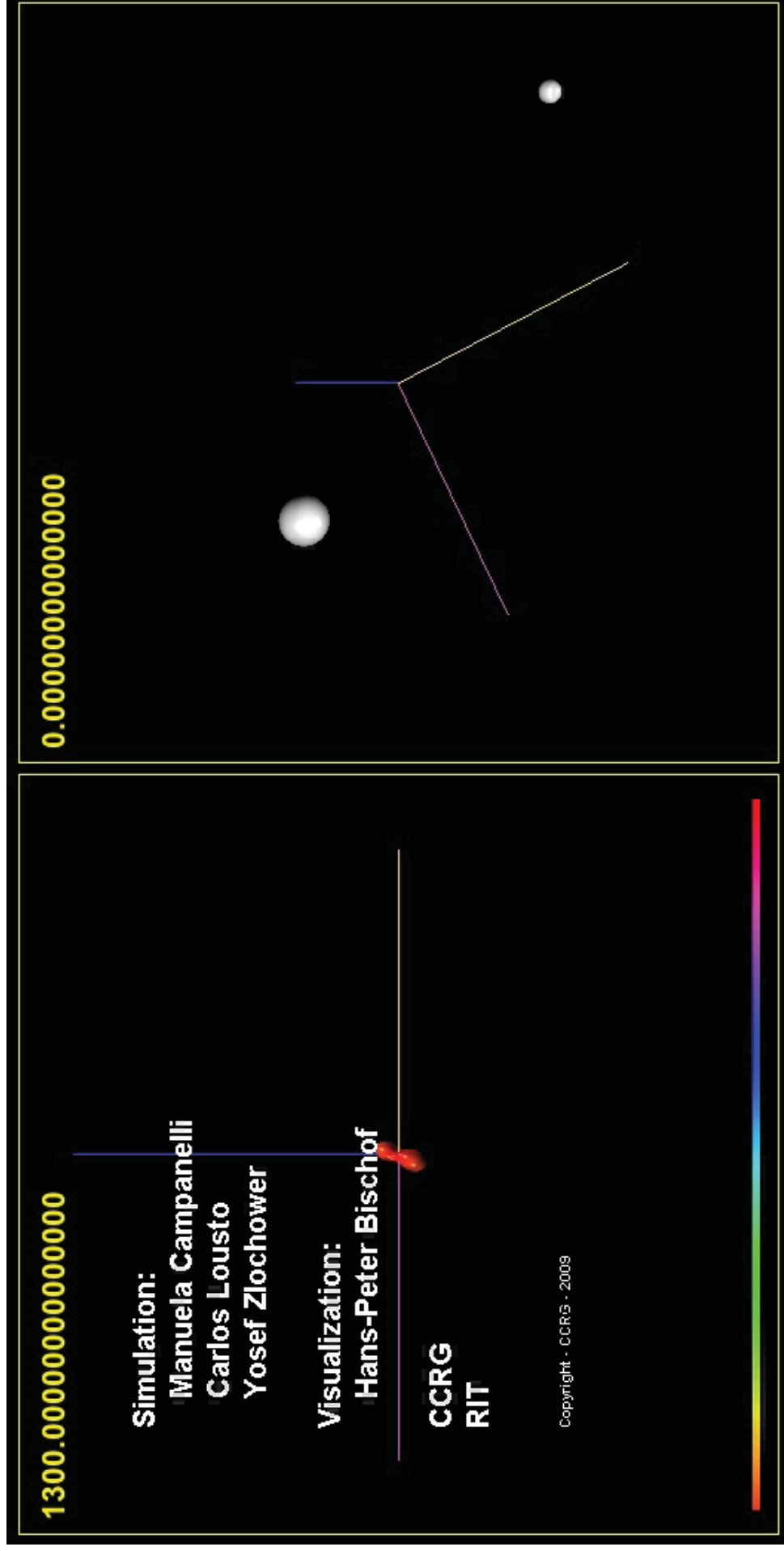
- Equal-mass BHBs
- $S_{1,2}/m^2 = \{-0.75, 0, 0.75\}$
- $M\Omega = 0.05$ (Starting "radius" the same).

- Hang-up effect due to repulsive spin-orbit interaction leaving behind a remnant with sub-maximal spin < 0.96 [Campanelli, Lousto, Zlochower, PRD 2006]: cosmic censorship respected!



Kick from a Generic Astrophysical BH binary

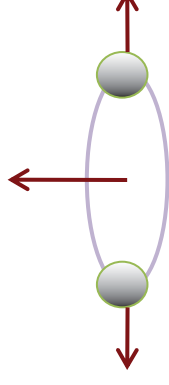
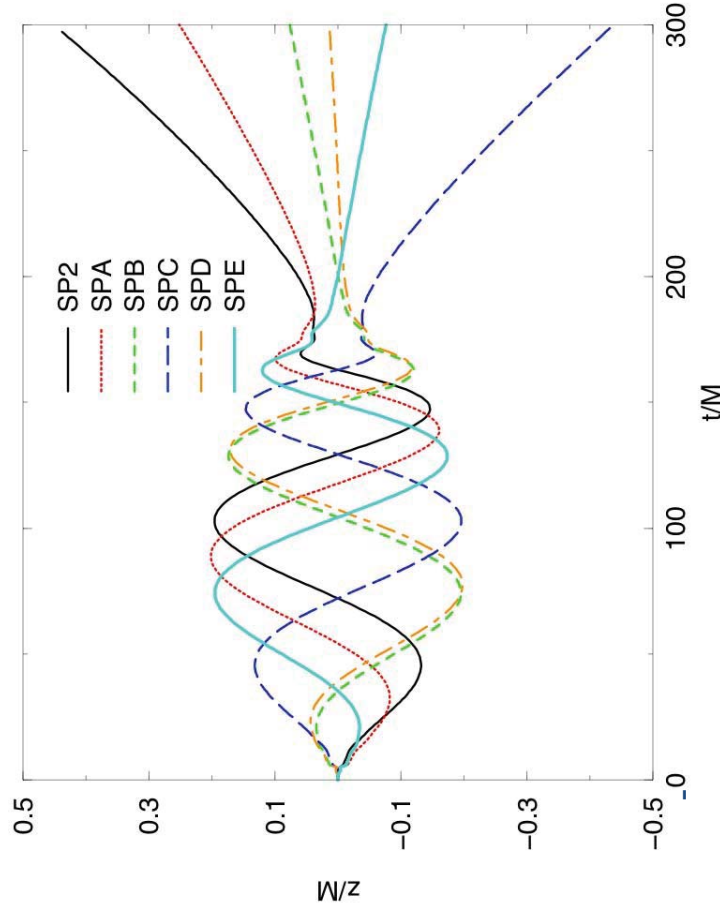
- In astrophysical, quasi-circular, spinning BH binaries mergers, such kicks can be as large as 3,680 km/s [Campanelli et al, ApJL 2007, Campanelli et al, PRL 2007, Lousto & Zlochower, PRD, 2010]



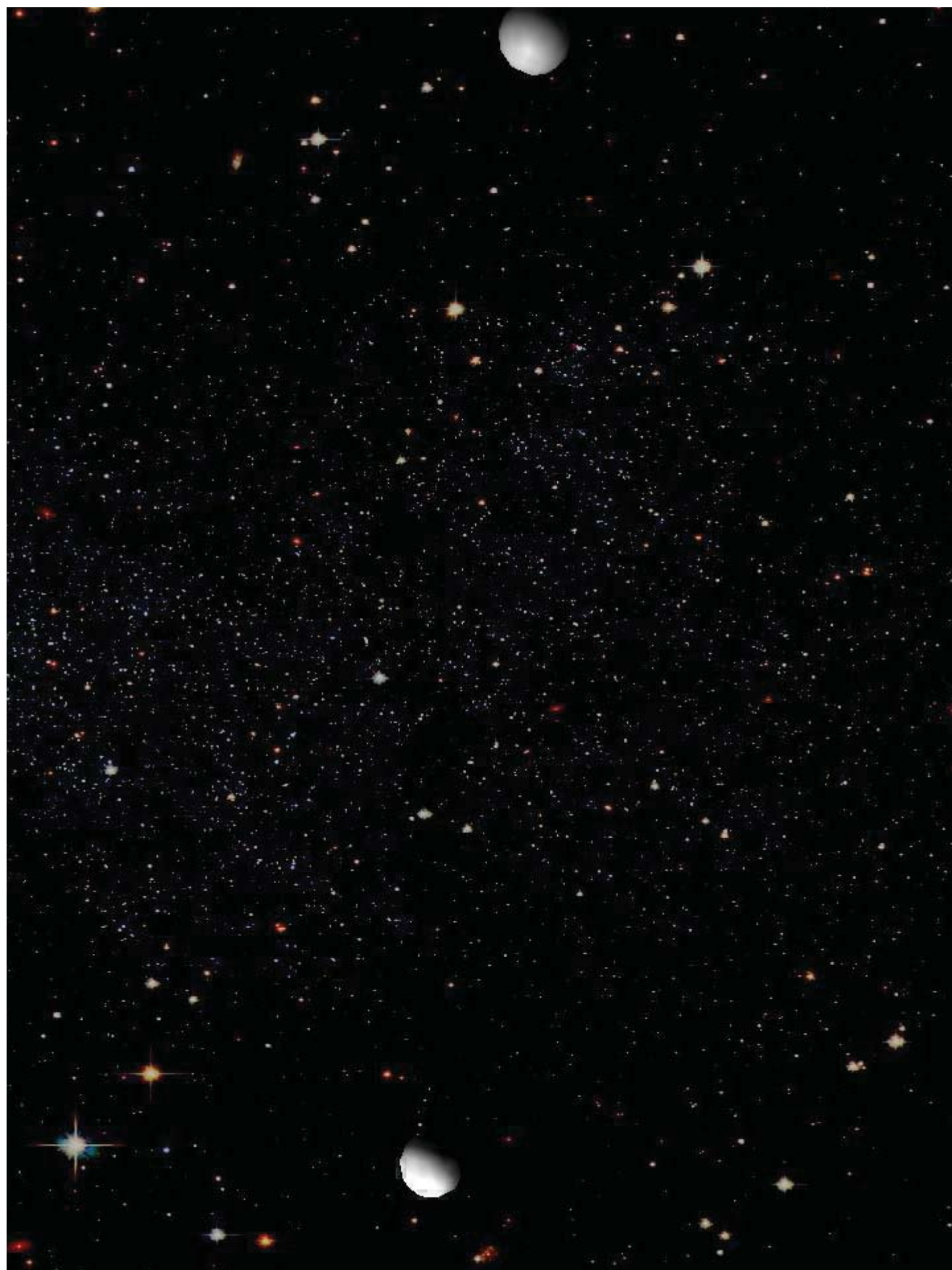
Radiated power (Left), Orbit and Kick (Right)

The Superkick Configurations

- In quasi-circular BH binaries, the largest kicks are obtained when BH spins within the orbital plane, equal in magnitude, but opposite in direction
- The resultant recoil velocity depends sinusoidally on the initial phase of the binary, and linearly (at leading order) on the magnitude of the individual BH spins [Campanelli et al, 2007]
- The "bobbing" motion of the orbit causes an asymmetric beaming of the radiation produced by the in-plane orbital motion of the binary, and the net asymmetry is balanced by a recoil [Sperhake et al, 2010]



- Schematic explanations: Effect due to frame dragging of one BH relative to the other and viceversa [Pretorius, 2008]; spin-curvature coupling [Keppel et al, 2009]; kinematical spin effect [Gralla et al, 2010]; tendexes and vortexes [Owen et al, 2011].



An empirical Formula for the merger kick

Empirical formula [Campanelli et al '07] for the radiation recoil of generic binary black-hole mergers originally motivated by PN formula [Kidder 1995]

$$\vec{V}_{\text{recoil}}(q, \vec{a}_i) = v_m \hat{e}_1 + v_{\perp} (\cos(\xi) \hat{e}_1 + \sin(\xi) \hat{e}_2) + v_{\parallel} \hat{e}_z,$$

$$q = m_1/m_2, \quad \eta = q/(1+q)^2, \quad \vec{a}_i = \vec{S}_i/m_i^2$$

$$v_m = A\eta^2 \sqrt{1 - 4\eta(1 + B\eta)}$$

in-plane kick < 175 km/s [Fitchett '83, Gonzalez et al 07]

$$v_{\perp} = H \frac{\eta^2}{(1+q)} \left(a_2^{\parallel} - q a_1^{\parallel} \right)$$

in-plane kick < 500 km/s [Baker et al 07, Hermann et al '07, Koppitz et al '07]. See [Pollney et al 2007] for quadratic corrections in the spins. Also, work in progress by RIT ...

$$v_{\parallel} = K \frac{\eta^2}{(1+q)} \cos(\Theta - \Theta_0) \left| \vec{a}_2^{\perp} - q \vec{a}_1^{\perp} \right|$$

out-of-plane kick < 4,000 km/s [Campanelli et al 07, Lousto et al '08]. See also [Baker et al '08] for a modification of mass scaling ($\eta^2 \rightarrow 4\eta^3$) ...

- ζ angle between unequal-mass and spin contributions to recoil in the orbital plane
- Θ angle between in-plane $\vec{\Delta} \equiv (m_1 + m_2)(\vec{S}_2/m_2 - \vec{S}_1/m_1)$ and infall direction at merger

Remnant BH masses and spins

What is the cosmological distribution of masses and spins? (related to distribution of recoils)

What is the final mass and spins from generic mergers?

Is cosmic-censorship obeyed?

- Models of remnant BH masses and spins:

- Early efforts via Lazarus [Baker et al. 2004]

- Empirical fitting models [Campanelli et al. 2006;

Rezzolla et al. 2009; Tichy & Marronetti 2008,

Healy et al, 2009]

- Physically motivated models with fitting functions

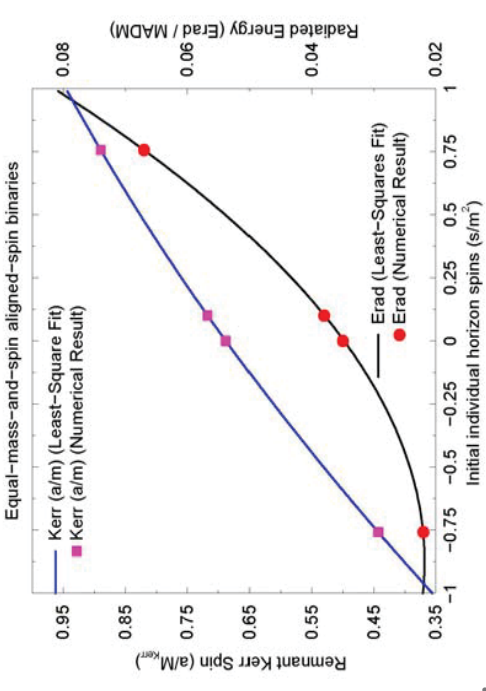
[Boyle & Kesden 2008, Buonanno et al. 2008, Kesden 2008, .

- Empirical formulas based on PN-inspired parametrization

[Lousto, Campanelli & Zlochower, arXiv: 0904.3541]:

$$\delta M/M = \eta \tilde{E}_{ISCO} + E_2 \eta^2 + E_3 \eta^3 + \frac{\eta^2}{(1+q)^2} \left\{ E_S (\alpha_2^\parallel + q^2 \alpha_1^\parallel) + E_\Delta (1-q) (\alpha_2^\parallel - q \alpha_1^\parallel) + E_A |\vec{\alpha}_2 + q \vec{\alpha}_1|^2 + E_B |\alpha_2^\perp + q \alpha_1^\perp|^2 (\cos^2(\Theta_+ - \Theta_2) + E_C) + E_D |\vec{\alpha}_2 - q \vec{\alpha}_1|^2 + E_E |\alpha_2^\perp - q \alpha_1^\perp|^2 (\cos^2(\Theta_- - \Theta_3) + E_F) \right\}, \quad (2)$$

- **Merging equal-mass BH binaries, with maximal spins aligned with orbital angular momentum produce a remnant BH with final spin $a/M \sim 0.96 - 0.98$ no naked singularities so far!**



[Campanelli et al, PRD73, 2006]

$$\vec{\alpha}_{\text{final}} = (1 - \delta M/M)^{-2} \left\{ \eta \tilde{J}_{ISCO} + (J_2 \eta^2 + J_3 \eta^3) \hat{n}_\parallel + \frac{\eta^2}{(1+q)^2} \left(\left[J_A (\alpha_2^\parallel + q^2 \alpha_1^\parallel) + J_B (1-q) (\alpha_2^\parallel - q \alpha_1^\parallel) \right] \hat{n}_\parallel + (1-q) |\vec{\alpha}_2^\perp - q \vec{\alpha}_1^\perp| \sqrt{J_\Delta \cos[2(\Theta_\Delta - \Theta_4)]} + J_{M\Delta} \hat{n}_\perp + |\vec{\alpha}_2^\perp + q^2 \vec{\alpha}_1^\perp| \sqrt{J_S \cos[2(\Theta_S - \Theta_5)]} + J_{MS} \hat{n}_\perp \right) \right\}. \quad (4)$$

Super Hang up configuration

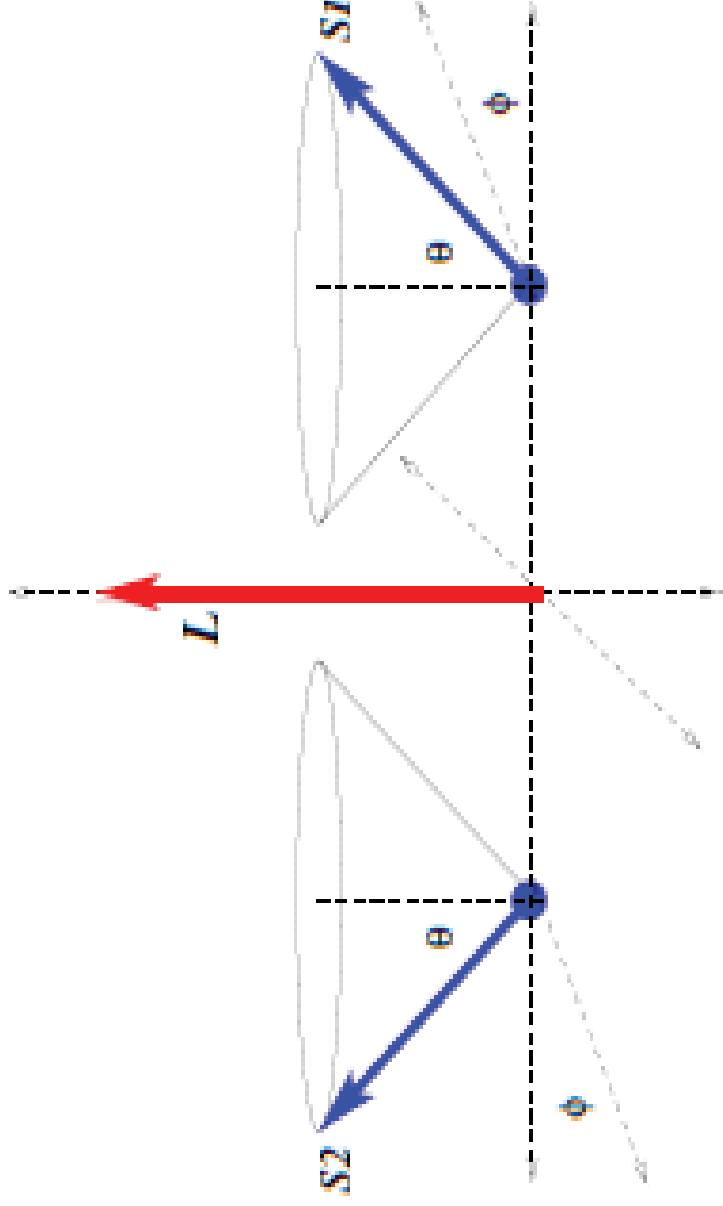
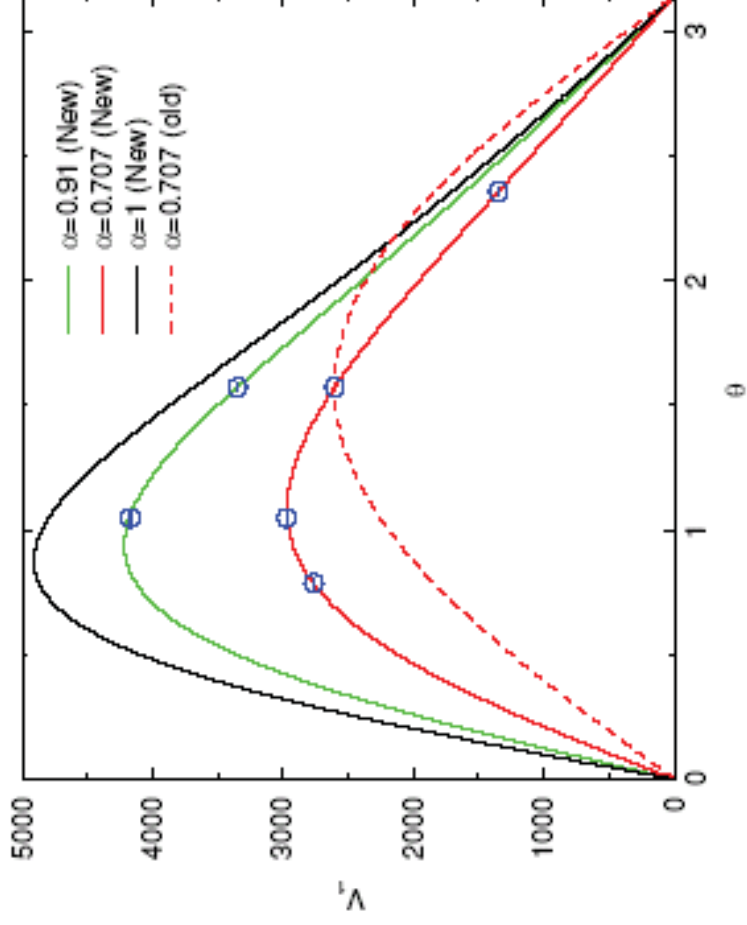


FIG. 1: Black-hole-binary configuration for hangup recoils.

Asymmetric recoils

FIG. 2: A fit of the recoil (V_1) to the form Eq. (3) for the $\alpha = 1/\sqrt{2}$ configurations, and predictions (based on this fitting) for the $\alpha = 0.91$ recoils. Note how well the $\alpha = 0.91$ curve matches the two measured values. For reference, curves corresponding to the old empirical formula prediction for $\alpha = 1/\sqrt{2}$ and the new formula for $\alpha = 1$ are also included. Note the skew in the velocity profile compared to the old predictions.



Statistical distribution with accretion

To test the effect of our new empirical formula on the predicted recoil rates, we consider a model distribution of BHBs with mass ratio distribution $P(q) \propto q^{-0.3}(1 - q)$ [30–32], spin distribution $P(\alpha) \propto (1 - \alpha)^{(b-1)}\alpha^{(a-1)}$, with parameters $a = 4.8808$ and $b = 1.72879$ (which models the distribution kindly provided by M. Volonteri [33] resulting from spin-up effects due to accretion), spin-direction distribution $P(\theta) \propto (1 - \theta)^{(b-1)}\theta^{(a-1)}$, with parameters $a = 2.5$ and $b = 7$ (which approximates the spin-direction distribution provided by M. Volonteri [33]),

$$v_{\parallel} = 16 \frac{\eta^2}{(1+q)} |\alpha_2^{\perp} - q\alpha_1^{\perp}| \times \left[V_{1,1} + A \left(\frac{2[\alpha_2^z + q^2\alpha_1^z]}{(1+q)^2} \right) + B \left(\frac{2[\alpha_2^z + q^2\alpha_1^z]}{(1+q)^2} \right)^2 + C \left(\frac{2[\alpha_2^z + q^2\alpha_1^z]}{(1+q)^2} \right)^3 \right] \cos(\phi_{\Delta} - \phi_1), \quad (4)$$

Results

FIG. 3: The recoil probability distribution using the new and old empirical formulas for the recoil, starting from a distribution of BHB configurations consistent with recent models for accreting binaries [33]. The new formula predicts a significantly larger probability for high recoils.

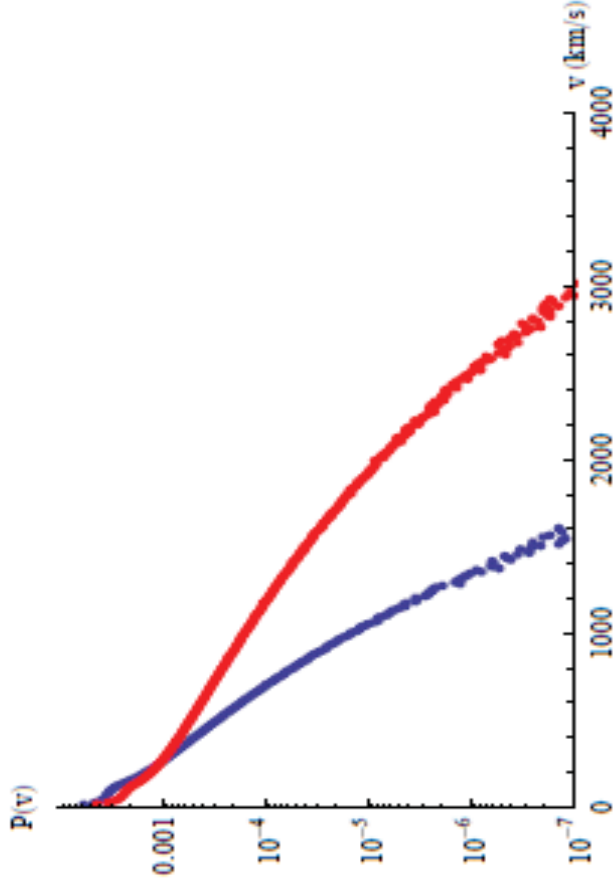


TABLE II: Probability that the recoil velocity will be in a given range P , and the probability that the recoil will be in a given range along the line of sight P_{obs} (to relate to possible redshift measurements in galaxies) for the new prediction (LEFT) and the old prediction (RIGHT)

range	P	P_{obs}	P old	P_{obs} old
0-500	79.027%	92.641%	94.888%	98.914%
500-1000	15.399%	6.177%	4.921%	1.067%
1000-2000	5.384%	1.164%	0.191%	0.019%
2000-3000	0.189%	0.018%	0	0
3000-4000	0.001%	0.0001%	0	0

Observational evidence

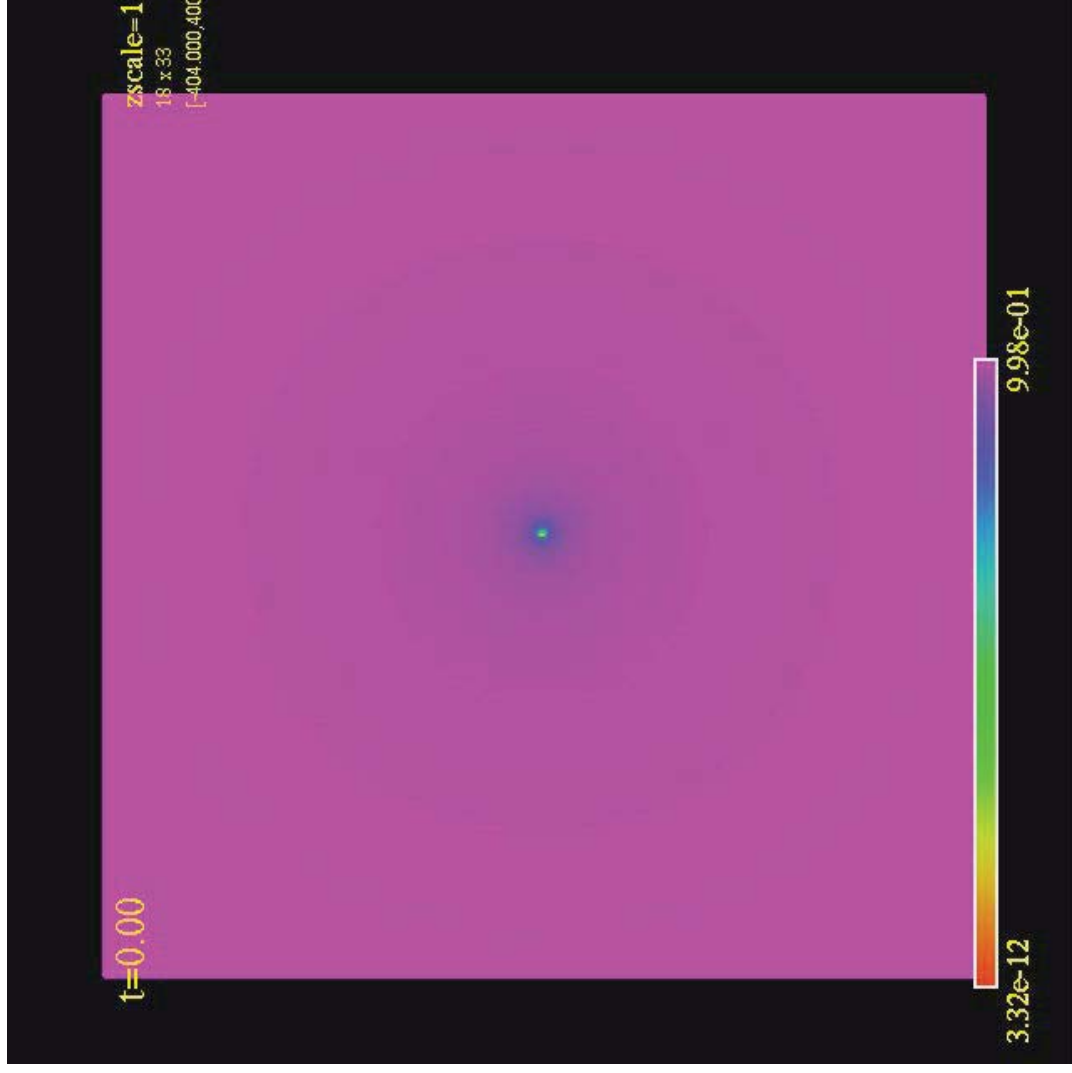
- Komosa et al (2008), Shield et al (2009), Civano et al (2010)
- Surveys
 - Eracleous et al (2011)
 - $DV > 1000 \text{ km/s}$. 88 objects, 68 spectra -14 binaries
 - Tsalmantza et al (2011)
 - SDSS, 32 objects -9 binaries

This observations could well be the first confirmation of highly dynamic, nonlinear (strong field), predictions of GR (NR)!

Exploring the extremes of the BBH parameter space

- High energy collision of BHs: Pretorius, Sperhake et al $\gamma > 1$
- Highly spinning black holes: Lovelace et al succeeded evolving 2BHs with spins=0.97 for 25 orbits.
- Small mass ratio: Two orbits from ISCO for $q=m_1/m_2=1/100$ (RIT group)

Simulation of 1:100 Black-Hole Binaries: Numerical Challenges



- Requires very large simulations on (up to) 1,024 processors
- AMR with 15 levels of refinement
- Use of perturbation theory (Regge-Wheeler-Zerilli potentials) to guide the setting up of the initial grids (AMR boundaries).
- Adapted gauge conditions
- Runs take 3 months

Grid Structure

[Lousto and Zlochower, Phys. Rev. Lett. arXiv: 1009.0292]

Results: Trajectories & Waveforms

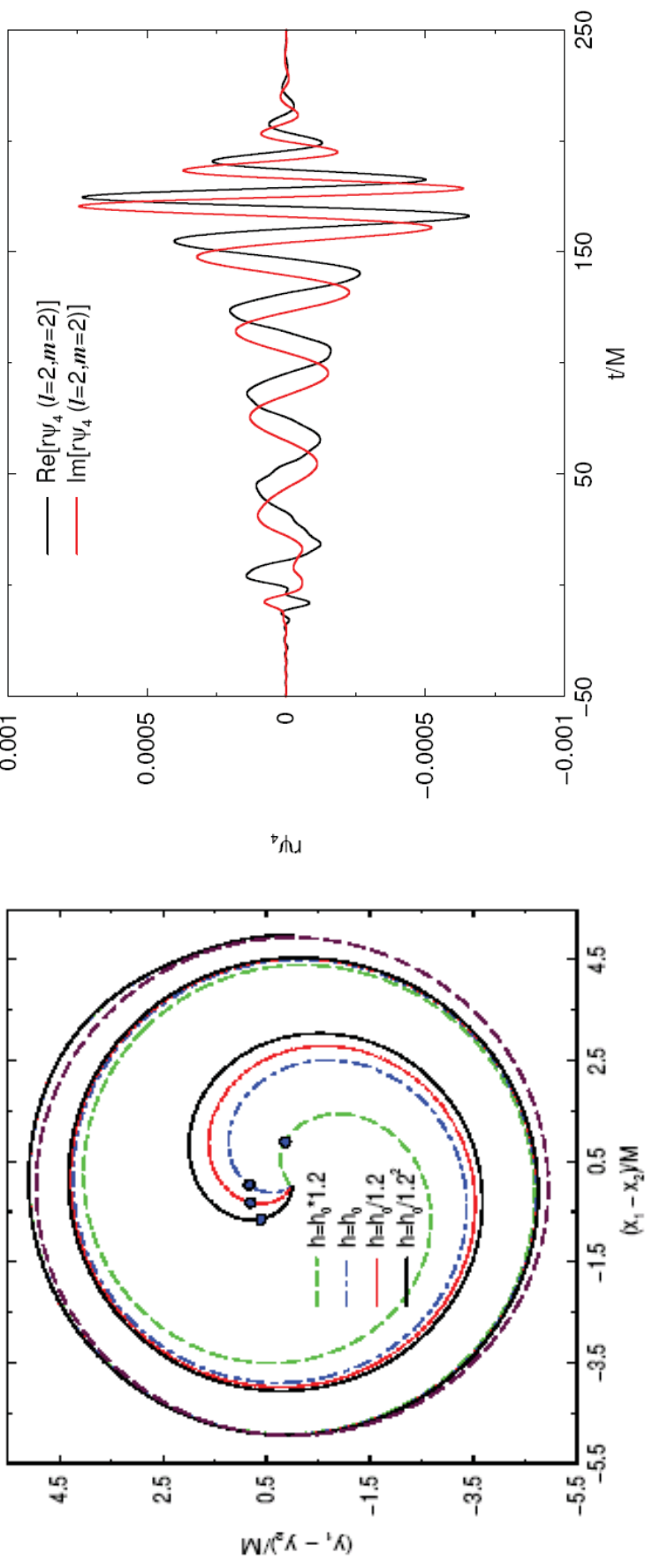


FIG. 1: An xy projection of the trajectories for the two highest resolutions of the $q = 1/100$ configuration. The dotted circle corresponds to the ISCO radius while the small filled in circle corresponds to the point on the trajectory where a common horizon is first detected. Note the initial “jump” in radius (see Fig. 2) due to the initial data radiation content.

[h2o with timeline and zoom](#)

Comparison with perturbations and $q \sim 1$ extrapolations

The amount of energy and angular momentum radiated when the $(2, 2)$ mode frequency is larger than $M\omega_{2,2} > 0.167$ is given by (adding up to $\ell = 4$ modes) $\delta E/M = 0.000047 \pm 0.000001$ and $\delta J/M^2 = 0.00034 \pm 0.00001$, which agrees to within 4% with the particle limit predictions of $\delta E/M = 0.47\eta^2$ and $\delta J/M^2 = 3.44\eta^2$ from $\omega = 0.167$ on (Nagar & Bernuzzi, 2010)

[h2o_with_time.mov](#)

[h2o_real_size.mov](#)

[h2o_fast.mov](#)

We also agree with the recoil prediction extrapolated from $q > 0.1$ to $Vr = 1.12$ km/s (predicted) and 1.07 ± 0.05 km/s (computed).

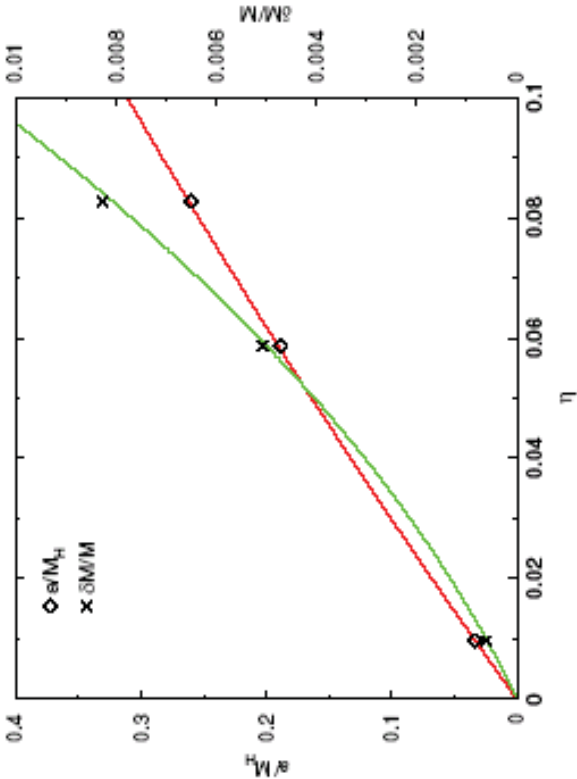


FIG. 4: The remnant spin a/M_H and the radiated mass from infinite separation $\delta M/M$, as a function of the symmetric mass ratio $\eta = q/(1 + q)^2$, for $q = 1/10, 1/15, 1/100$, as well as the empirical formula prediction.

TABLE III: Remnant spin and total radiated mass (starting from infinite separation) as a function of mass ratio q as measured in our simulations and as predicted by our empirical formulae [32].

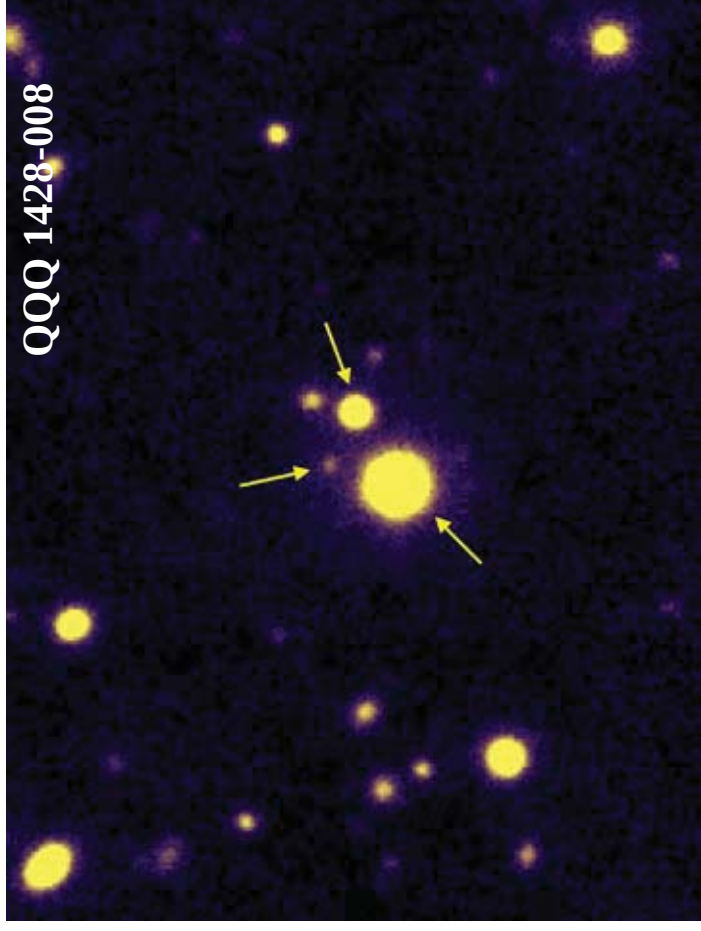
q	1/10	1/15	1/100
α (Computed)	0.2603	0.18875	0.0333
α (Predicted)	0.2618	0.1903	0.03358
δM (Computed)	0.00826	0.00507	0.000618
δM (Predicted)	0.00806	0.00498	0.000604

Beyond the BBH problem

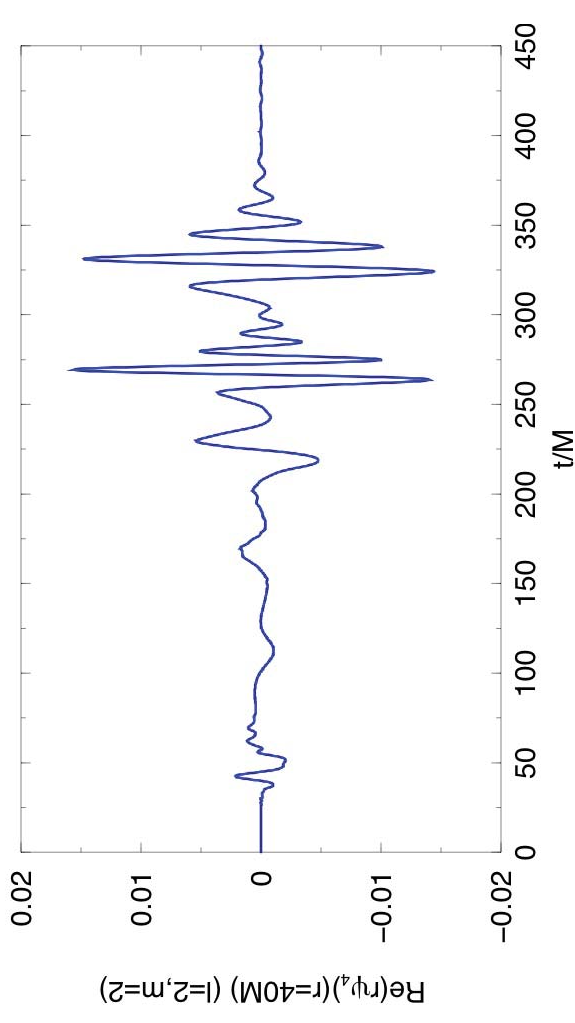
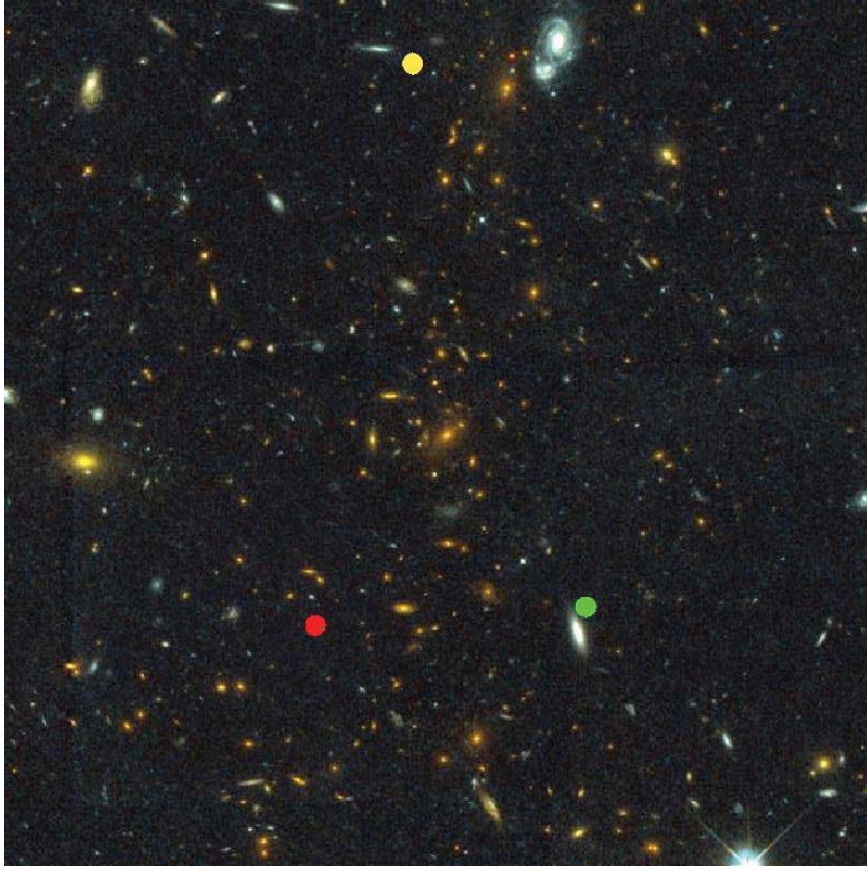
- **Matter:** BH+NS, BH+Disk, NS+NS→BH
 - Shibata, Shapiro, Lehner, Rezzolla et al
- **D-dimensions:** BBH collisions
 - Lisbon group, Shibata next talk
- **N-Black Holes:** 3BHs already very rich!
 - RIT group

Close encounters of three BHs: Motivation

- 3-body and 4-body interactions are expected to be important in globular clusters and in galaxy cores hosting SMBHs
- Newtonian simulations show that 3-body encounters generically lead to the breakup of the system into an elliptical binary and the 3-body that escapes.
- Explore 3-body interactions in GR due to the effect of gravitational radiation



Triple Merger I



- The waveform is generically distinctive.
- 3BHs interactions are prevalently grazing type of collisions that lead to less gravitational radiation, here 1/3 that of a typical binary.

BH1 and **BH2** form a quasi circular orbit, which is almost immediately disrupted by **BH3**. **BH1** and **BH3** merge at point M1 and that merger remnant merges with **BH2** at M2.

Triple Merger II

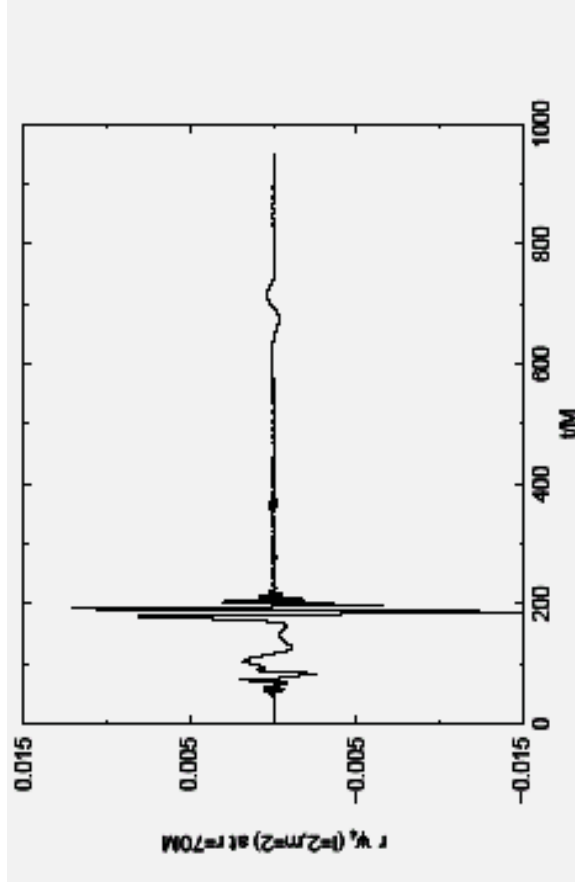
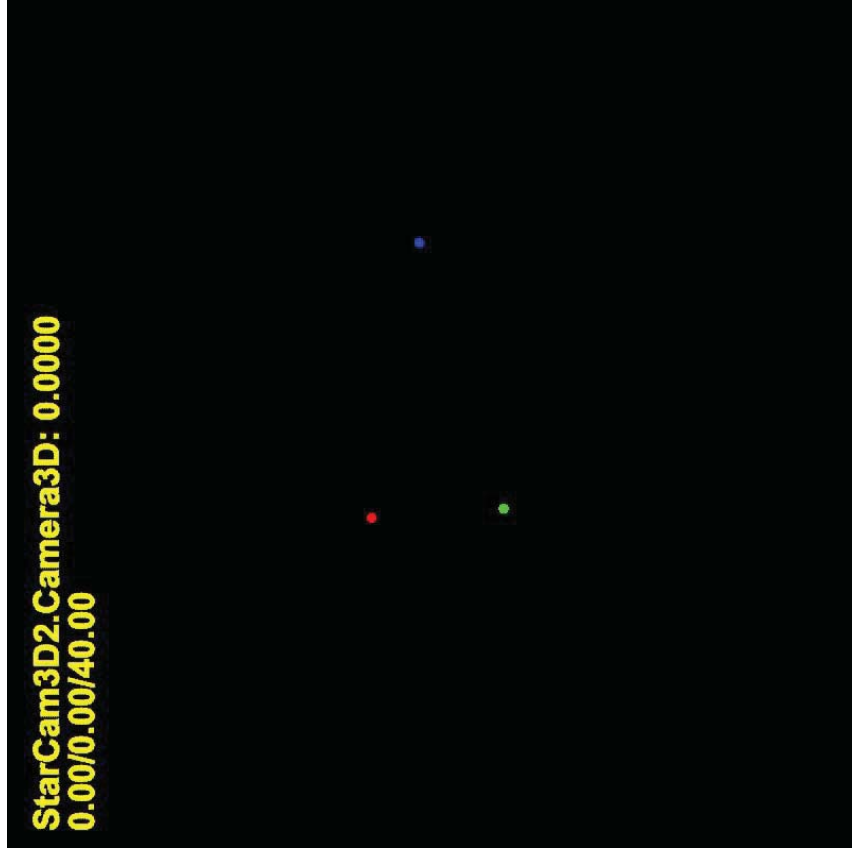
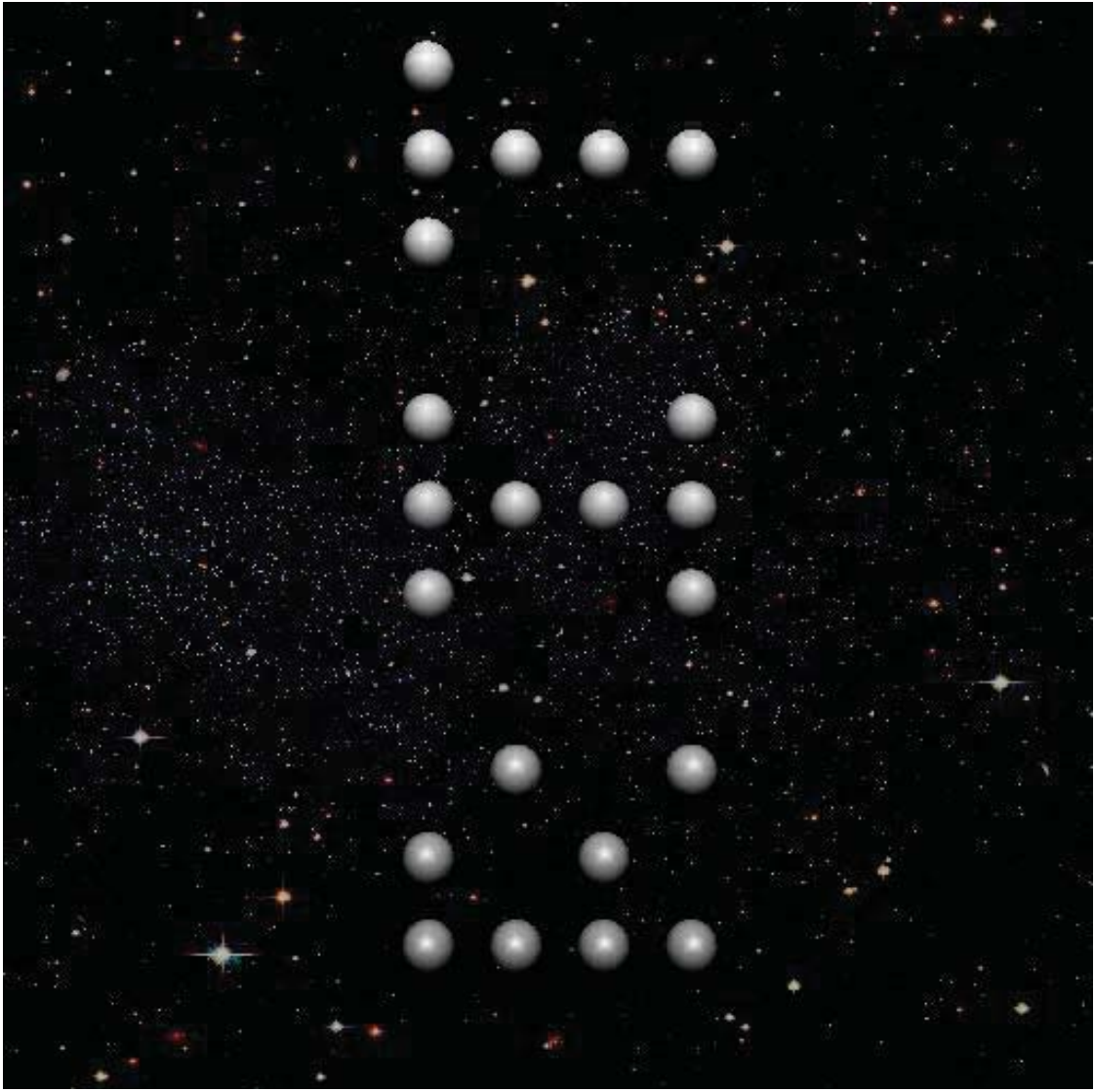


FIG. 3: The $(l = 2, m = 2)$ mode of ψ_4 for 3BH101. The BH2-BH3 merger waveform is centered at $t \sim 185M$, the small pulse at $t \sim 700$ was produced by the close approach of the BH2-BH3 merger product to BH1.

Provide a mechanism for producing
highly-elliptical close-binaries

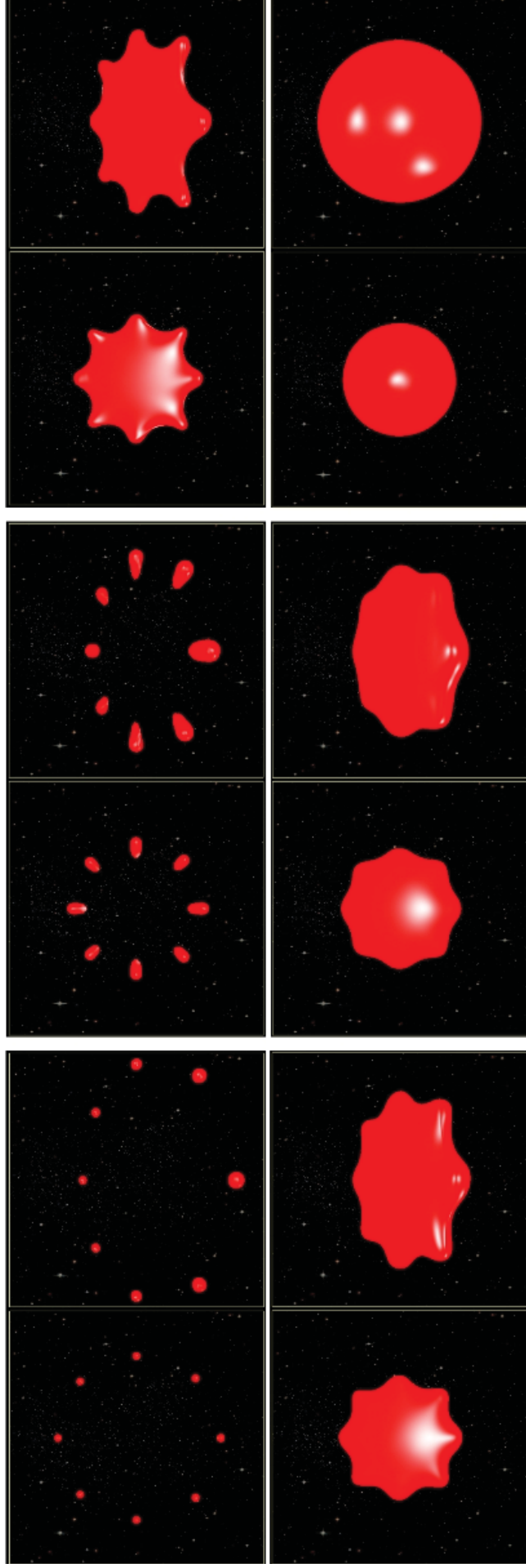


General Relativistic RIT

vs.

[Newtonian RIT](#)

Topological censorship in $D=4$



[Event horizons of 8 Merging BHs](#)

Numerical Relativity applications

- **Gravitational Waves:** Computation of waveforms. Sparse bank of templates to assist GW observatories in first detection and then parameter extraction of sources
- **Astrophysics:** Large gravitational recoil predictions and final remnant masses and spins.
- **Mathematical Relativity:** Horizon dynamics, cosmic and topological censorship, no-hair theorem.
- **HEP:** As discussed in this workshop.
 - What would AdS/CFT duality would prefer?
 - Tow spinning BHs hanging up for a long time before merger
 - N-BHs in relativistic cluster, thermalizing, not thermalizing?
- **Next:** Computer science, engineering, taking over the world, etc.

Fin