Perturbation theory of black holes

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Introduction

Perturbation analysis (analytical/numerical)

- GW emission from a particle (plunging into or orbiting) around a BH
- Stability problem

  Stable → final state of gravitational collapse

  Unstable → New branch of solutions

- Information about the geometry: Quasi-Normal Modes
- Insights into Uniqueness/non-uniqueness
- Attempt to find new, approximate solutions (by deforming an existing solution)

Two major issues when formulating perturbation theory

- Fixing gauge ambiguity
  - Imposing suitable gauge conditions

    or
  - constructing manifestly gauge-invariant variables

- Reduction of perturbation equations to a simple, tractable form (Master equations)
  - classifying perturbation variables into (to be) mutually decoupled groups
  - separating variables

Master equations

4D **Static** asymptotically flat vacuum case Regge-Wheeler 57

Zerilli 70

charge case Moncrief

-- Stability Regge-Wheeler 57, Veshveshwara 70 …

asymptotically AdS/dS case Cardoso-Lemos

--- set of decoupled *self-adjoint* ODEs

Stationary Rotating vacuum (Kerr) case Teukolsky 72

--- Stability Press-Teukolsky 73 --- Whihting 89 …

Talk by Ferrari

4D classification (Uniqueness)

- BH Thermodynamics: $$\Delta M = \frac{1}{8\pi}\kappa \Delta A + \Omega_H \Delta J$$

Stationary black holes obey equilibrium thermodynamic law (i.e. 1st law)

➔ described by merely a small numbers of parameters

$$(M\ ,\ J\ ,\ Q)$$

(Israel-Carter-Robinson-Mazur-Bunting-Chrusciel)

– *vacuum rotating black hole spacetime* $\Longrightarrow$ *Kerr-metric*

Physical implications

- If weak cosmic censorship (Penrose) holds, gravitational collapse always forms a black hole

- The Kerr-metric is stable (Press-Teukolsky 73, Whiting 89)

  $\Rightarrow$ *describes a possible final state of dynamics*

The Kerr-metric describes–to a very good approximation–black holes, formed via gravitational collapse in our universe

Uniqueness + **Stability** + **Cosmic Censorship**

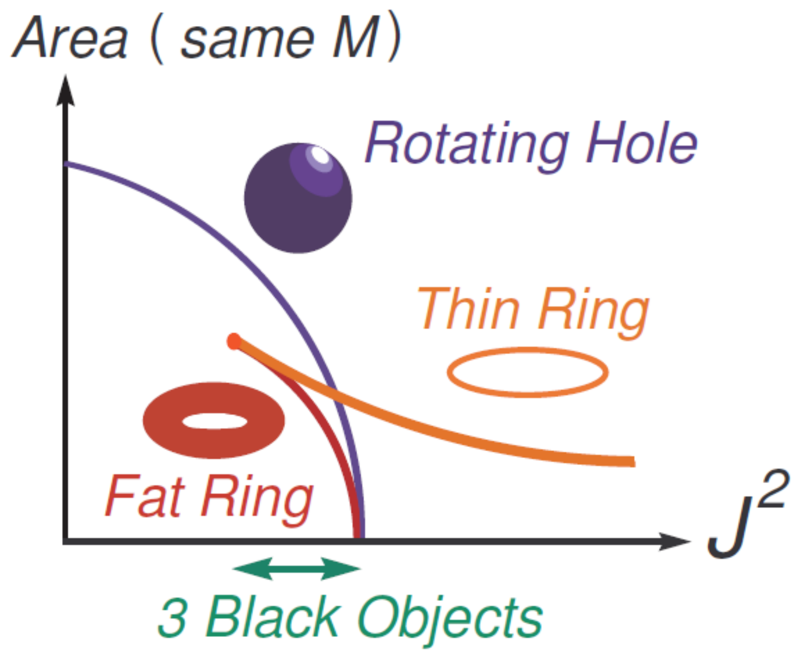
"In my entire scientific life ... the most shattering experience has been the realization that an exact solution of general relativity, discovered by the New Zealand mathematician Roy Kerr, provides the absolutely exact representation of untold numbers of massive black holes that populate the Universe"

Chandrasekhar

Classification Problem in Higher Dimensions

- *D*>4 General Relativity

  No uniqueness like 4*D* GR



Many unstable black (rotating) objects

Stability results so far

- *4D* Static: vacuum, charged, Λ → stable
  Rotating: vacuum (Kerr) → stable
  Λ(<0) (Kerr-AdS) → unstable ($\ell\Omega_H > 1$)

- *D>4* Static: vacuum → stable
  charged (D=5…11) w. Λ → can be unstable

  Rotating: → many unstable cases

Gregory-Laflamme instability
Superradiant instability

Perturbations in Higher Dimensional General Relativity

- Rotating BH case ➔ Not separable in general (Talk by Godazgar)
  still a long way from having a complete perturbation theory

  Progress in some special cases
  cohomogeneity-one (odd-dim.)
  Kunduri-Lucietti –Reall 07, Murata-Soda 08
  Kundt spacetimes (e.g. Near-horizon geom)
  Durkee-Reall 11

  Numerical approach
  e.g. Talks by Shibata, Dias,

- Static BH case ➔ simpler and tractable:
  -- can reduce to a set of decoupled ODEs

In this talk

-- focus on *Static* black holes in General Relativity

-- discuss Gauge-invariant formulation of linear perturbations (metric pert. Approach)

Master equations

Stability analysis

Generalizations

Open issues

See. e.g. latest review AI & Kodama 1103.6148

Background geometry

$$\mathcal{M}^D = \mathcal{N}^m \times \mathcal{K}^n \qquad ds^2 = g_{ab}(y)dy^a dy^b + r^2(y)d\sigma_n^2$$

$g_{ab}(y)$: m – dim spacetime metric

$d\sigma_n^2 = \gamma_{ij}(z)dz^i dz^j$: n – dim Einstein metric $\quad R_{ij} = (n-1)K\gamma_{ij}$

e.g. n-sphere $\quad K = \pm 1,\ 0$

This metric describe a fairly generic class of metrics

$m=1$ $\quad y^a \to t$ $\quad$ FLRW universe $\quad ds^2 = -dt^2 + r(t)^2 d\sigma_n^2$

$m=2$ $\quad y^a \to (t,r)$ $\quad$ Static black hole

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\sigma_n^2$$

$m \geq 3$ $\quad y^a \to (t,r,\mathbf{y})$ $\quad$ Black-brane

$$ds^2 = d\mathbf{y}^2 - f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\sigma_n^2$$

$m=4$ $\quad y^a \to (t,r,\theta,\phi)$ $\quad$ Myers-Perry black hole (w/ single rotation)

$r \to r\cos\theta$

$$ds^2 = \langle\!\langle \text{4-dim. Kerr type metric} \rangle\!\rangle + r^2 \cos^2\theta\, d\sigma_n^2$$

Cosmological perturbation theory

$$ds^2 = -dt^2 + r(t)^2 d\sigma_n^2$$: FLRW background metric

$r(t)$: scale factor

$d\sigma_n^2 = \gamma_{ij}(z) dz^i dz^j$: homogeneous isotropic time-slice

$n = 3$

Perturbations $\delta g_{\mu\nu}$ $\delta T_{\mu\nu}$ are decomposed into 3 types according to its tensorial behaviour on time-slice $(\mathcal{K}^n, \gamma_{ij})$

Tensor-type: transverse-traceless (possible only when $n \geq 3$)
➔ Gravitational Waves

Vectro-type: div-free vector ➔ couple to matter
e.g. velocity perturbations

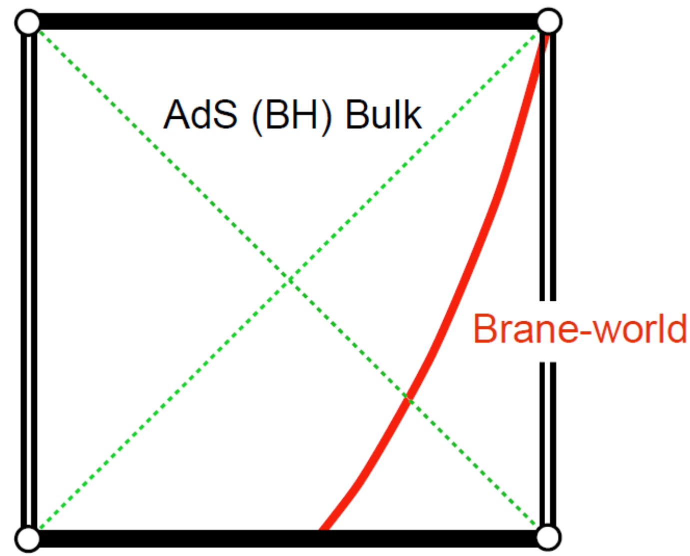
Scalar-type: scalar ➔ couple to matter
e.g. density perturbations

Gauge-invariant formulation Bardeen 80 Kodama-Sasaki 84

Brane-world cosmology

- AdS - (Black Hole)-Bulk spacetime

$$ds^2_{2+n} = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\sigma_n^2$$



- Brane-world $\quad f(r)\dot{t}^2 - \frac{1}{f(r)}\dot{r}^2 = 1$

$$ds^2_{1+n} = -d\tau^2 + r^2(\tau)d\sigma_n^2$$

Bulk perturbations induce brane-world cosmological perturbations

--- need to develop a formula for AdS-Black Hole perturbations

--- convenient to decompose bulk perturbations into

Tensor-, Vector-, Scalar-type wrt $\quad d\sigma_n^2 = \gamma_{ij}(z)dz^i dz^j$

Kodama – AI – Seto '00

Background geometry

Static solutions of Einstein-Maxwell + cosmological constant in $D = 2 + n$

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\sigma_n^2$$

$$f(r) = K - \frac{2M}{r^{n-1}} + \frac{Q^2}{r^{2(n-1)}} - \lambda r^2$$

$K = \pm 1,\ 0$

M ADM-mass

Q charge

$\lambda \propto \Lambda$ Cosmological constant

Basic strategy to derive master equations

(1) Mode-decompose $\delta g_{\mu\nu}$ as

- Tensor-type ⟷ new component in $D > 4$ case
- Vector-type ⟷ axial - mode in $D = 4$ case
- Scalar-type ⟷ polar - mode in $D = 4$ case

(2) Expand $\delta g_{\mu\nu}$ by tensor harmonics $\mathbb{T}_{ij}$ $\mathbb{V}_i$ $\mathbb{S}$ defined on $\mathcal{K}^n$

(3) Write the Einstein equations in terms of the expansion coefficients in *2*-dim. spacetime $\mathcal{N}^2$ spanned by $y^a = (t, r)$

Decomposition theorem

(i) Any (dual) vector field on compact Riemaniann manifold can be uniquely decomposed as

$$v_i = V_i + \hat{D}_i S, \qquad \text{where } \hat{D}^i V_i = 0.$$

(ii) Any 2nd-rank symmetric tensor t_{ij} on compact Einstein manifold can be uniquely decomposed as

$$t_{ij} = t^{(2)}_{ij} + 2\hat{D}_{(i}t^{(1)}{}_{j)} + t_L\gamma_{ij} + \hat{L}_{ij}t_T$$

$$\hat{L}_{ij} := \hat{D}_i\hat{D}_j - \frac{1}{n}\gamma_{ij}\hat{\triangle},$$

Metric perturbations : decomposed as

$$h_{MN}dx^M dx^N = h_{ab}dy^a dy^b + 2h_{ai}dy^a dz^i + h_{ij}dz^i dz^j$$

$$h_{ai} = \hat{D}_i h_a + h^{(1)}_{ai},$$

$$h_{ij} = h^{(2)}_T{}_{ij} + 2\hat{D}_{(i}h^{(1)}_T{}_{j)} + h_L\gamma_{ij} + \hat{L}_{ij}h^{(0)}_T$$

Tensor-type perturbations

$$\delta g_{\mu\nu} = \begin{array}{c} \left(\begin{array}{c:c} \mathbf{0} & \mathbf{0} \\ \hdashline \mathbf{0} & r^{(4-n)/2}\,\Phi(t,r)\ \mathbb{T}_{ij} \end{array}\right) \begin{array}{l} \} \ y^a = (t,r) \\ \} \ z^i \end{array} \end{array}$$

- $\mathbb{T}_{ij}$: Transverse-Traceless harmonic tensor on $\mathcal{K}^n$

$$(\hat{\triangle}_n + k_T^2)\mathbb{T}_{ij} = 0 \qquad \mathbb{T}^i{}_i = 0, \quad \hat{D}_j \mathbb{T}^j{}_i = 0$$

- $\Phi(t,r)$ is a gauge-invariant variable

- Einstein's equations reduce to Master equation $\mathcal{N}^2$

$$\boxed{\left(\Box - \frac{V_T}{f}\right)\Phi = 0}$$

$$V_T \equiv \frac{f}{r^2}\left[\frac{n(n+2)}{4}f + \frac{n(n+1)M}{r^{n-1}} + k_T^2 - (n-2)K\right]$$

Vector-type perturbations

$$\delta g_{\mu\nu} = \begin{pmatrix} 0 & h_a(t,r)\,\mathbb{V}_i \\ * & H(t,r)\,D_{(i}\mathbb{V}_{j)} \end{pmatrix} \begin{matrix} \}\ y^a = (t,r) \\ \}\ z^i \end{matrix}$$

- $\mathbb{V}_i$: Div.-free vector harmonics on $\mathcal{K}^n$: $(\hat{\Delta}_n + k_V^2)\mathbb{V}_i = 0\,, \quad \hat{D}_i\mathbb{V}^i = 0$
- Gauge-invariant variable: $F^a := r^{n-2}h^a - \frac{r^n}{2}D^a\left(\frac{H}{r^2}\right)$
- Einstein's equations reduce to $\begin{cases} D_aF^a = 0 & \cdots \quad (1) \\ \Box F^a + \cdots = 0 & \cdots \quad (2) \end{cases}$

(1) ➡ There exists $\Phi(t,r)$ such that $F^a = \epsilon^{ab}D_b\left(r^{n/2}\Phi\right)$

(2) ➡ Einstein's equation reduces to Master equation

$$\boxed{\left(\Box - \frac{V_V}{f}\right)\Phi = 0} \qquad V_V \equiv \frac{f}{r^2}\left[k_V^2 - (n-1)K + \frac{n(n+2)}{4}f - \frac{n}{2}r\frac{df}{dr}\right]$$

-- corresponds to the Regge-Wheeler equation in *4D*

Scalar-type perturbations

- Expand $\delta g_{\mu\nu}$ by scalar harmonics $\mathbb{S}$ on $\mathcal{K}^n$: $(\hat{\Delta}_n + k_S^2)\mathbb{S} = 0$

- Construct gauge-invariant variables: X , Y , Z on $\mathcal{N}^2$

- After Fourier transf. wrt 't' Einstein's equations reduce to

  - Set of 1st –order ODEs for X , Y , Z
  - A linear algebraic relation among them

--- such a system can be reduced to a single wave equation

- For a certain linear combination $\Phi(t,r)$ of X , Y , Z

Einstein's equations reduce to
$$\boxed{\left(\Box - \frac{V_S}{f}\right)\Phi = 0}$$

-- corresponds to the Zerilli equation in *4D*

Thus, for each type of perturbations, we obtain
a single master wave equation :

$$\boxed{\frac{\partial^2}{\partial t^2}\Phi = \left(\frac{\partial^2}{\partial r_*^2} - V\right)\Phi}$$

V : effective potential $\qquad r_* := \int \frac{dr}{f(r)}$

- Degrees of freedom:

Tensor harmonics	$\mathbb{T}_{ij}$:	$\frac{(n-2)(n+1)}{2}$	-- indpdt. components	$(n \geq 3)$
Vector harmonics	$\mathbb{V}_i$:	$n-1$	***	$(n \geq 2)$
Scalar harmonics	$\mathbb{S}$:	1	***	

Total $\Rightarrow$ $\frac{(n+2)(n-1)}{2} = \frac{D(D-3)}{2}$ components

- *4D* case:

  Intertwining between vector (axial) and scalar (polar) perturbations

$$\Phi_S = p\Phi_V + q\Phi'_V$$

$$V_S, V_V = \pm f\frac{dF}{dr} + F^2 + cF$$

- *D*>4 case:

  No such intertwining among Tensor-, Vector-, and Scalar-type perturbations

Stability analysis

- Master equation takes the form:

$$\boxed{\frac{\partial^2}{\partial t^2}\Phi = -A\Phi} \qquad A = -\frac{d^2}{dr_*^2} + V$$

$\Phi \propto \exp(-i\omega t)$

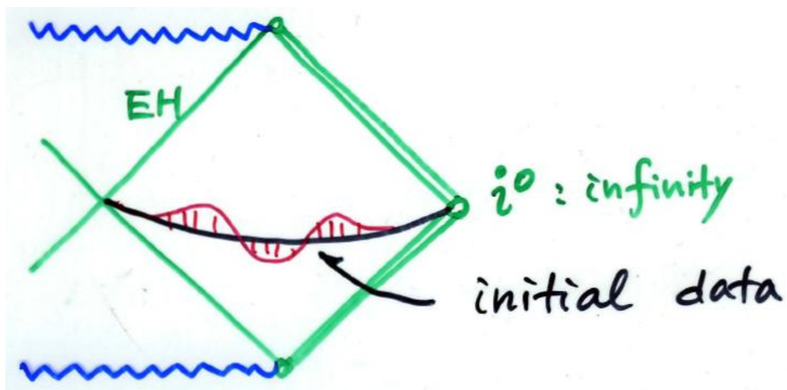
$$\omega^2 \int dr_* |\Phi|^2 = \int dr_* \Phi^* A \Phi$$

If "A" is a *positive* self-adjoint operator, the master equation does *not* admit *"unstable"* solutions

--- The black hole is stable

Boundary conditions

- Asymptotically flat case: $r_* \to \begin{cases} \infty & \text{at infinity} \\ -\infty & \text{at Horizon} \end{cases}$

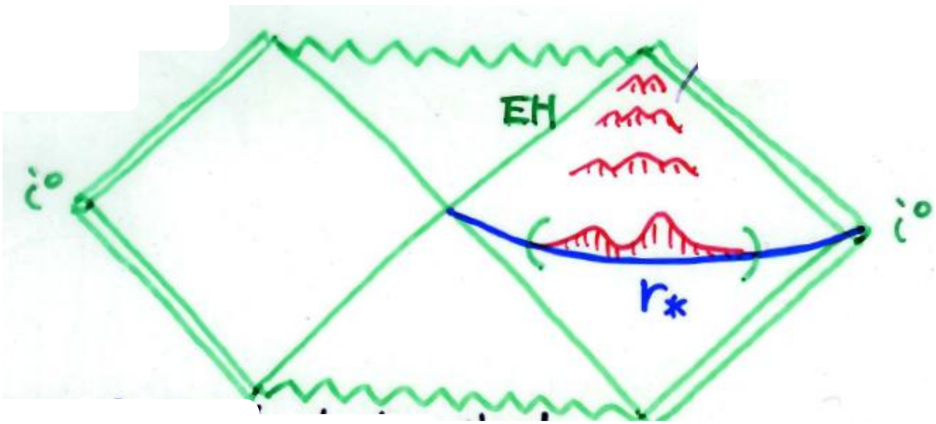


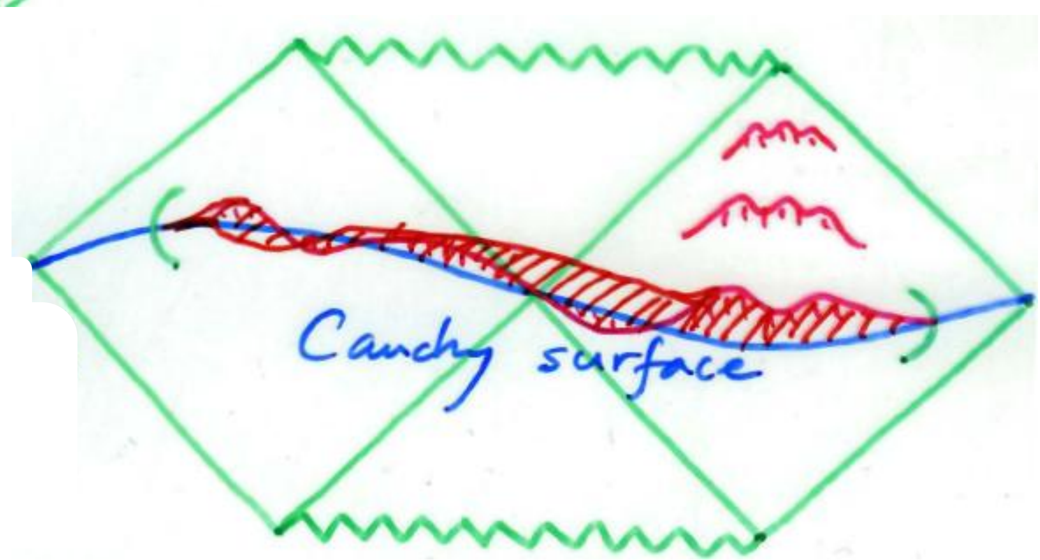
Show positivity of $A = -\frac{d^2}{dr_*^2} + V$ under the boundary condition

$\Phi = 0$ at $\begin{cases} \text{infinity} \\ \text{Horizon} \quad \text{(-- can be removed)} \end{cases}$

If stable wrt pert. w/ Dirichlet condition at Horizon, then
stable wrt pert. on extended Schwarzschild (Kruskal) spacetime

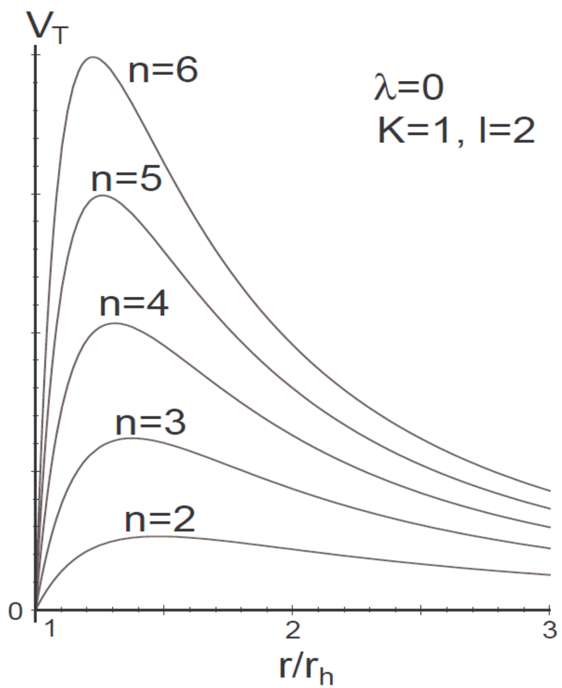
Kay-Wald 87





Stability wrt Tensor-type

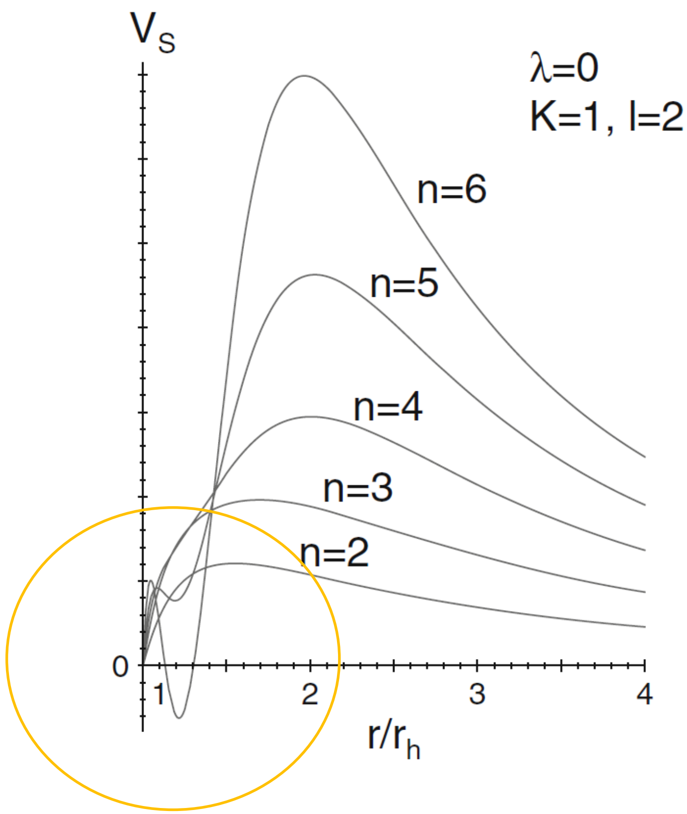
$$V_T \equiv \frac{f}{r^2}\left[\frac{n(n+2)}{4}f + \frac{n(n+1)M}{r^{n-1}} + k_T^2 - (n-2)K\right] > 0$$

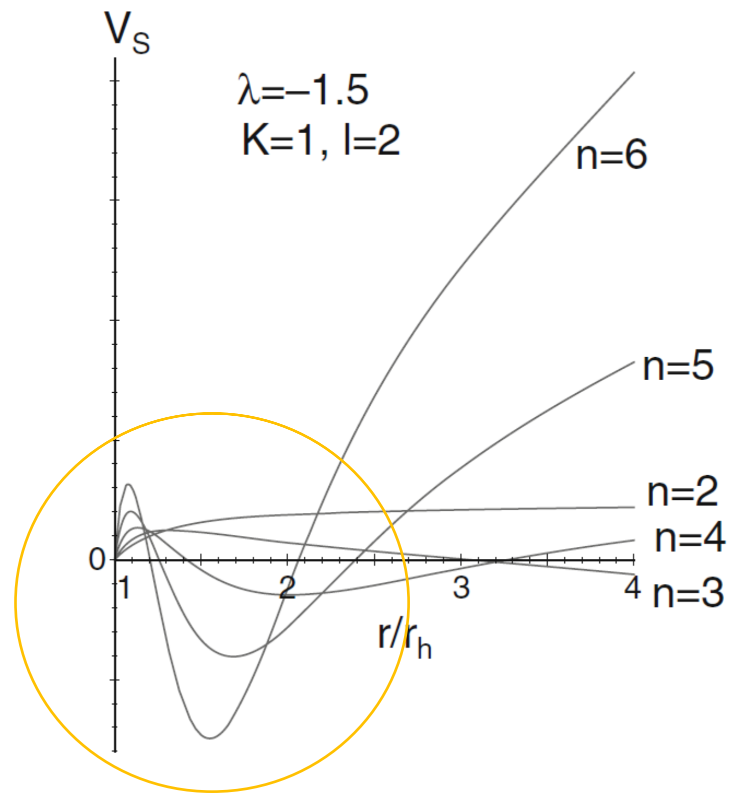


$$A = -\frac{d^2}{dr_*^2} + V > 0$$

Stable

Stability wrt Scalar-type





The potential is *NOT* positive definite in $D > 4$

- Not obvious to see whether $A = -\frac{d^2}{dr_*^2} + V$ is positive or not …

Stability proof

- Define $D := \frac{d}{dr_*} + S$ w. some function $S(r)$

$$(\Phi, A\Phi) = -\Phi^* D\Phi|_{\text{bndry}} + \int dr_* \, |D\Phi|^2 + \tilde{V} \, |\Phi|^2$$

where $\tilde{V} := V + \frac{dS}{dr_*} - S^2$

Boundary terms vanish under our Dirichlet conditions $\Phi = 0$

$\boxed{\textbf{Task:} \quad \text{Find } S(r) \text{ that makes } \tilde{V} \text{ positive definite}}$

Then, A is uniquely extended to be a positive self-adjoint operator

When the horizon manifold $\mathcal{K}^n$ is maximally symmetric

		Tensor		Vector		Scalar	
		$Q = 0$	$Q \neq 0$	$Q = 0$	$Q \neq 0$	$Q = 0$	$Q \neq 0$
$K = 1$	$\lambda = 0$	OK	OK	OK	OK	OK	$D = 4, 5$ OK $D \geq 6$?
	$\lambda > 0$	OK	OK	OK	OK	$D \leq 6$ OK $D \geq 7$?	$D = 4, 5$ OK $D \geq 6$?
	$\lambda < 0$	OK	OK	OK	OK	$D = 4$ OK $D \geq 5$?	$D = 4$ OK $D \geq 5$?
$K = 0$	$\lambda < 0$	OK	OK	OK	OK	$D = 4$ OK $D \geq 5$?	$D = 4$ OK $D \geq 5$?
$K = -1$	$\lambda < 0$	OK	OK	OK	OK	$D = 4$ OK $D \geq 5$?	$D = 4$ OK $D \geq 5$?

“OK” → “Stable”

WRT Tensor- and Vector-perturbations → Stable over entire parameter range

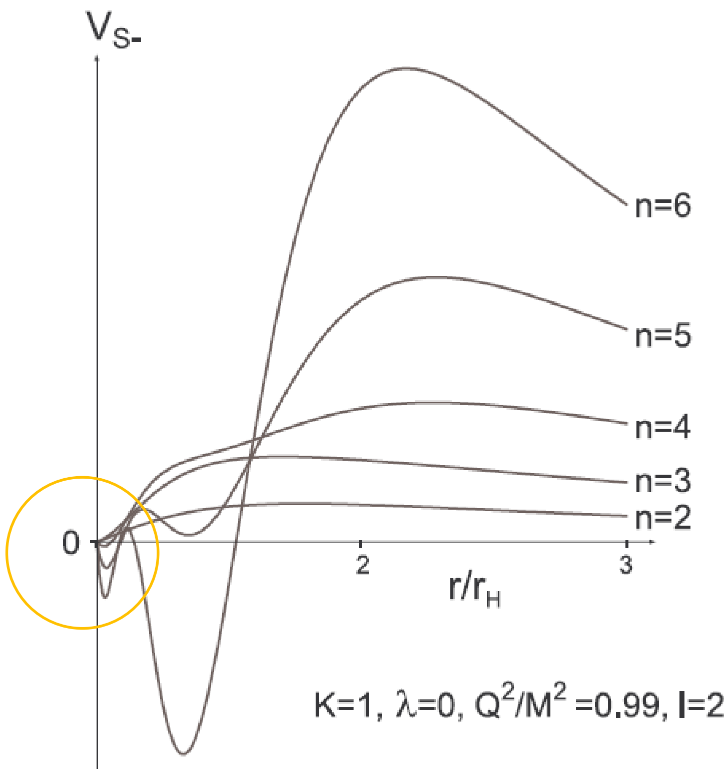
WRT Scalar-perturbations → ??? when $Q \neq 0$ $\Lambda \neq 0$

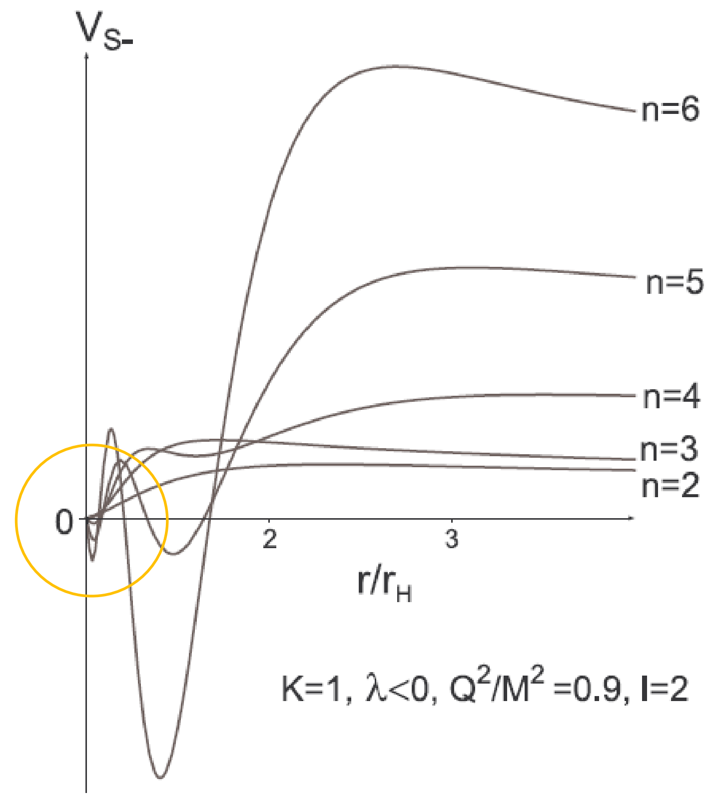
Numerical study

Konoplya-Zhidenko 07 09

- Schwarzschild-de Sitter in $D = 5, \ldots, 11$ ➔ stable
- Charged (RN) de Sitter can be unstable in $D \geq 7$ when Q and Λ large enough

Potential for Scalar-type pert. w. non-vanishing Q , Λ



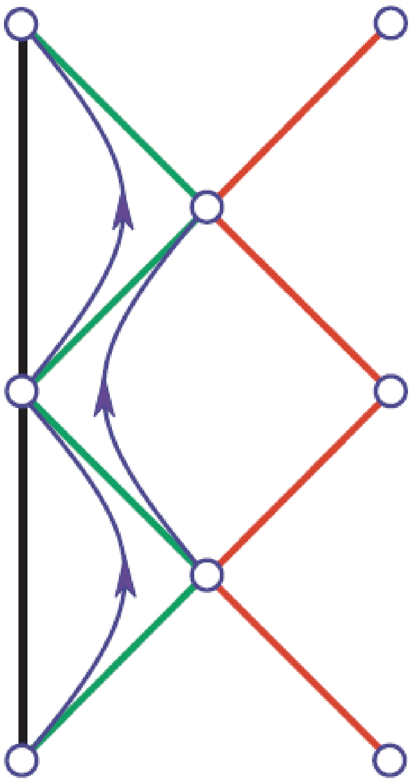


For extremal and near-extremal case, the potential becomes *negative* in the *immediate vicinity* of the horizon

Not fully studied yet

Extremal BHs

- The effective potential has a negative ditch near the horizon
- Numerical (QNMs) study indicates instability



- No bifurcate surface: Kay-Wald's theorem concerning boundary conditions at horizon does not apply

What boundary conditions are appropriate at near-horizon throat?

Some generalizations and open problems

Static black holes in Lovelock theory

Higher curvature terms involved

$$L = \sum_{m=0}^{k} c_m \mathcal{L}_m \qquad \mathcal{L}_m = \frac{1}{2^m} \delta^{\lambda_1\sigma_1\cdots\lambda_m\sigma_m}_{\rho_1\kappa_1\cdots\rho_m\kappa_m} R_{\lambda_1\sigma_1}{}^{\rho_1\kappa_1} \cdots R_{\lambda_m\sigma_m}{}^{\rho_m\kappa_m}$$

Equations of motion contain only up to 2nd-order derivatives

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\sigma_n^2$$

$$f(r) = K - X(r)r^2$$

- Master equations in generic Lovelock theory   Takahashi – Soda 10
  in Gauss-Bonnet theory   Dotti – Gleiser 05

- Asymptotically flat, small mass BHs are unstable wrt
  Tensor-type perturbations (in even-dim.)
  Scalar-type perturbations (in odd-dim.)

- Instability is stronger in higher multipoles rather than low-multipoles

$$(\Phi, A\Phi) = \int dr_* \, |D\Phi|^2 + \ell(\ell+n-1) \int dr_* N(r) \, |\Phi|^2$$

If $N(r) < 0$, then $(\Phi, A\Phi) < 0$ for sufficiently large ℓ

c. f. Cohomogeneity-1 Myers-Perry BHs

$$D = \mathrm{odd},\ J_1 = J_2 = \cdots J_{[(D-1)/2]}$$

(not in the class of metrics of $ds^2 = g_{ab}(y)dy^a dy^b + r^2(y)d\sigma_n^2$)

enhanced symmetry: $\mathbb{R} \times U((D-1)/2)$

Perturbation equations reduce to *ODEs*

Kunduri-Lucietti –Reall 07, Murata-Soda 08

Talk by Dias

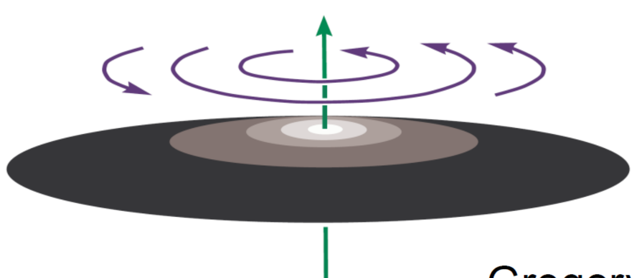
Rotating case: Cohomogeneity-2 Myers-Perry BHs

$$m = 4 \qquad ds^2 = \overbrace{\langle\langle \text{4-dim. Kerr type metric} \rangle\rangle}^{\mathcal{N}^4} + \overbrace{r^2 \cos^2\theta d\sigma_n^2}^{\mathcal{K}^n}$$

symmetry enhance $U(1)^N \Rightarrow U(1) \times SO(D-3)$

Numerical approach to stability analysis   Talks by Shibata, Dias

--- include the *ultra-spinning* case



Talk by Dias

Gregory-Laflamme modes

Cohomogeneity-2 MP case: Analytic formulation?

$$m = 4 \qquad ds^2 = \overbrace{\langle\langle \text{4-dim. Kerr type metric} \rangle\rangle}^{\mathcal{N}^4} + \overbrace{r^2 \cos^2\theta d\sigma_n^2}^{\mathcal{K}^n}$$

Tensor-type perturbations: A single master scalar variable Φ on $\mathcal{N}^4$

satisfy the same equation for a massless Klein-Gordon field

$n \geq 3$

How about **vector-type** and **scalar-type** perturbations?

KK-reduction along $\mathcal{K}^n$ ➔ Equations for *massive vector/ tensor fields* on $\mathcal{N}^4$: 4-dim. Kerr –type spacetime

Not known how to (analytically) deal with even in the standard 4-dim. Kerr background

For vector-type: ➔ 3 master scalar variables on $\mathcal{N}^4$

For scalar-type: ➔ 6 master scalar variables on $\mathcal{N}^4$ ($n \geq 3$)

Metric perturbation approach in $D = m + n$

$$ds^2 = \overbrace{g_{ab}(y)dy^a dy^b}^{\mathcal{N}^m} + r^2(y)\overbrace{d\sigma_n^2}^{\mathcal{K}^n}$$

Tensor-type: ➔ 1 master scalar variable $(n \geq 3)$

Vector-type: ➔ $m-1$ gauge-invariant scalar variables $(n \geq 2)$

Scalar-type: ➔ $\dfrac{m(m-1)}{2}$ gauge-invariant scalar variables

--- intricately coupled on $\mathcal{N}^m$

F_{ab} , F : Gauge-invariant variables in $\mathcal{N}^m$

$$
\begin{aligned}
&-\Box F_{ab}+D_aD_cF^c_b+D_bD_cF^c_a+n\frac{D^cr}{r}(-D_cF_{ab}+D_aF_{cb}+D_bF_{ca})+{}^mR^c_aF_{cb}+{}^mR^c_bF_{ca}-2\,{}^mR_{acbd}F^{cd}\\
&\quad+\left(\frac{k^2}{r^2}-\bar{R}+2\Lambda\right)F_{ab}-D_aD_bF^c_c-2n\left(D_aD_bF+\frac{1}{r}D_arD_bF+\frac{1}{r}D_brD_aF\right)-\left[D_cD_dF^{cd}+\frac{2n}{r}D^crD^dF_{cd}\right.\\
&\quad+\left(-\,{}^mR^{cd}+\frac{2n}{r}D^cD^dr+\frac{n(n-1)}{r^2}D^crD^dr\right)F_{cd}-2n\Box F-\frac{2n(n+1)}{r}Dr\cdot DF\\
&\quad\left.+2(n-1)\frac{k^2-nK}{r^2}F-\Box F^c_c-\frac{n}{r}Dr\cdot DF^c_c+\frac{k^2}{r^2}F^c_c\right]g_{ab}=2\kappa^2\Sigma_{ab}\,,
\end{aligned}
$$

$$
\frac{k}{r}\left[-\frac{1}{r^{n-2}}D_b(r^{n-2}F^b_a)+rD_a\left(\frac{1}{r}F^b_b\right)+2(n-1)D_aF\right]=2\kappa^2\Sigma_a
$$

$$
\begin{aligned}
&-\frac{1}{2}D_aD_bF^{ab}-\frac{n-1}{r}D^arD^bF_{ab}+\left(\frac{1}{2}\,{}^mR^{ab}-\frac{(n-1)(n-2)}{2r^2}D^arD^br-(n-1)\frac{D^aD^br}{r}\right)F_{ab}\\
&\quad+\frac{1}{2}\Box F^c_c+\frac{n-1}{2r}Dr\cdot DF^c_c-\frac{n-1}{2n}\frac{k^2}{r^2}F^c_c+(n-1)\Box F+\frac{n(n-1)}{r}Dr\cdot DF\\
&\quad-\frac{(n-1)(n-2)}{n}\frac{k^2-nK}{r^2}F=\kappa^2\Sigma,
\end{aligned}
$$

$$
-\frac{k^2}{2r^2}[2(n-2)F+F^a_a]=\kappa^2\tau_T
$$

Perturbation analysis and Classification problem

- In $4D$ classification: Uniqueness + Stability of the Kerr metric
  ⇒ important (astro-)physical implications

Uniqueness Theorem was established by assembling

Topology results, Symmetry (Rigidity) results, etc. and all comp. of the Einstein equations *all together*

- How about $D>4$ classification?

Role of symmetry in Stability problem

- Stability of an extremal black hole and its near-horizon geometry

  Examine perturbations of the near-horizon geometry that respect the symmetry of the full BH solution

  Durkee - Reall 11

- Symmetric perturbation at Topology changing point

c.f. critical behavior
and critical exponent

Summary: linear perturbation

- Static BHs: Complete formulation for perturbations

- Rotating BHs:
  -- Still a long way from having a complete formulation
  -- Considerable progress recently made for some special cases

- Interesting:
  Interplay between Symmetry /Topology properties
  and Perturbation analysis