# Gauge/gravity duality

Veronika Hubeny
Durham University

NR/HEP workshop
Madeira, August 31, 2011

# Motivation:

The gauge/gravity duality (a.k.a. AdS/CFT correspondence) relates strongly coupled non-abelian gauge theory in d dimensions to string theory, which in certain regime reduces to classical gravity, on (d+1)-dimensional asymptotically Anti de Sitter spacetime.

Key aspects:

- Gravitational theory maps to non-gravitational one!
- Holographic: gauge theory `lives on boundary' of AdS.
- Strong/weak coupling duality.

Invaluable tool to:

~ Use the gauge theory to define & study quantum gravity in AdS

~ Use gravity on AdS to learn about strongly coupled field theory

# References:

Original work:

- Maldacena: hep-th/9711200
- Gubser, Klebanov, Polyakov: hep-th/9802109
- Witten: hep-th/9802150, hep-th/9803131

Early reviews:

- MAGOO: hep-th/9905111
- D'Hoker & Freedman: hep-th/0201253
- Horowitz & Polchinski: gr-qc/0602037

More recent reviews:

- McGreevy: 0909.0518
- VH & Rangamani: 1006.3675
- Polchinski: 1010.6134
- Casalderrey-Solana, Liu, Mateos, Rajagopal, Wiedemann: 1101.0618
- Maldacena: 1106.6073

and many many more....

# OUTLINE

- Demystification / `derivation' of gauge/gravity duality
- Elements of AdS/CFT correspondence dictionary
    - Probes of bulk geometry
    - UV/IR (scale/radius) duality
    - Black holes in AdS
    - AdS/CFT with curved boundary
    - Time dependence
- Open problems

# Demystifying gauge/gravity:

[Horowitz & Polchinski]

Assertion:

Every non-abelian gauge theory hides a quantum theory of gravity.

- Spin-2 graviton as composite of two spin-1 gauge bosons?
- ∃ no-go thm of [Weinberg&Witten]; but evade by allowing graviton and gauge bosons to propagate in different spacetimes...
- Suggested by Holographic principle ['t Hooft, Susskind]: gravitational theory in d+1 dimensions can be fully described by a non-gravitational theory in d dimensions.
- For gauge theory to capture an extra dimension (apart from the background on which it lives), it should be local WRT another parameter.
- Wilsonian RG => gauge theory is local WRT energy scale (cf. color transparency), hence identify with the extra dimension.

# Demystifying gauge/gravity:

- To accommodate extra dimension's worth of information, gauge theory must have sufficiently many degrees of freedom.
- In SU($N$) Yang-Mills gauge theory, ['t Hooft ] identified natural limit:

$$\text{let } N \to \infty \text{ keeping } \lambda \equiv g^2_{\rm YM}\, N \text{ fixed.}$$

- Since classical Yang-Mills is very different from classical gravity, the only hope of equivalence lies in highly quantum regime:

$$\lambda \to \infty$$

- So we need gauge theory with many fields and at strong coupling; we then expect gravity to `emerge' as effective classical degrees of freedom.

# Demystifying gauge/gravity:

- To avoid potential instabilities of strongly coupled QFT, impose supersymmetry which ensures energy bounded from below (can relax this later).
- Most supersymmetric case in 3+1 dim: $\mathcal{N} = 4$ SU(N) SYM = conformal field theory (CFT)
- => coupling does not run with energy scale, so strongly-coupled over arb. large region => allows macroscopic 5-d ST.
- => CFT on $R^{3,1}$ ( $ds^2_{\rm CFT} = \eta_{\mu\nu}\, dx^\mu\, dx^\nu$ ) is scale invariant under:

$$x^\mu \to \alpha\, x^\mu \ , \qquad r \to \alpha^{-1}\, r$$

- Most general geometry consistent with these symmetries = AdS:

$$ds^2 = \frac{r^2}{\ell^2}\,(\eta_{\mu\nu}\, dx^\mu\, dx^\nu) + \frac{\ell^2}{r^2}\, dr^2$$

# Demystifying gauge/gravity:

- Natural conjecture so far:

  $\mathcal{N}$=4 SU(N) SYM on $R^4$ = a gravitational theory on $AdS_5$

- LHS: contains non-abelian gauge field, 6 scalars, & 4 spinors
- RHS: corresponding supersymmetric extension of gravity: IIB supergravity in 10-d w/ field content: {metric, 2 scalars, 2-form potentials, 4-form potential w/ self-dual field strength, & fermionic partners}; this has $AdS_5 \times S^5$ solution.

- Summary: we have realized a regime of gauge theory $(N, \lambda \gg 1)$ from which classical gravity can emerge.
- But note: strings, D-branes, extra dimensions, etc. also emerge!
  Cf. ['t Hooft]: planar structure of large-N gauge thy → theory of strings

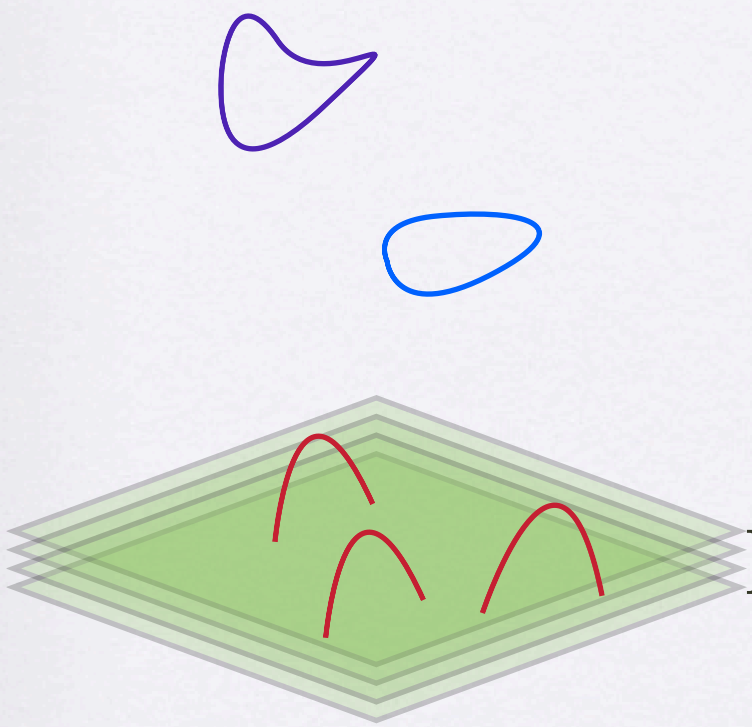
# Maldacena's construction

(in IIB string theory)

* Consider a stack of $N$ coincident D3-branes in 2 regimes:

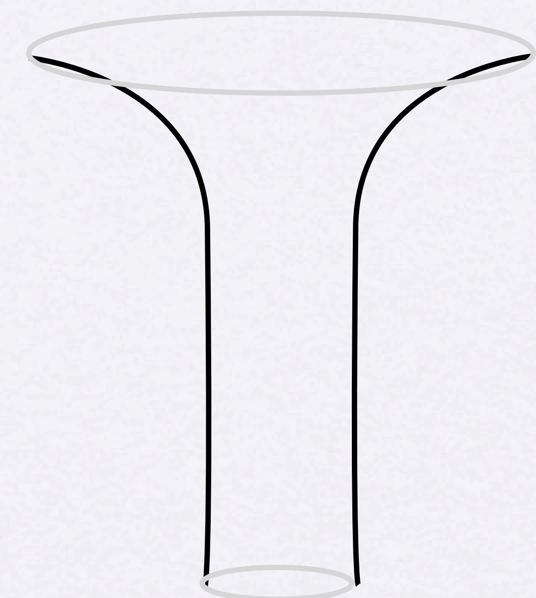
$g_s\, N \ll 1$ (perturbation theory valid)

$g_s\, N \gg 1$



} $N$ D3-branes

closed + open string excitations in $R^{10}$

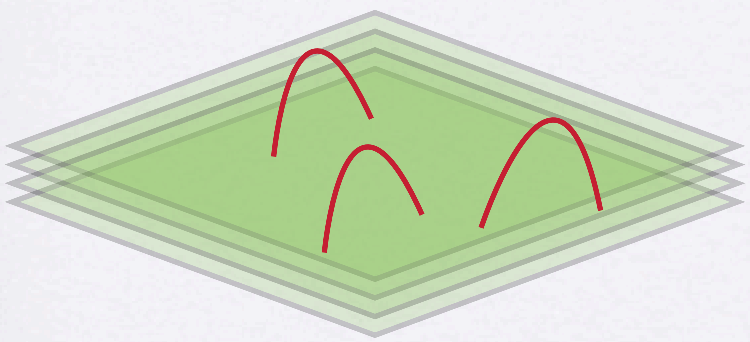


curved geometry: extremal black 3-brane

# Maldacena's construction:

* take ``decoupling'' (low-energy) limit:

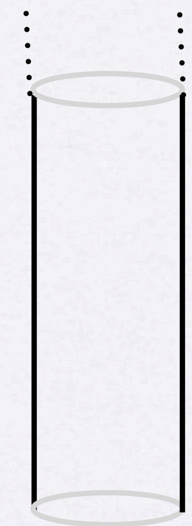
$g_s N \ll 1$



closed strings decouple;
we're left with low-energy
open string modes:

SU(N) gauge theory

$g_s N \gg 1$



Asymptotic modes decouple;
we're left with:

closed string theory on
near-horizon geometry:
$AdS_5$ x $S^5$

# Maldacena's conjecture:

$g_s N \ll 1$

SU(N) gauge theory

$g_s N \gg 1$

closed string theory on $AdS_5 \times S^5$

* but gauge theory is defined for all coupling, so identify the two descriptions:

→ AdS/CFT correspondence:

Four-dimensional $\mathcal{N} = 4$ SU(N) SYM gauge theory
is fully equivalent to
IIB string theory with $AdS_5 \times S^5$ boundary conditions.

# Parameters:

$\mathcal{N} = 4$ SU(N) SYM:

$N$ , $\lambda \equiv g^2_{\rm YM}\, N$

rank of gauge group

't Hooft coupling

string theory on $AdS_5 \times S^5$:

$g_s$ , $\dfrac{\ell}{\ell_s}$

string coupling

string length

AdS size

$$g^2_{\rm YM} = 4\pi\, g_s$$

$$\lambda^{1/4} = (4\pi\, g_s\, N)^{1/4} = \frac{\ell}{\ell_s}$$

- large $\lambda$ ⇒ small stringy corrections
- large $N$ ⇒ small quantum corrections
- Hence $N \gg 1\,,\ \lambda \gg 1$ ⇒ classical gravity on $AdS_5 \times S^5$

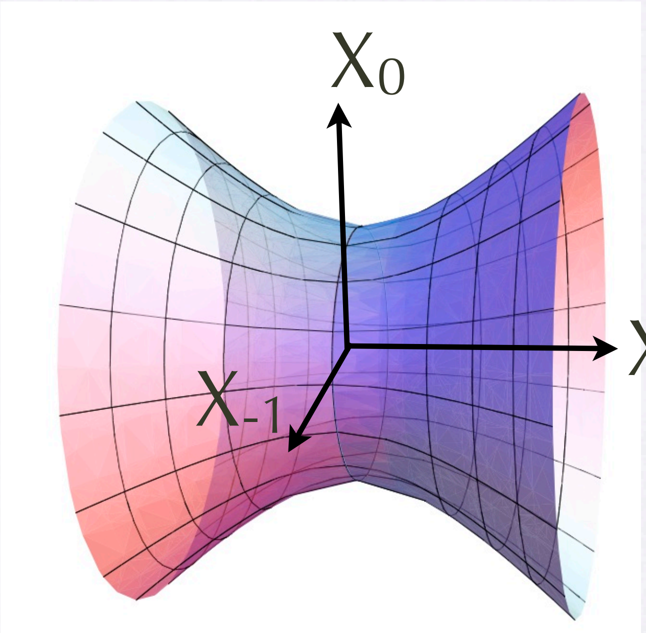
# Geometry of AdS:

$AdS_{d+1}$ is a constant negative curvature Lorentzian spacetime;
to manifest the full SO(d,2) symmetry group,
write as the hyperboloid

$$-X_{-1}^2 - X_0^2 + X_1^2 + \ldots + X_d^2 = -\ell^2$$

embedded in flat space $R^{d,2}$,

$$ds^2 = -dX_{-1}^2 - dX_0^2 + dX_1^2 + \ldots + dX_d^2$$



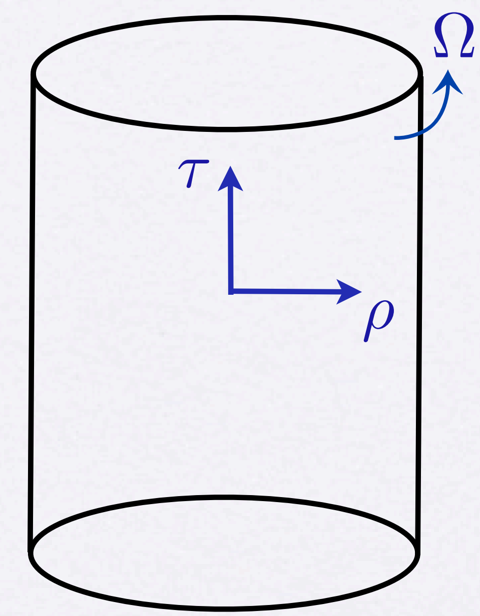

$\ell$ is the `size' (radius of curvature) $AdS_{d+1}$
which we can WLOG set =1, and measure all distances in AdS units.

# Geometry of AdS:

Useful coordinate systems obtained by different parameterizations of embedding surface.

## Global AdS:

$$ds^2 = -(\rho^2 + 1)\, d\tau^2 + \frac{d\rho^2}{\rho^2 + 1} + \rho^2\, d\Omega^2$$

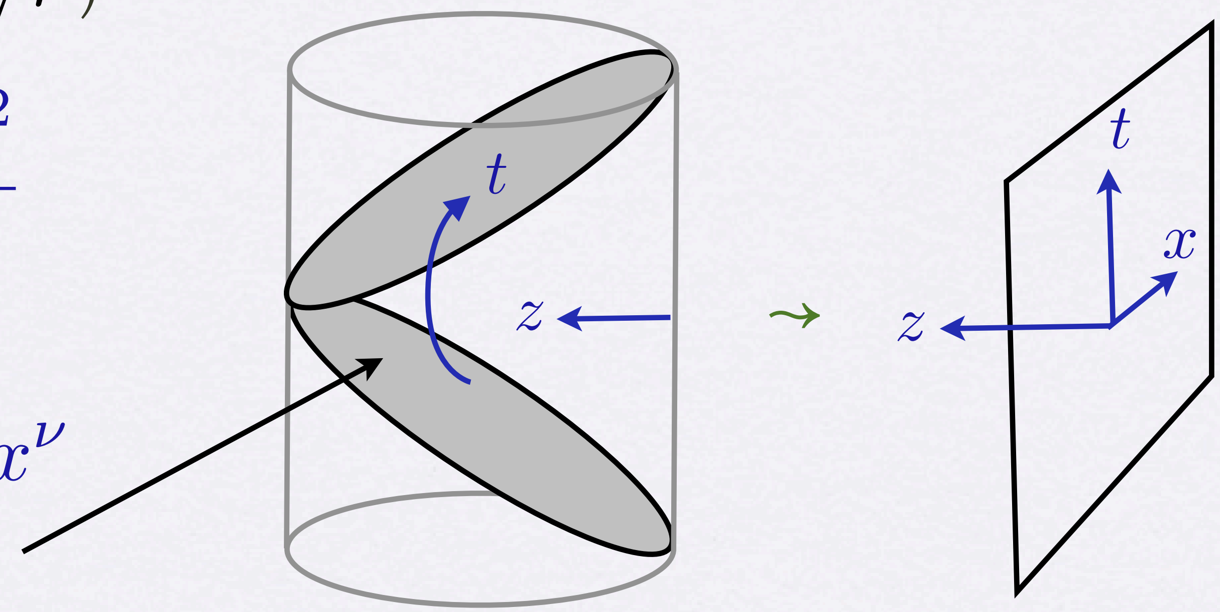



timelike boundary at $\rho = \infty$
conformal bdy metric: $ds^2 = -d\tau^2 + d\Omega^2$

## Poincare AdS: (with $z = \ell^2/r$)

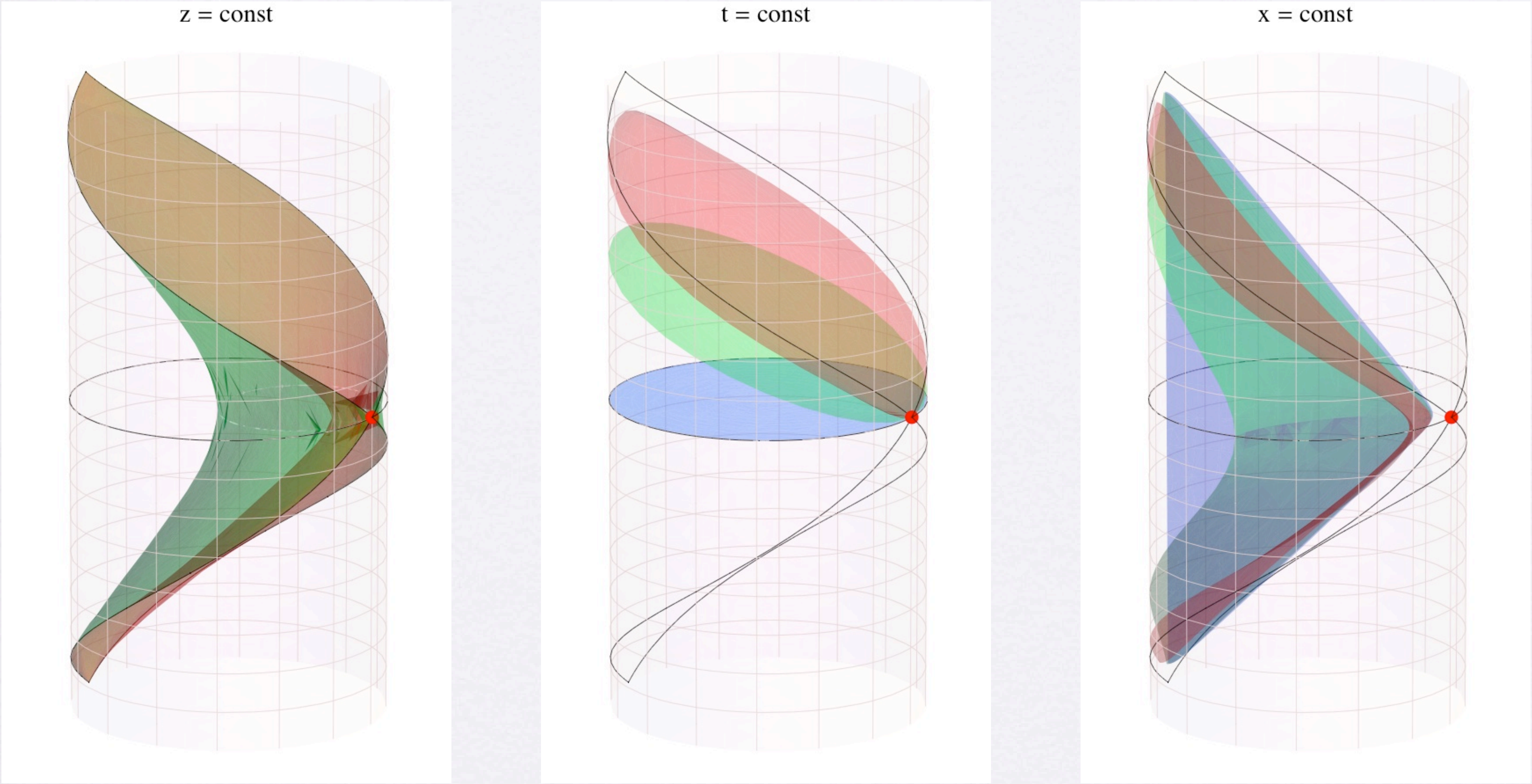
$$ds^2 = \frac{-dt^2 + dx_i\, dx^i + dz^2}{z^2}$$

bdy at $z = 0$
bdy metric: $ds^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu$
Poincare horizon at $z = \infty$



# Geometry of AdS:

Poincare coordinates plotted on global AdS:



# Extensions

The D3-brane construction can be extended in diverse directions; e.g.:

- ✱ Other branes:
  - ~ M2 branes in 11-d M-theory → $AdS_4 \times S^7$ → 3-d SCFT
  - ~ M5 branes in 11-d M-theory → $AdS_7 \times S^4$ → 6-d SCFT
  - ~ D1-D5 branes: $AdS_3 \times S^3 \times T^4$
  - ~ other Dp branes: non-AdS / non-CFT
- ✱ Other asymptotically AdS x S geometries
  (UV of the CFT is determined only by the asymptotics)
  & global completion of AdS
- ✱ Other asymptotically locally AdS x S geometries
  → CFT on arbitrary non-dynamical background $g_{\mu\nu}$

etc...

# Tests

Though the gauge/gravity duality has not been rigorously proven, $\exists$ overwhelming body circumstantial evidence:

* Symmetries match:

  between gravity & gauge theory

  ~ SO(4,2): from $AdS_5$ geometry & from conformal invariance

  ~ SO(6): from $S^5$ geometry & from rotation of scalars

  ~ supersymmetries match → superconformal PSU(2,2|4)

* Spectrum of supersymmetric states (e.g. all modes of graviton) match
* Matching long string states can be identified on both sides
* Amplitudes which are protected by SUSY match
* Higher symmetries on both sides match (integrability)

# Tests

* In large number of situations, geometry realizes gauge theory expectations
  - When (conformal or super-) symmetry is broken, phase transitions, confinement, ...
  - Dynamical evolution, when calculable...
  - Causality tests, ...
* To the extent they have been tested, gravity predictions agree with numerical calculations in strongly coupled gauge theory (e.g. lattice simulations)
* Similarly, predictions `agree with experiment'...

No counter-examples have been found, despite intense efforts...

# Implication for QG:

- Since the gauge theory is fully quantum mechanical and consistent, and since it includes all the graviton states, it constitutes a quantum theory of gravity.
- Hence

  if we understand the gauge/gravity correspondence dictionary sufficiently well, we can recast long-standing quantum gravitational questions into non-gravitational language, and

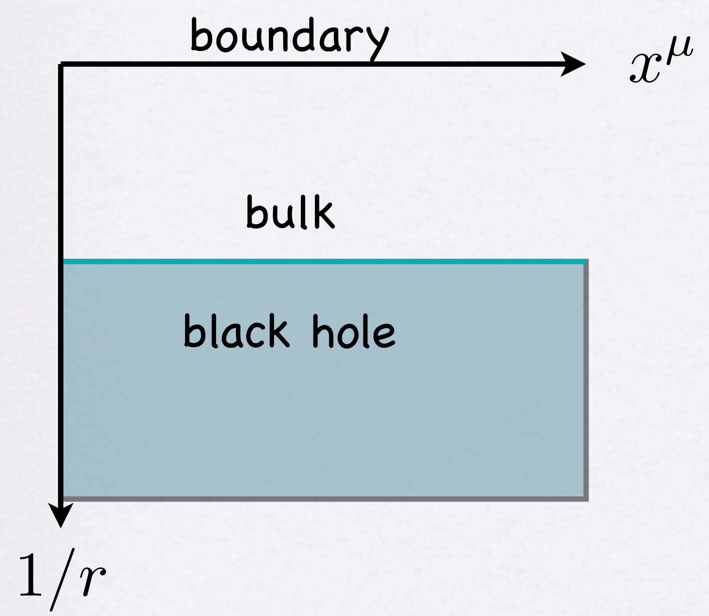
  if we have sufficient handle on the gauge theory, we can answer these QG questions.

=> Much of the effort in the AdS/CFT program has been directed toward understanding the dictionary.

# Basics of AdS/CFT dictionary:

- Distinct asymptotically AdS (bulk) geometries
  → distinct states in (boundary) gauge theory; e.g.:
  - AdS bulk geometry → vacuum state of the gauge theory
  - Schwarzschild-AdS black hole → thermal state of gauge theory
- Fields in AdS → (single-trace chiral primary) operators in CFT
- Bulk geometry induces boundary stress tensor (a la [Brown&York])
  ∃ explicit prescription: [Balasubramanian et.al, Skenderis et.al.].
  which captures the essential physics of the gauge theory state
  (eg. local energy density, pressure, temperature, entropy current, etc.)

# Basics of AdS/CFT dictionary:




boundary fluid specified by $T_{\mu\nu}(x^\mu)$

bulk geometry specified by $g_{ab}(r, x^\mu)$

$$ds^2 = g_{ab}\, dX^a\, dX^b \qquad X^a = \{r, x^\mu\}$$

- Bulk dynamics is specified by Einstein's equations.

$$E_{ab} \equiv R_{ab} - \frac{1}{2} R\, g_{ab} + \Lambda\, g_{ab} = 0$$

- Boundary dynamics is specified by stress tensor conservation.

$$\nabla_\mu T^{\mu\nu} = 0$$

# Probes of bulk geometry

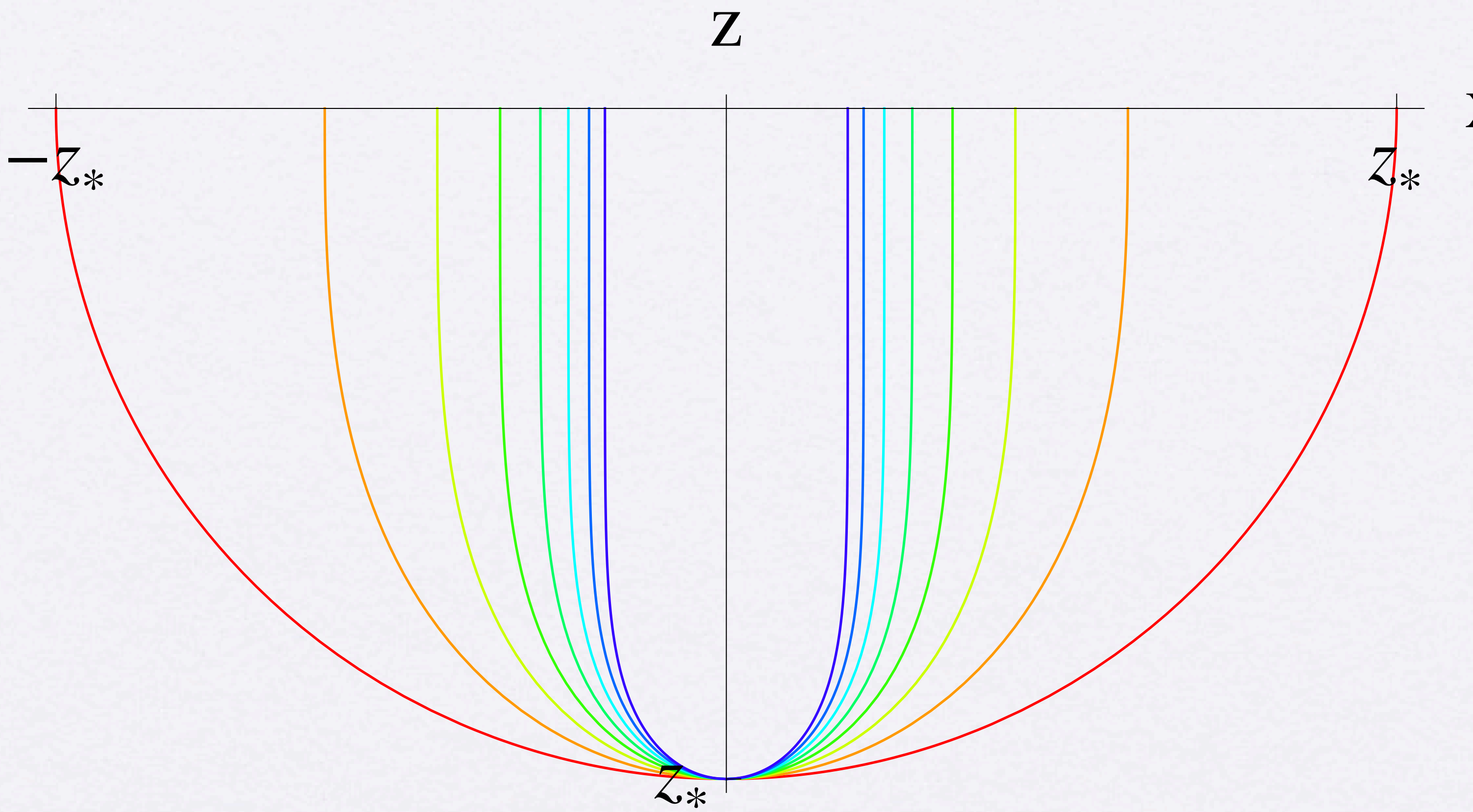
The bulk metric can be extracted using various CFT probes (which are described by geometrical quantities in the bulk:

Examples:

| CFT probe | bulk quantity |
|---|---|
| ✴ expectation values of local gauge-invariant operators | asymptotic fall-off of corresponding conjugate field |
| ✴ correlation functions of local gauge-invariant operators | in WKB approx., proper length of corresponding geodesic |
| ✴ Wilson loop exp. vals. | area of string worldsheet |
| ✴ entanglement entropy | vol of extremal co-dim.2 surface |

# Probes of bulk geometry

Minimal surfaces of various dimensionalities
(cross-sectional profiles for n-surfaces anchored on a strip):



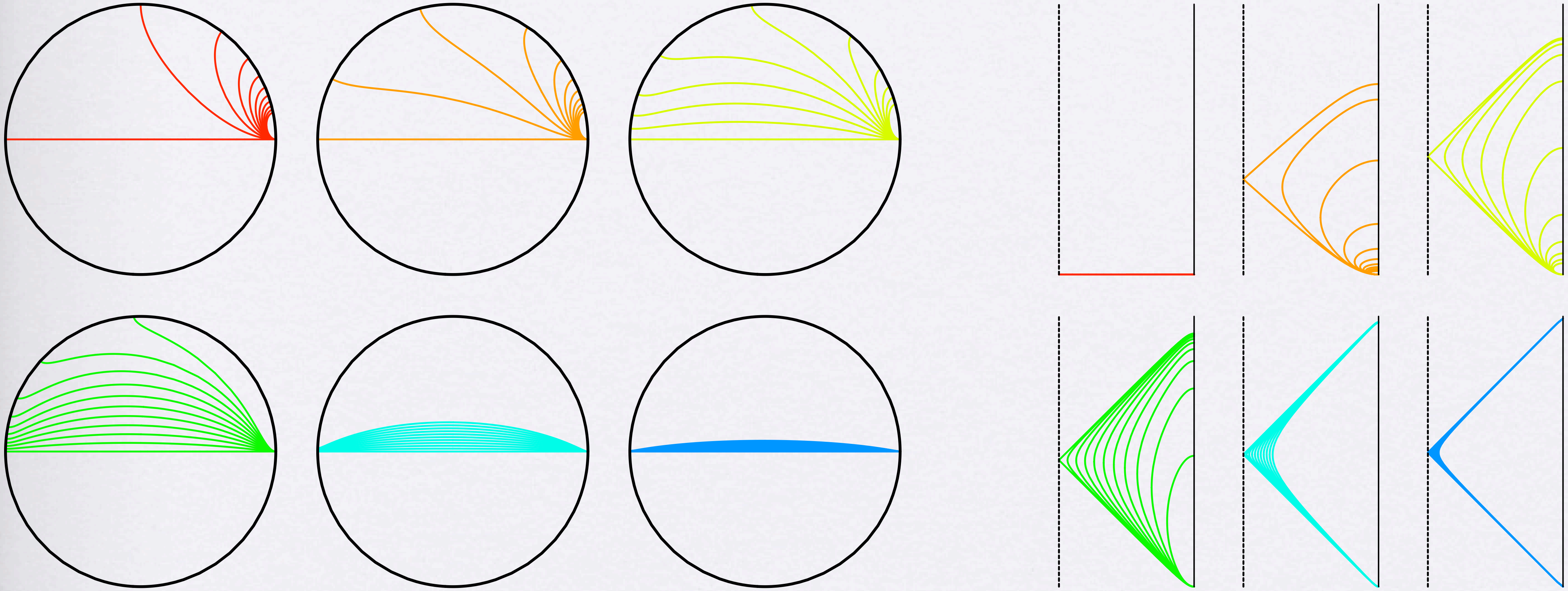

- to reach same bulk depth $z^*$ need different width strip on bdy
- not true for n-ball rather then n-strip: indep of n.

# Probes of bulk geometry

Spacelike geodesics in AdS (for varying E,L):

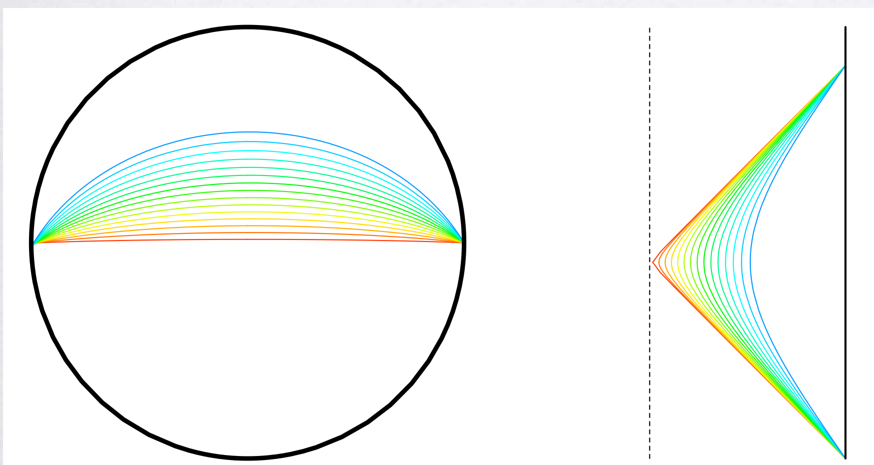
$$ds^2 = -(\rho^2+1)\,d\tau^2 + \frac{d\rho^2}{\rho^2+1} + \rho^2\,d\varphi^2$$

$(\rho, \varphi)$ plane: $(\rho, \tau)$ plane:

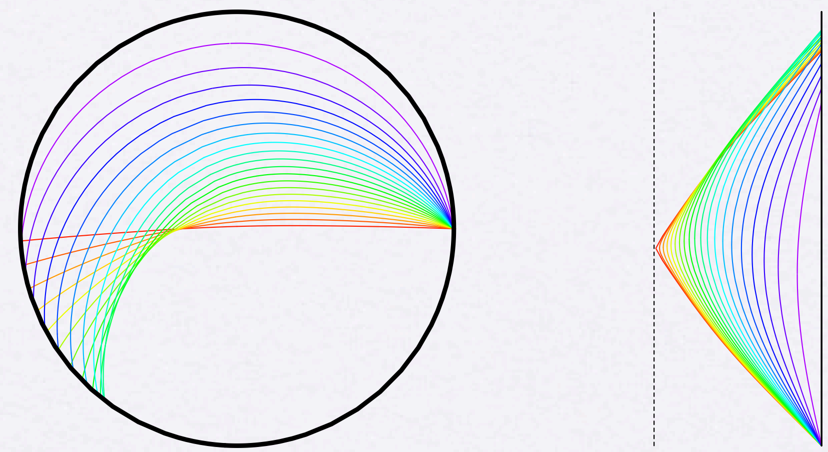


# Probes of bulk geometry

Null geodesics in AdS:



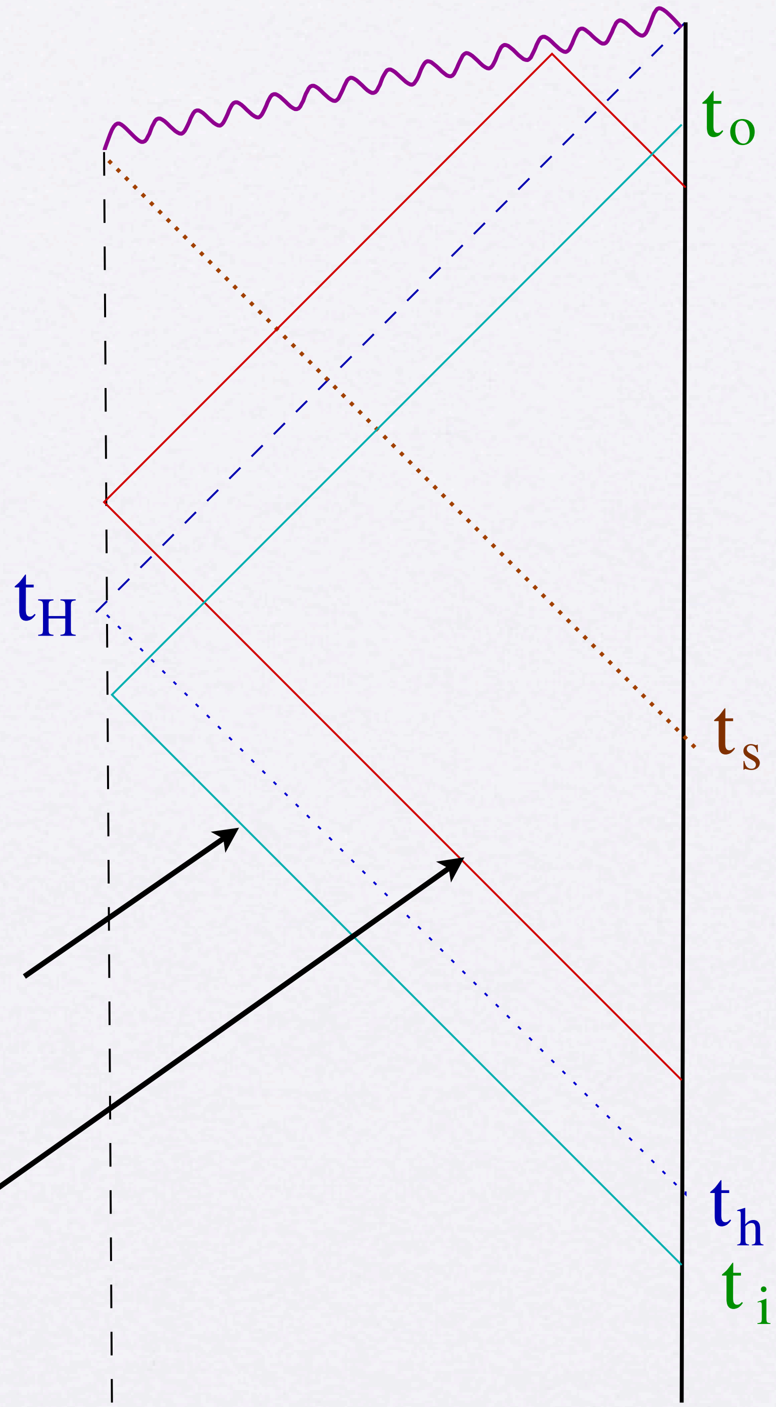
cf. Null geodesics in AdS `star' geometry:



- Geodesic endpoints depend on the bulk geometry
- Hence can `read-off' bulk metric from CFT correlators (bulk cone singularities) [VH, Liu, Rangamani]
- only down to null circular orbit
- nevertheless, can capture signals of event horizon

# Probes of bulk geometry

Event horizon formation is visible via bulk cone singularities:




bulk cone singularity $\langle \mathcal{O}(t_i)\, \mathcal{O}(t_o) \rangle$ at longer time separations $t_o - t_i$ as $t_i \to t_h$

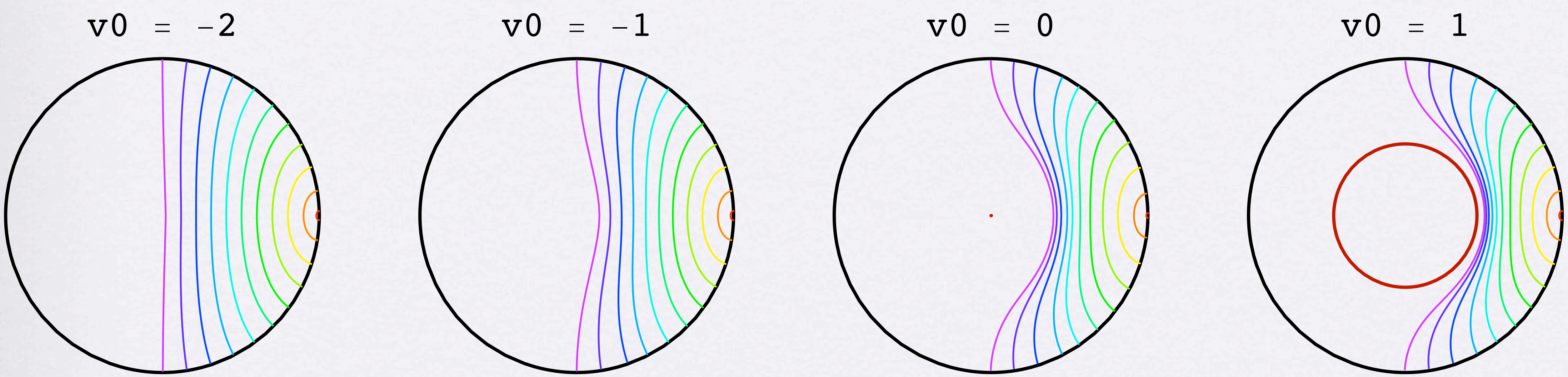
more subtle signal once $t_i > t_h$

# Probes of bulk geometry

Spacelike geodesics (= timelike string WS = entanglement entropy in 3-d.) for Vaidya-$AdS_3$ at various times v0: [VH, Takayanagi, Rangamani]

$$ds^2 = -\left(\rho^2 + 1 - \frac{m(v)}{\rho^{d-2}}\right) dv^2 + 2\, dv\, d\rho + \rho^2\, d\Omega_{d-1}^2$$

take e.g. $m(v) = \tanh v$




~ As BH starts to collapse, geodesics get `repelled' by horizon

# Probes of bulk geometry

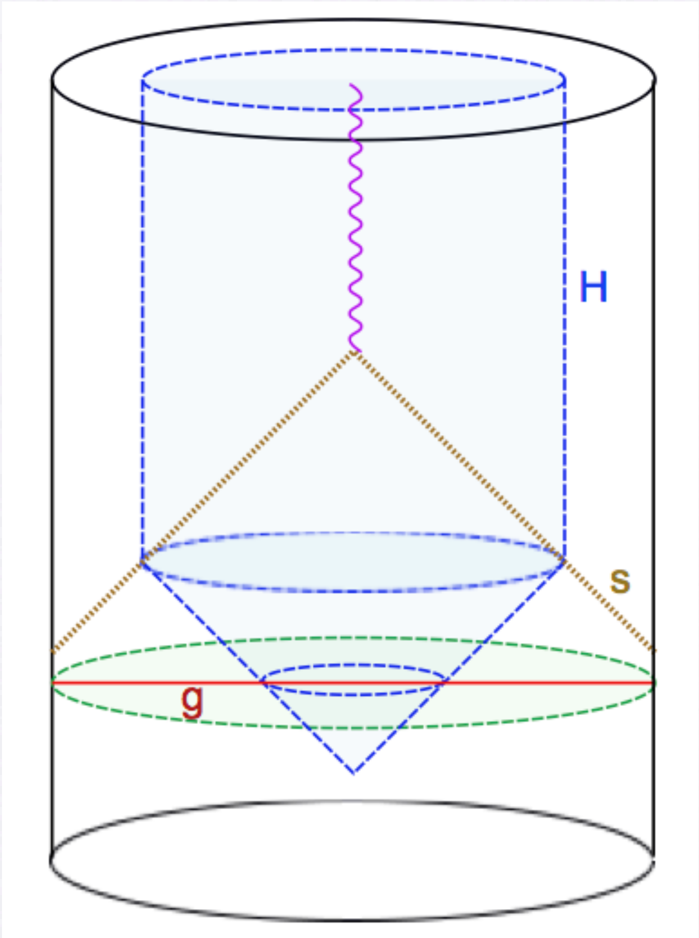
- in static BH geometry, spacelike geodesics (or surfaces cannot probe past event horizon.
- This however need not be the case for dynamical BHs

  (due to the teleological nature of event horizon)

Precursors can see inside black hole:

- measure precursor g at some time
- later collapse a shell s
- if s sufficiently soon & energetic, event horizon H can stretch to past of g.

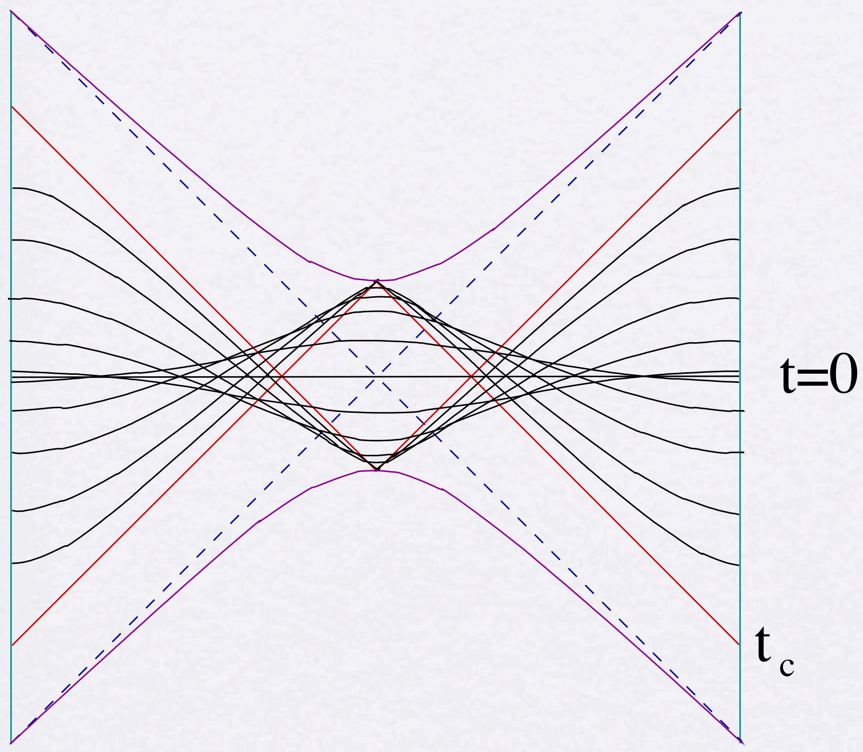
[VH]



# Probes of bulk geometry

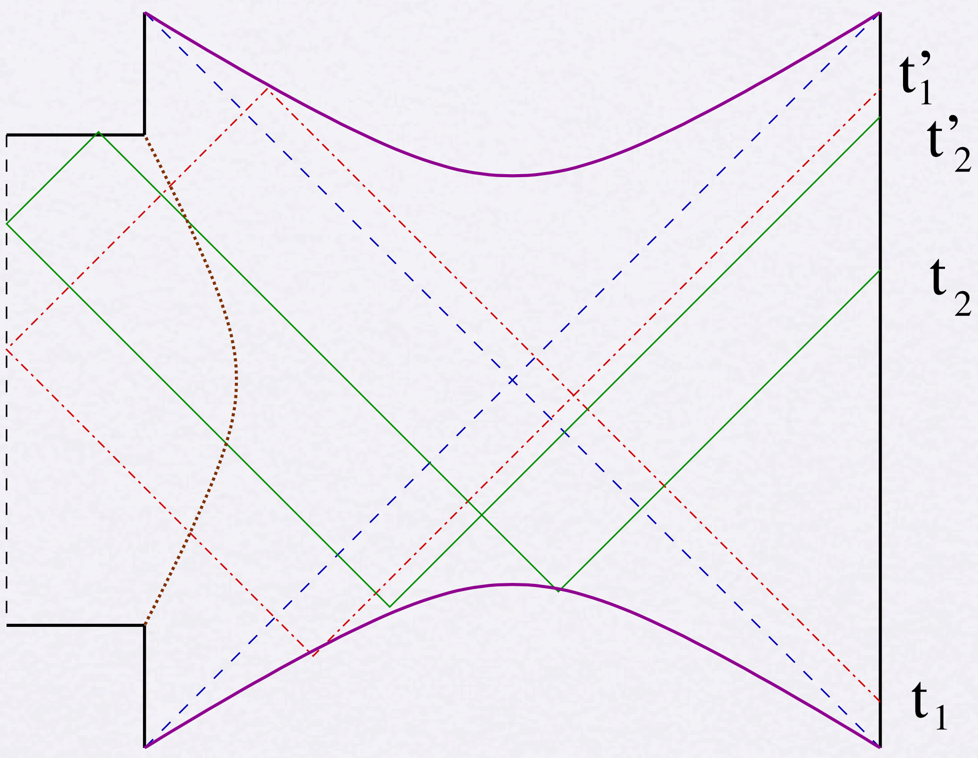
Analytically continued correlators can probe beyond horizons

Schw-AdS$_5$:




[Fidkowski, VH, Kleban, Shenker]

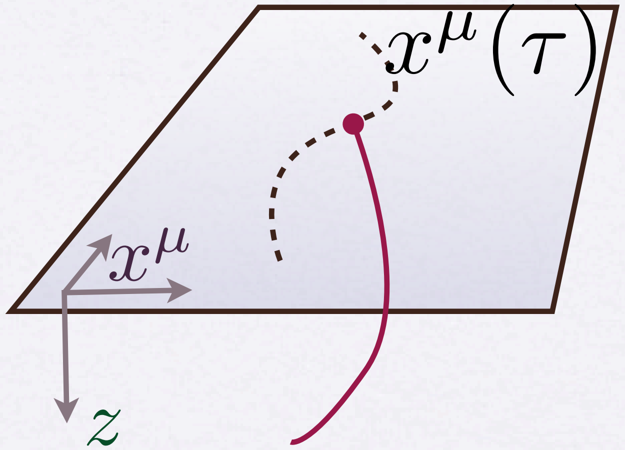
more contrived geom
with dS behind horizon:




[Freivogel et.al.]

# Quarks and trailing strings

- Consider a quark in 4-d strongly coupled CFT in R$^4$ at zero temperature following some trajectory $x^\mu(\tau)$.
- For general accelerated trajectory, the quark would radiate. This is captured by the 4-d stress tensor.
- In the gravity dual this is described by a string (ending on the quark) in AdS



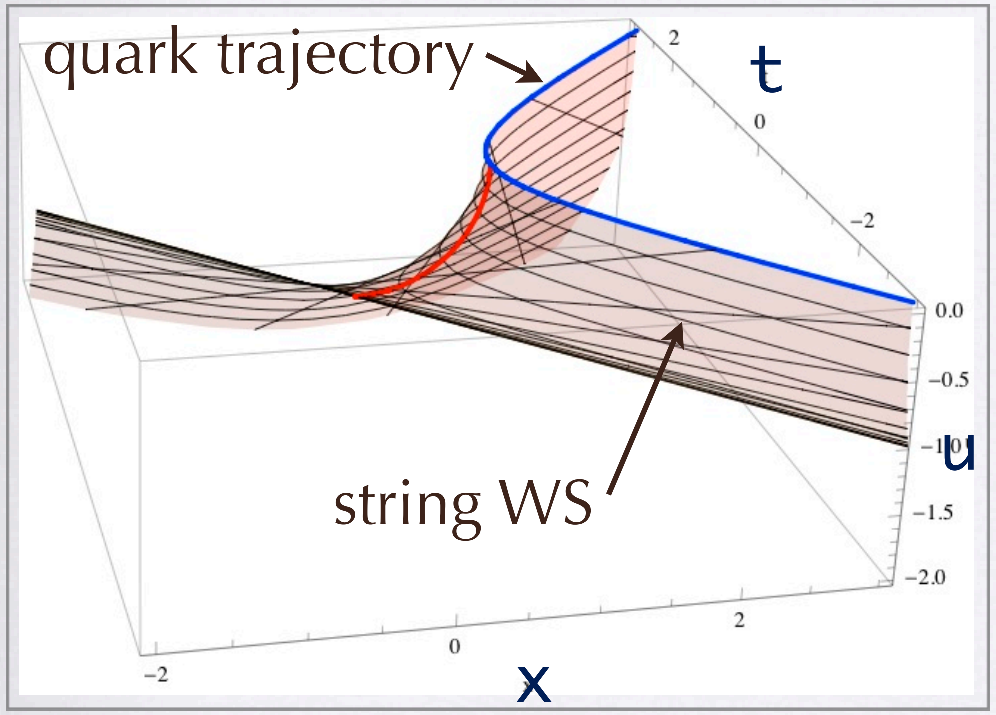

$$ds^2_{\rm AdS} = \frac{1}{z^2}\left(\eta_{\mu\nu}\, dx^\mu\, dx^\nu + dz^2\right)$$

- The string carries energy and backreacts on the spacetime; the asymptotic fall-off of this 5-d metric deformation induces 4-d bdy stress tensor.
- Brownian motion

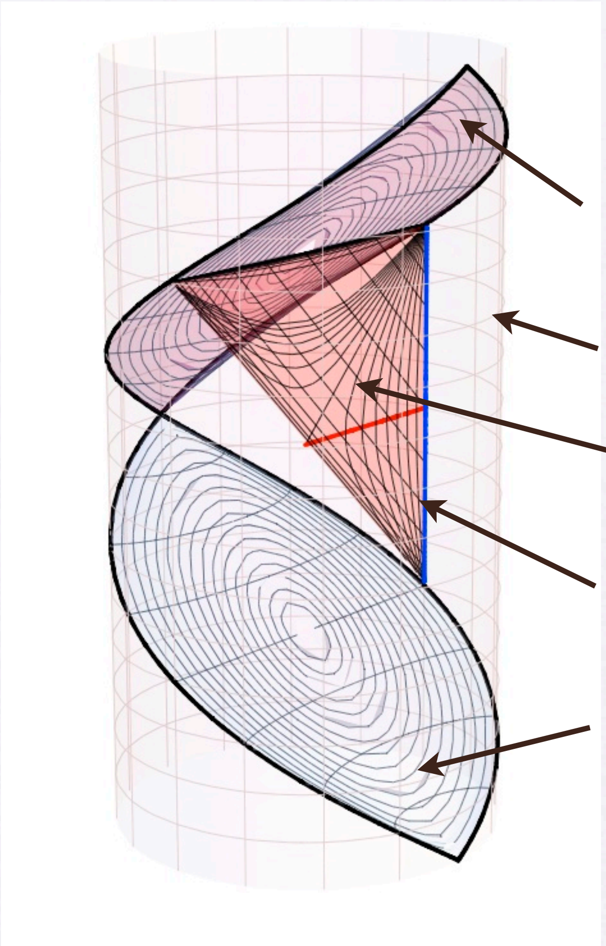
# Quarks and trailing strings

Example: string worldsheet for uniformly accelerated quark

In Poincare coordinates:



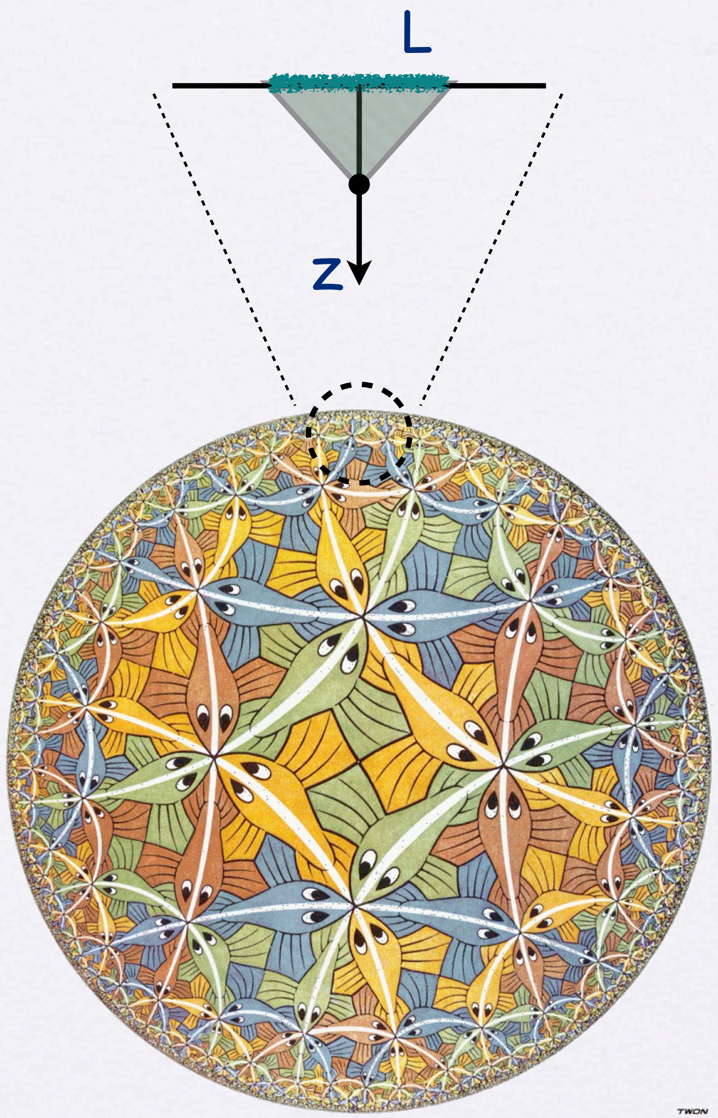

In global coordinates:




(even on $R^4$, induces a `mirror' antiquark...)

# UV/IR (scale/radius) duality:

- statement of scale/radius duality:
  bulk excitation at radial position z in AdS is
  manifested by CFT excitation on scale L ~ z
  [Susskind & Witten]
- UV cutoff in CFT ~ large radius cutoff in AdS
  - and RG flow ~ bulk radial `evolution'
- tells when interaction is possible:
  - e.g. different-scale CFT excitations at same position
    don't interact (since in bulk dual, radially separated)
    => color transparency
- provides useful intuition:
  eg. object falling into a black hole ↔
  CFT excitation spreads and thermalizes
  [Banks, Douglas, Horowitz, Martinec]



# Black holes in AdS:

Consider generic smooth initial data which collapses to a BH.
In CFT such generic initial data will thermalize.

- ✶ Collapse to black hole in gravity $\iff$ thermalization in CFT
- ✶ Stationary black hole $\iff$ thermal equilibrium (at same T)
- ✶ Quasinormal modes $\iff$ approach to thermal equilibrium
  [Horowitz, VH]
- ✶ Horizon response properties $\iff$ transport coefficients in CFT
  [Kovtun, Son, Starinets]

- ✶ Long-wavelength, small frequency deformations $\iff$ fluid flows
- ✶ Fluid/gravity correspondence [Bhattacharyya, VH, Minwalla, Rangamani]:

Einstein's equations in bulk contain relativistic Navier-Stokes equations
for the boundary conformal fluid.

# Spherical black hole in AdS

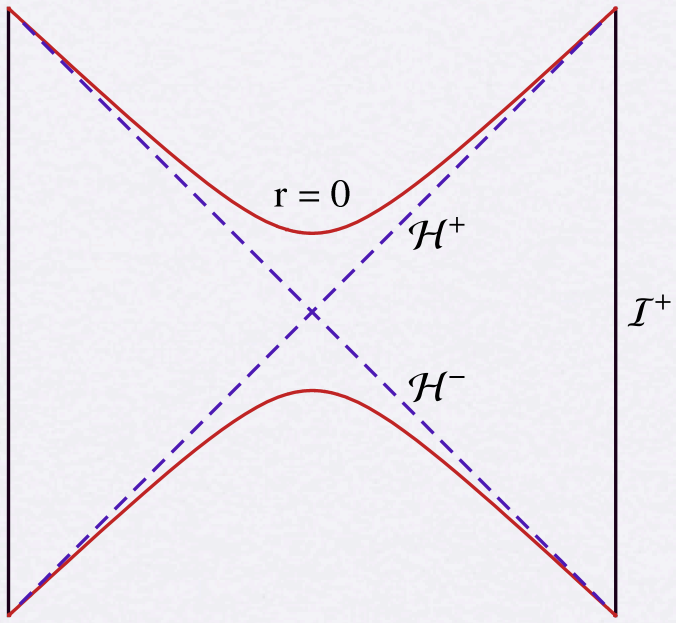
global Schwarzschild-AdS$_{d+1}$ black hole metric:

$$ds^2 = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2\, d\Omega_{d-1}^2$$

with $$f(r) = 1 + \frac{r^2}{\ell^2} - \frac{r_+^{d-2}}{r^{d-2}} \left(1 + \frac{r_+^2}{\ell^2}\right)$$

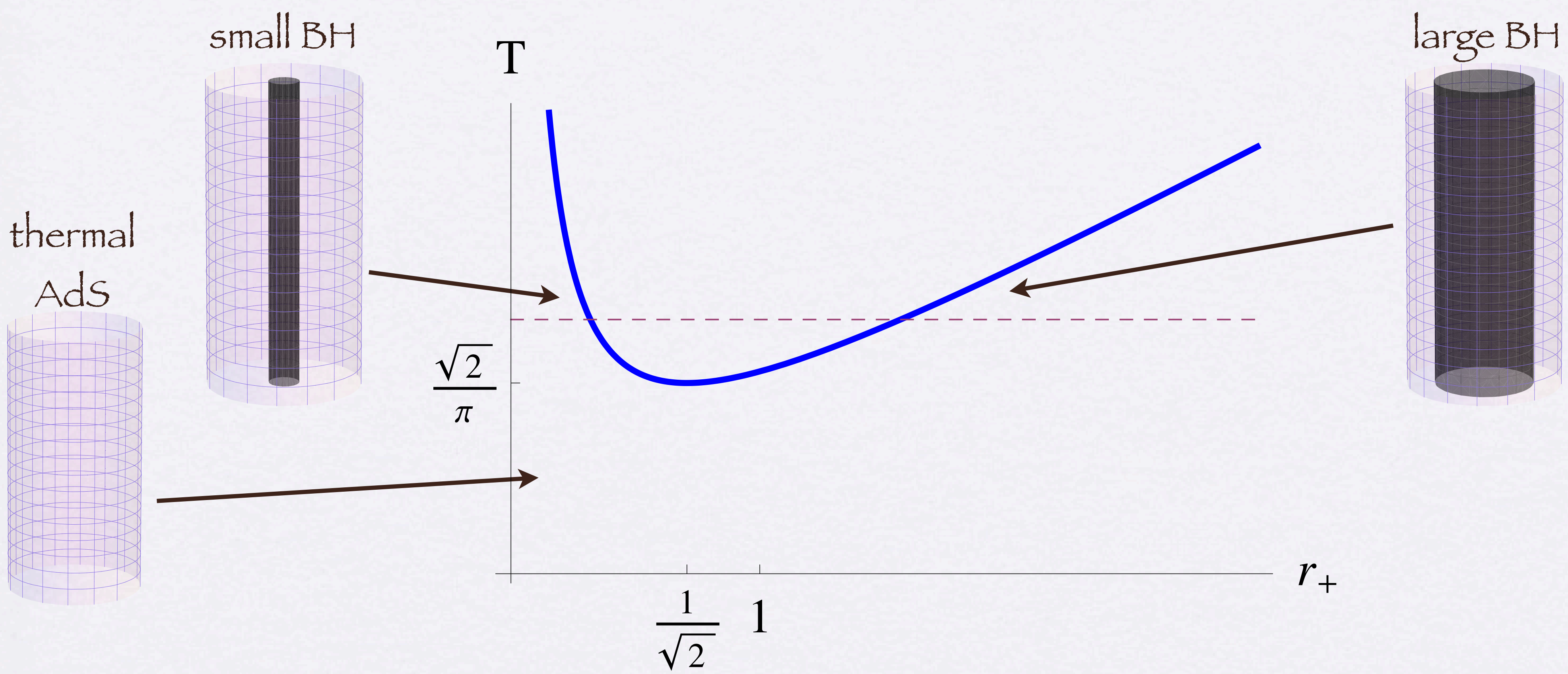
Causal structure:

- spacelike curvature singularity at r=0
- regular event horizon at r=r$_+$
- timelike AdS boundary at r=∞



# BH temperature:

- two static `thermal' geometries in bulk:
  - thermal AdS & Schwarzschild-AdS black hole
  - geometries exchange dominance when BH radius $r_+$ = AdS radius =1
  - free energy jumps from O(1) at low T to O($N^2$) at high T




- Hawking-Page transition $\longleftrightarrow$ confinement-deconfinement transition

# Planar black hole in AdS

$$ds^2 = r^2 \left( -f(r)\, dt^2 + \sum_i (dx^i)^2 \right) + \frac{dr^2}{r^2\, f(r)} \qquad \text{with} \quad f(r) = 1 - \frac{r_+^4}{r^4}$$

Arises as

- near-horizon geometry of near-extremal D3 branes
- limit of Schw-AdS when $r_+ \gg \ell$

in fact = 4-parameter family of stationary black hole solutions, parameterized by BH temperature $T = r_+/\pi$ and horizon velocity $u^\mu$.

## Induced boundary stress tensor:

$$T^{\mu\nu} = \pi^4\, T^4\, \left(\eta^{\mu\nu} + 4\, u^\mu\, u^\nu\right)$$

= Perfect fluid at temperature $T$ moving with velocity $u^\mu$ (such that $u^\mu\, u_\mu = -1$ ).

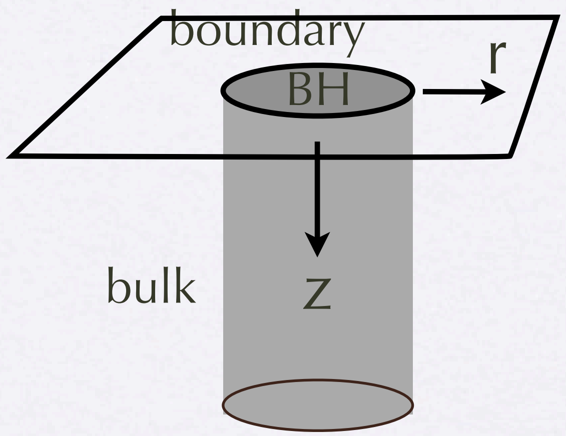
Note: this describes a conformal fluid ( $T^\mu_\mu = 0$ ) with no dissipation.

# AdS/CFT with curved boundary

- cf. Randall-Sundrum brane-world models:
  bulk: locally $AdS_5$ + brane; induced 4-d gravity on the brane
- here we take the limit as brane → AdS bdy
  ⇒ CFT on fixed, non-dynamical background
  - bdy metric = boundary condition on bulk metric
  - this bdy metric need not satisfy any field eqns (for a black hole, we can set temperature T and horizon size R independently)
  - bulk equations of motion: vacuum Einstein's equations w/ $\Lambda<0$
  - there can be multiple bulk solutions, distinguished by induced boundary stress tensor (i.e. different phases of the CFT)
- Particularly interesting case: $\text{Schwarzschild}_4$ bgd

# CFT on black hole background

- ∃ one well-known solution: AdS black string (= Schw warped in $AdS_5$) [Chamblin, Hawking, Reall]
  - exact bulk solution, singular on Poincare horizon
  - dynamically unstable [Gregory]
  - but induced stress tensor on the boundary does not grow with N: $T_{\mu\nu} \sim \mathcal{O}(1)$
  - corresponds to a confined phase of the CFT.




- we want to find other solutions with non-zero induced stress tensor $T_{\mu\nu} \sim \mathcal{O}(N^2)$ (deconfined phase)

# CFT on black hole background

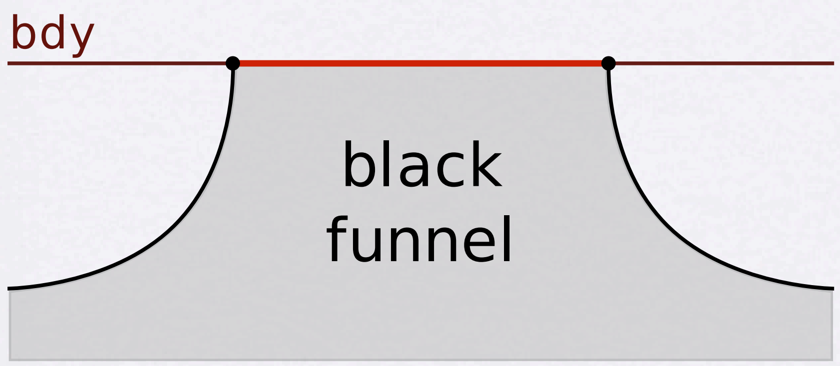
Deconfined phase should look thermal far away

→ planar Schw-AdS ...

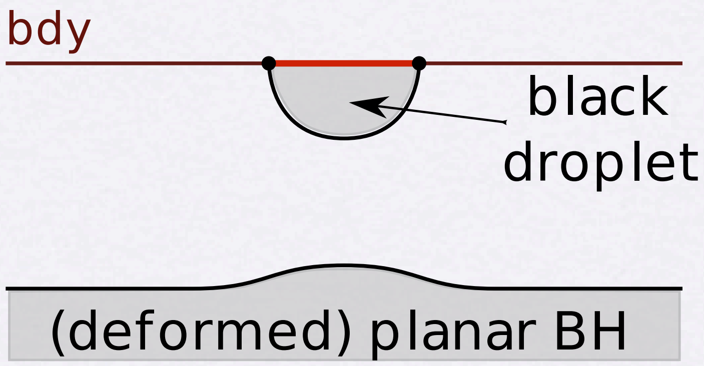
⇒ conjecture:

[VH, Marolf, Rangamani]

Deconfined phase in CFT on BH background has corresponding bulk dual solutions described by black funnels, or black droplets suspended over deformed planar black hole (depending on TR).




for $TR > 1$




for $TR < 1$

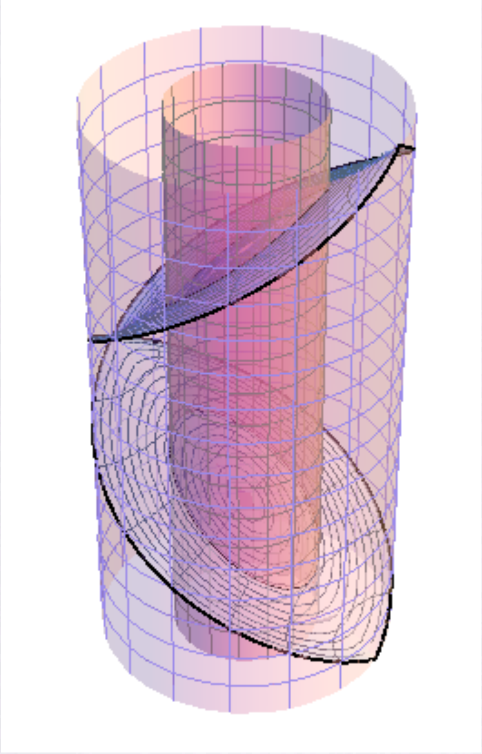
Would be interesting to find such solutions explicitly...

# Conformal soliton geometry

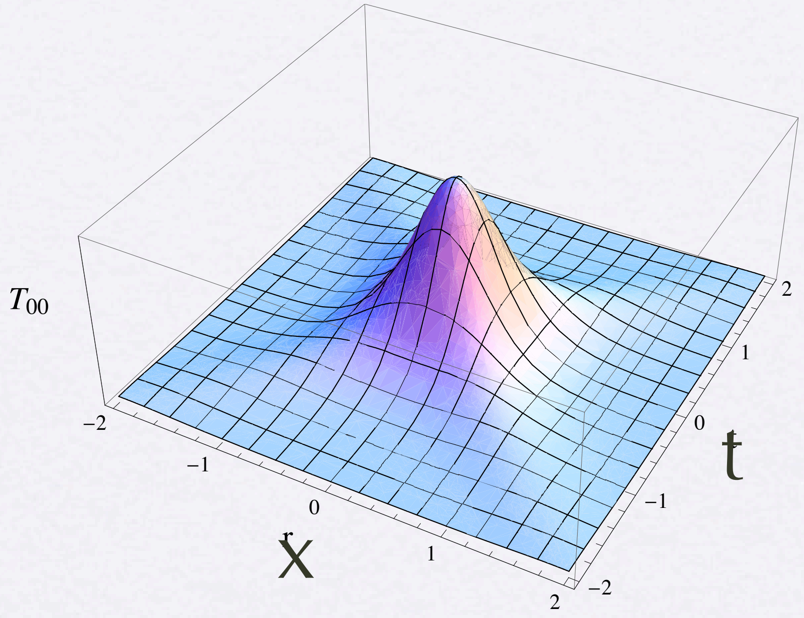
= spherical Schw-AdS BH in Poincare patch; useful for studying (mock) time dependence.

[Friess et.al.]

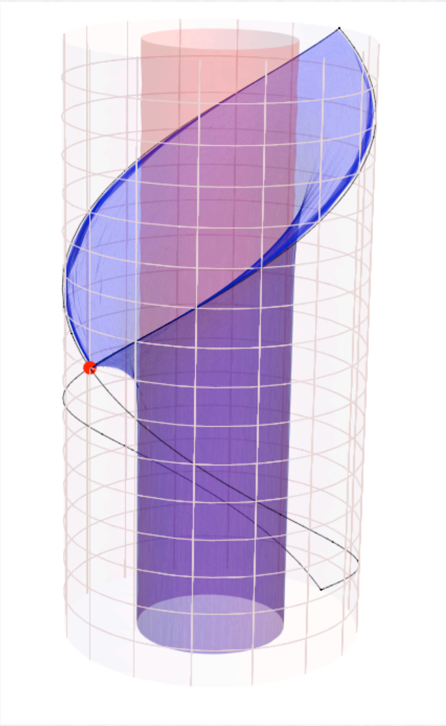
bulk geometry:



bdy energy density:




apparent vs. event horizon:



[Figueras et.al.]

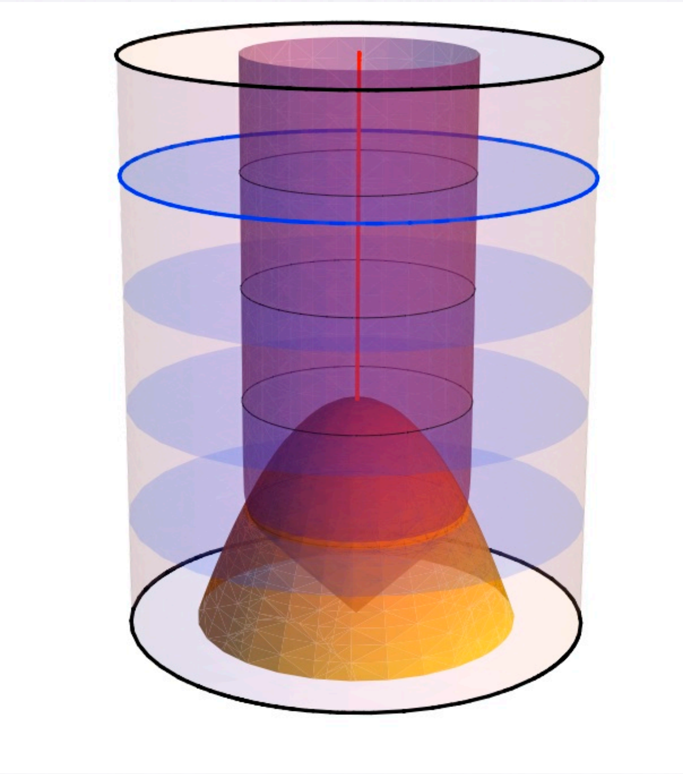
time-reversal invariant: no entropy production

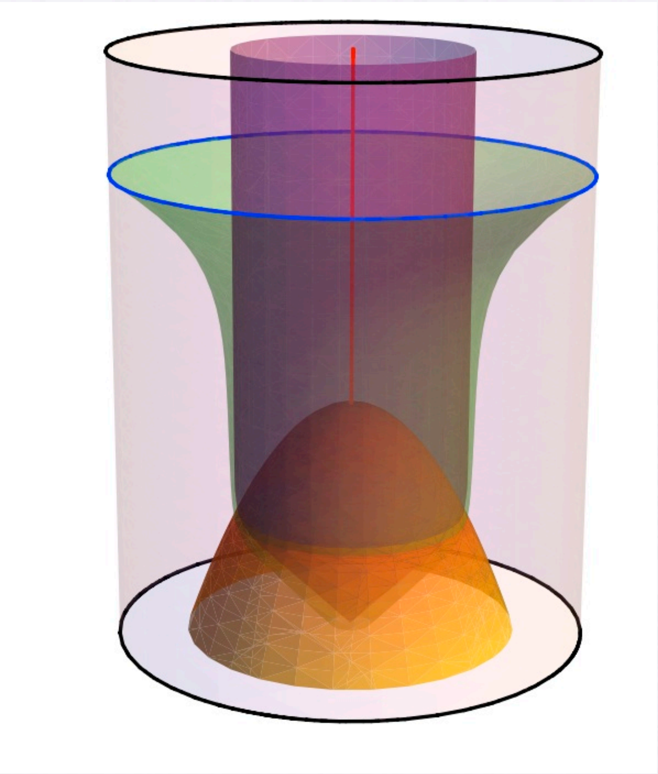
However: event horizon for Poincare patch ends on Poincare horizon, with ∞ area...

# Entropy dual?

What is the bulk dual of CFT entropy?
Area of some geometric surface? (suggestive due to 2nd Law.)

- Event horizon doesn't work (& is too teleological)
- Apparent horizon works better (but is foliation-dependent)





- What about causally trivial spacetimes such as `stars' in AdS?

# `Easy' open problems

- Details of thermalization (e.g. entropy growth) in CFT
  - in heavy ion collisions
  - by heating the system in different ways

  (& can we get linear growth in entropy as commonly observed?)

- Other interesting dynamics & its manifestation in CFT?
  - Critical behaviour & self-similarity (e.g. Choptuik scaling?)
  - Chaotic behaviour (e.g. Mixmaster singularity?)
- Cosmic censorship violation: what happens to CFT?
- New possible phases and phase transitions in CFT
  - with various matter content
  - in various backgrounds
- When are there interesting emergent symmetries in far IR?

# Hard open problems

* How does spacetime emerge?
  * how does one encode a local bulk observer in the CFT?
  * what is the relation between bulk and boundary `time'?
  * what is the CFT dual of bulk causal structure?
* How does CFT encode (& `resolve') curvature singularities?
* BH evaporation & Information paradox resolution
* Which QFTs (and which states) admit a classical gravity dual?
* Behaviour at finite $N$ and/or $\lambda$
* Can one extend AdS/CFT to gravity with other asymptotics?
  * e.g. for asymp.flat spacetimes, de Sitter, or other cosmologies
  * more complicated causal structure
* & if so, what is the underlying fundamental principle?

# THE END...