Decoupling perturbations *à la* Teukolsky in higher dimensions

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Motivation

- Soon after the discovery of Kerr metric, the status of its classical stability became an active area of research.

The full non-linear problem remains unsolved to this day.

Even if we consider the linear problem, one is left with solving the linearised Einstein equation on the Kerr background. Significant progress in proving the linear stability of the Kerr solution was made by Teukolsky.

- For algebraically special solutions (including the Kerr class), one is able to derive from the original perturbation equation, a decoupled equation, satisfied by the gauge invariant perturbed Weyl scalar $\delta\Psi_0 = \delta(\ell^a m^b \ell^c m^d C_{abcd})$ [Teukolsky 1972, 1973].

That is to say, there exists operator $\mathcal{S}$ such that

$$\mathcal{S}\left(\mathcal{E}(h)\right) = \mathcal{O}(\delta\Psi_0).$$

The decoupled equation,

$$\mathcal{O}(\delta\Psi_0) = 0,$$

is much easier to study. Furthermore, the existence of a decoupled equation allows one to find all solutions of the original perturbation equation [Cohen, Kegeles 1974; Chrzanowski 1975; Wald 1978].

- A study of the decoupled equation is sufficient for an analysis of the linear stability of the solution.

- The classical stability of higher dimensional black holes has been at the focus of much recent research.

The stability of the Schwarzschild-Tangherlini solution has been demonstrated using a formalism analogous to that used to find the Regge-Wheeler and Zerilli equations for the 4d Schwarzschild solution [Ishibashi, Kodama 2003].

It has been expected for some time that ($d \geq 6$) Myers-Perry solutions in the ultraspinning regime suffer from Gregory-Laflamme type instabilities [Emparan, Myers 2003].

There has been a great deal of recent progress in using numerical methods to tackle and confirm such expectations [Dias *et al.* 2009; Shibata, Yoshino 2009; Dias *et al.* 2010; Shibata, Yoshino 2010; Dias *et al.* 2010...]

- There is evidence of a relation between the uniqueness and stability of black hole solutions.

Aim

- Find a Teukolsky like formalism in higher dimensions that allows us to decouple perturbations of (a particular class of) Myers-Perry solutions.

Such a framework would facilitate a much simpler study of linear perturbations of Myers-Perry black holes.

Also, it may allow a study of regimes that are currently inaccessible to numerical investigations.

Decoupling perturbations in higher dimensions [Durkee, Reall 2010]

The Teukolsky framework for decoupling perturbations makes use of the Newman-Penrose (NP) formalism [Newman, Penrose 1962], in which one works with a null frame that is constructed using privileged null vector fields (PNDs in this case). Thus, general covariance is fully broken.

The Geroch-Held-Penrose (GHP) formalism [Geroch *et al.* 1972] is like the NP formalism, expect that some covariance is restored by allowing Lorentz transformations that preserve the null directions.

The status of Teukolsky's result was studied by Durkee and Reall using a higher dimensional generalisation of the GHP formalism [Durkee *et al.* 2010].

In higher dimensions,

$$
\begin{aligned}
(\ell, n, m, \bar{m}) &\rightarrow (\ell, n, m_i), \\
g_{ab} = 2\ell_{(a}n_{b)} - 2m_{(a}\bar{m}_{b)} &\rightarrow g_{ab} = 2\ell_{(a}n_{b)} + m_{i\,a}m_{i\,b}, \\
(\Psi_0; \Psi_1; \Psi_2) &\rightarrow (\Omega_{ij}; \Psi_{ijk}; \Phi_{ijkl}, \Phi^{\mathrm{A}}_{ij}), \\
\text{Algebraically special} &\rightarrow \Omega_{ij} = \Psi_{ijk} = 0.
\end{aligned}
$$

- Teukolsky: for algebraically special solutions, Weyl Scalar $\delta\Psi_0$ decouples, i.e. $\mathcal{O}(\delta\Psi_0) = 0$.

Higher dimensional analogue of this statement would be that for solutions for which algebraically special solutions $(\Omega^{(0)}_{ij} = \Psi^{(0)}_{ijk} = 0)$, of which the Myers-Perry solution is an example, the gauge invariant perturbed object $\delta\Omega_{ij}$ satisfies some decoupled equation $\mathcal{O}(\delta\Omega_{ij}) = 0$.

However, this is not the case [Durkee, Reall 2010].

- Ω_{ij} only decouples for Kundt solutions.

A solution is Kundt if it admits a null geodesic congruence with vanishing optics. These solutions are necessarily algebraically special.

Examples include gravitational wave solutions and the near-horizon geometry of extremal black holes.

One important reason for this result is that algebraic specialness gives more constraints on the optics in 4d than it does in higher dimensions (Goldberg-Sachs theorem).

Decoupling perturbations of Schwarzschild-Tangherlini

- Although, Ω_{ij} does not decouple on a black hole background, it could be that some other gauge invariant quantity formed from Ω_{ij} does.

That is we may be able to add some other gauge invariant quantity to Ω_{ij} so that it decouples.

Consider this possibility for the Schwarzschild-Tangherlini solution as a simple class within the Myers-Perry family.

In order to find what gauge invariant quantity needs to be added to Ω_{ij} such that it decouples, we need to investigate what obstructs the decoupling of Ω_{ij}.

Using the Bianchi identities as written in the higher dimensional GHP formalism, one can derive an equation of the form

$$\mathcal{O}\Omega_{ij} \sim \Phi^{\mathrm{S}}{}^{t}_{ij}.$$

Thus, $\Phi^{\mathrm{S}}{}^{t}_{ij}$, which is gauge invariant, obstructs the decoupling of Ω_{ij}.

Now, what obstructs the decoupling of $\Phi^{\mathrm{S}}{}^{t}_{ij}$?

Repeating this iterative process, one finds that the obstruction is always one of three basic gauge invariant quantities.

- Ansatz: The gauge invariant quantity that decouples must we a linear combination of these three quantities.

However, we find that such a quantity can never decouple.

- Thus a gauge invariant quantity formed from completing Ω_{ij} cannot decouple.

One can consider other gauge invariant quantities constructed from components of the Weyl tensor. Most of these do not decouple.

However,

- Φ^{A}_{ij} does decouple.

Metric perturbations of the Schwarzschild-Tangherlini solution have been studied by Ishibashi and Kodama [Ishibashi, Kodama 2003].

Decomposing Φ^{A}_{ij} into a transverse and vector part as is done by Ishibashi and Kodama, suggests that the transverse part will lead to trivial perturbations, while the vector part corresponds to the vector modes of Ishibashi and Kodama.

It is not clear how this decoupling result can be extended to the general Myers-Perry case, since Φ^{A}_{ij} is no longer gauge invariant in the general rotating case.

It is conceivable that there exists a gauge invariant quantity constructed from Weyl components that decouples in the general rotating case and reduces to Φ^{A}_{ij} once rotation is turned off.

We have only considered gauge invariant quantities constructed from components of the Weyl tensor here. There could, of course, be other gauge invariant quantities, whose relation to components of the Weyl tensor is not clear, that decouple.

One could also consider the decoupling of non-gauge invariant quantities, but what this would mean is not so clear.

Thank you.