Oscillations of BHs & Stars

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Numerical Relativity and High Energy Physics,
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Oscillations of BHs & Stars

"The aim of this workshop is to bring together world experts in two extremely active and successful, but up to now essentially disjoint, research fields: numerical relativity and high energy physics. …"

Stellar oscillations provide an examples of how general relativity, its numerical implementations, and some fundamental issues in high energy physics are parts of a problem which is, by its own nature, interdisciplinary.

I will investigate compact object oscillations using gravitational radiation as a probe.

Remind: some basic results of the theory of black hole perturbations

Perturbations of Schwarzschild black holes

$$\frac{d^2 Z_\ell^\pm}{dr_*^2} + \left[\omega^2 - V_\ell(r)\right] Z_\ell^- = 0 \qquad r_* = r + 2M log(r/2M - 1)$$

$$V_\ell^-(r) = \frac{1}{r^3}\left(1 - \frac{2M}{r}\right)\left[\ell(\ell+1)r - 6M\right]$$

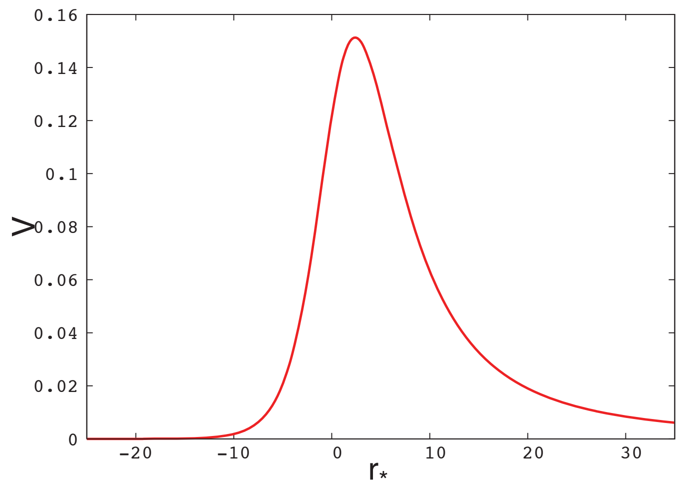
Regge&Wheeler 1957

$$V_\ell^+(r) = \frac{2(r-2M)}{r^4(nr+3M)^2}\left[n^2(n+1)r^3 + 3Mn^2r^2 + 9M^2nr + 9M^3\right]$$

F. Zerilli 1970

$$n = (\ell+1)(\ell-2)/2$$

$Z_\ell^\pm$ - axial (odd) perturbations
+ polar (even) perturbations



A wave equation also for the perturbations of Kerr black holes

$$\Delta R_{lm,rr} + 2(s+1)(r-M)R_{lm,r} + V(\omega,r)R_{lm} = 0$$
$$\Delta = r^2 - 2Mr + a^2$$

S.Teukolsky 1972 Phys. Rev. Lett. 29, 1114
S.Teukolsky 1973 Ap. J. 185, 635

$$\psi_s(t,r,\theta\varphi) = \frac{1}{2\pi}\int e^{-i\omega t}\sum_{l=|s|}^{\infty}\sum_{m=-l}^{l} e^{im\varphi}S_{lm}(\cos\theta)R_{lm}(r)d\omega$$

$S_{lm}(\cos\theta)$ satisfies the equations of the oblate spheroidal harmonics

s= is the spin-weight parameter, s=0, ± 1, ± 2,
for scalar, electromagnetic and gravitational perturbations

the potential is complex and depends on m and on frequency

$$\begin{aligned} V(\omega,r) &= \frac{1}{\Delta}\left[(r^2+a^2)^2\omega^2 - 4aMrm\omega + a^2m^2 + \right. \\ &+ \left. 2is(am(r-M) - M\omega(r^2-a^2))\right] + \left[2is\omega r - a^2\omega^2 - A_{lm}\right] \end{aligned}$$

- Black hole perturbations are described by wave equations, with one-dimensional potential barrier generated by the spacetime curvature

- Black hole perturbations can be studied as a scattering problem.

- Black holes oscillate at some characteristic, complex frequencies: the Quasi-Normal Mode frequencies.

  Standard methods used in quantum mechanics can be used to find the quasi-normal mode frequencies: they are the singularities of the scattering cross-section associated the wave equation

In quantum mechanics the equation which expresses the symmetry and unitarity of the S-matrix

$$\boxed{|R|^2 + |T|^2 = 1}$$

is an energy conservation law: if a wave of unitary amplitude is incident on one side of the potential barrier, it gives rise to a reflected and a transmitted wave such that the sum of the square of their amplitudes is still one.

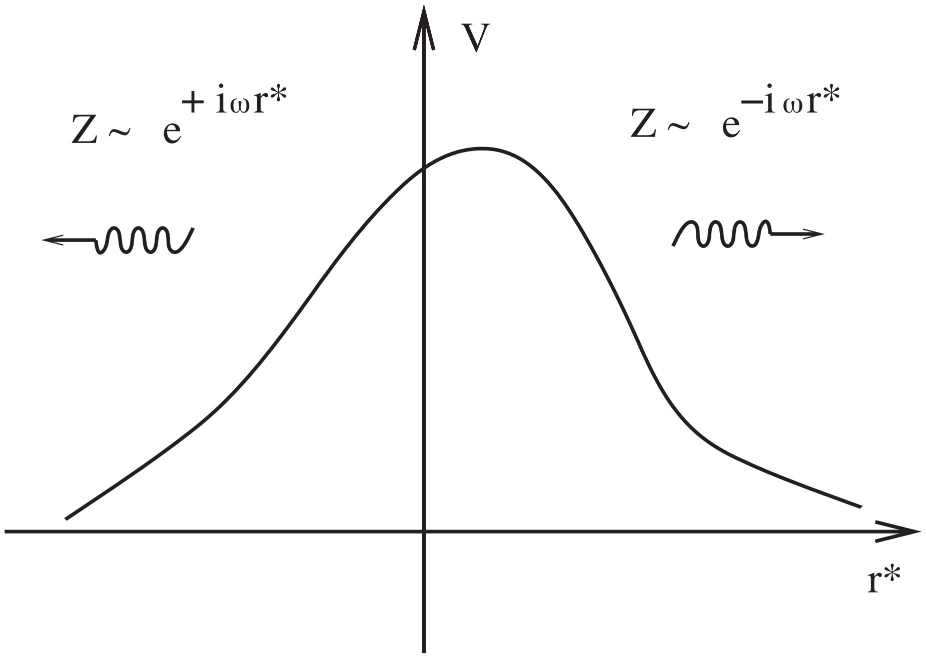
This conservation law is a consequence of the constancy of the Wronskian of pairs of independent solutions of the Schroedinger equation.

Same is for black holes: the constancy of the Wronskian of two independent solutions of the black holes wave equation, allows to write the same equation relating the reflection and transmission coefficients of the potential barrier.

Energy conservation also governs phenomena involving gravitational waves emitted by perturbed black holes.

Black hole Quasi-Normal modes

The wave equation for the functions $Z^{\pm}$ (+ for polar, - for axial) allows complex frequency solutions which satisfy the following boundary conditions



These boundary conditions are satisfied for a discrete set of frequencies

$$\omega_k + i\ 1/\tau_k$$

$$Z^{\pm}_{\ell} \to e^{i\omega r_*}, \quad r_* \to -\infty, \quad \text{(pure ingoing wave at the black hole horizon)}$$

$$Z^{\pm}_{\ell} \to e^{-i\omega r_*}, \quad r_* \to +\infty, \quad \text{(pure outgoing wave at infinity)}$$

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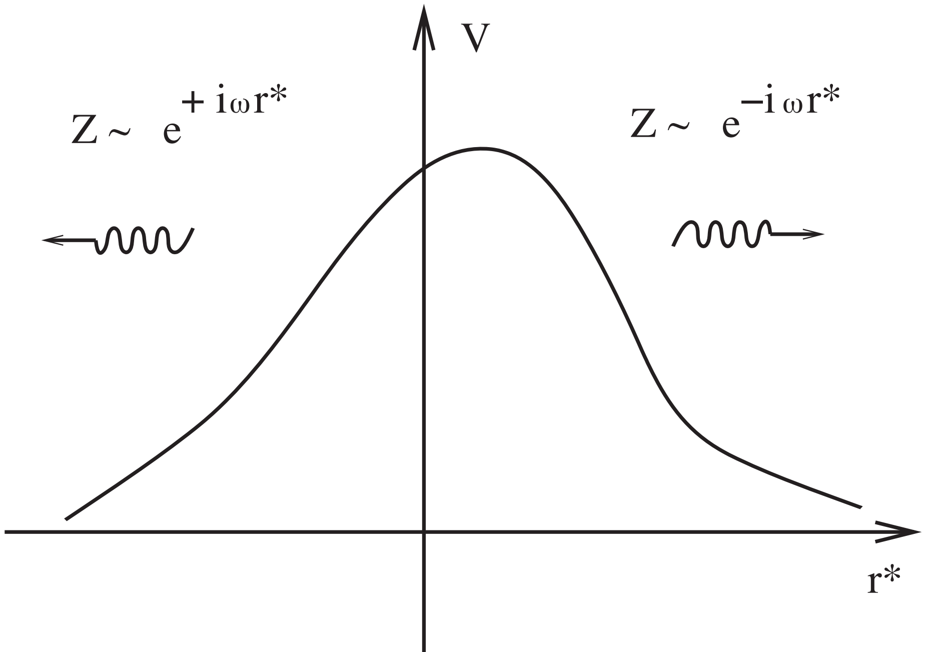

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$$\omega_k + i\ 1/\tau_k$$

When the black hole oscillates in these modes, the solution at radial infinity is a damped, outgoing wave

$$\boxed{Z^{\pm}_{\ell} \to A_\ell e^{i\omega_k(t-r_*)} \equiv A_\ell e^{-\frac{(t-r*)}{\tau_k}} e^{i\omega_k(t-r_*)},}$$

Firstly calculated by Chandrasekhar & Detweiler in 1975

	$M\omega_k + iM/\tau_k$		$M\omega_k + iM/\tau_k$
$\ell = 2$	0.3737+i0.0890	$\ell = 3$	0.5994+i0.0927
	0.3467+i0.2739		0.5826+i0.2813
	0.3011+i0.4783		0.5517+i0.4791
	0.2515+i0.7051		0.5120+i0.6903

The frequencies and damping times of the axial and polar quasi normal modes are equal, i.e.

Axial and Polar Quasi-Normal modes of non rotating black holes are

isospectral

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Assuming that the BH mass is n times the mass of the Sun, converting to physical units we find

For the lowest mode

$$\nu = \frac{c \times (M\omega_k)}{2\pi n M_\odot} = \frac{\mathbf{32.26}}{\mathbf{n}} (\mathbf{M\omega_k})\ \mathrm{kHz}$$

$$\tau = \frac{nM_\odot}{(M/\tau_k)c} = \frac{\mathbf{n \times 0.49 \cdot 10^{-5}}}{(\mathbf{M/\tau_k})}\ \mathrm{s}.$$

$$\nu = \frac{32.26}{n}(0.3737)\ \mathrm{kHz} \sim \frac{\mathbf{12}}{\mathbf{n}}\ \mathbf{kHz}$$

$$\tau = \frac{n \times 0.49 \cdot 10^{-5}}{(0.089)}\ \mathrm{s} \sim \mathbf{n \times 5.5 \cdot 10^{-5}\ s}$$

Do the QNM-frequencies of black holes fall in the bandwidth of gravitational wave detectors?

For the lowest mode

$$\boxed{\begin{aligned} \nu &= \frac{32.26}{n}(0.3737)\ \text{kHz} \sim \frac{\mathbf{12}}{\mathbf{n}}\ \mathbf{kHz} \\ \tau &= \frac{n \times 0.49 \cdot 10^{-5}}{(0.089)}\ \text{s} \sim \mathbf{n \times 5.5 \cdot 10^{-5}\ s} \end{aligned}}$$

Virgo/LIGO bandwidth: ~ [10 Hz - (1-2) kHz]

These detectors can detect BH oscillations if the BH mass is in the range

$$10\ M_\odot < M < 10^3\ M_\odot$$

corresponding to a frequency in the range

$$\nu \in [12\ \text{Hz},\ 1.2\ \text{kHz}]$$

(provided the signal is sufficiently strong).

For the lowest mode

$$\boxed{\begin{aligned} \nu &= \frac{32.26}{n}(0.3737)\ \mathrm{kHz} \sim \frac{\mathbf{12}}{\mathbf{n}}\ \mathbf{kHz} \\ \tau &= \frac{n \times 0.49 \cdot 10^{-5}}{(0.089)}\ \mathrm{s} \sim \mathbf{n \times 5.5 \cdot 10^{-5}\ s} \end{aligned}}$$

LISA bandwidth: ~ [10^{-4} - 10^{-1} Hz]

LISA will see oscillating black holes with mass in the range

$$1.2 \cdot 10^5\ M_\odot < M < 1.2 \cdot 10^8\ M_\odot$$

For instance, LISA should detect signals emitted by the massive black hole at the center of our Galaxy SGR A*, whose mass is

$$M = (3.7 \pm 0.2) \cdot 10^6\ M_\odot.$$

Kerr Quasi-Normal mode frequencies

from *S. Chandrasekhar The Mathematical Theory of Black Holes Oxford University Press 1983*

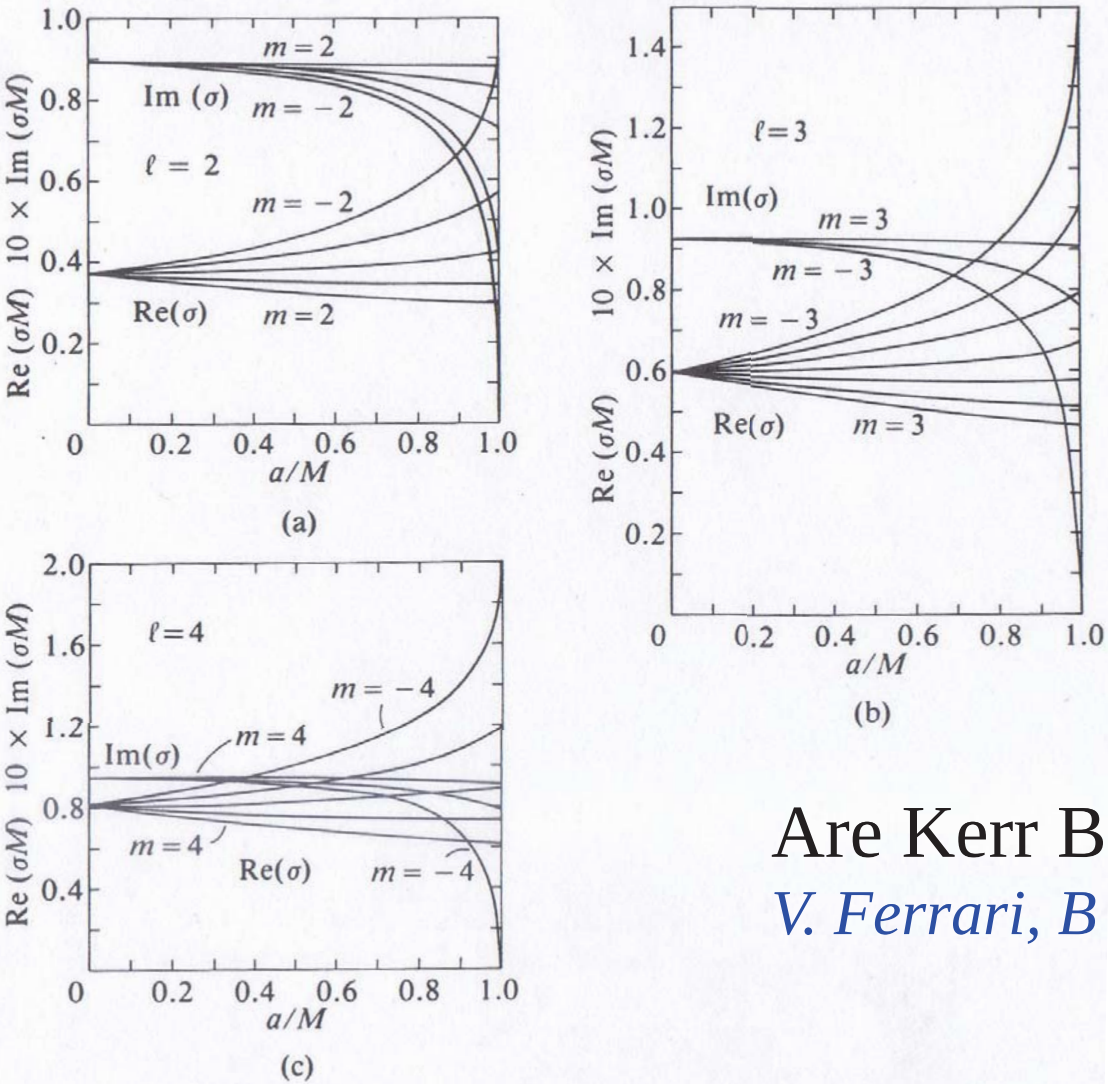


FIG. 45. The real and imaginary parts of the resonant frequency of a Kerr black-hole as a function of the parameter, a, for various values of l and m. (a). The case $l = 2$; for all values of m between -2 and 2. (b). The case $l = 3$; the imaginary part only for $m = -3$, 0, and 3 are illustrated. (c). The case $l = 4$; the real part only for even values of m are illustrated; and the imaginary part only for $m = -4$, 0, and 4 are illustrated.

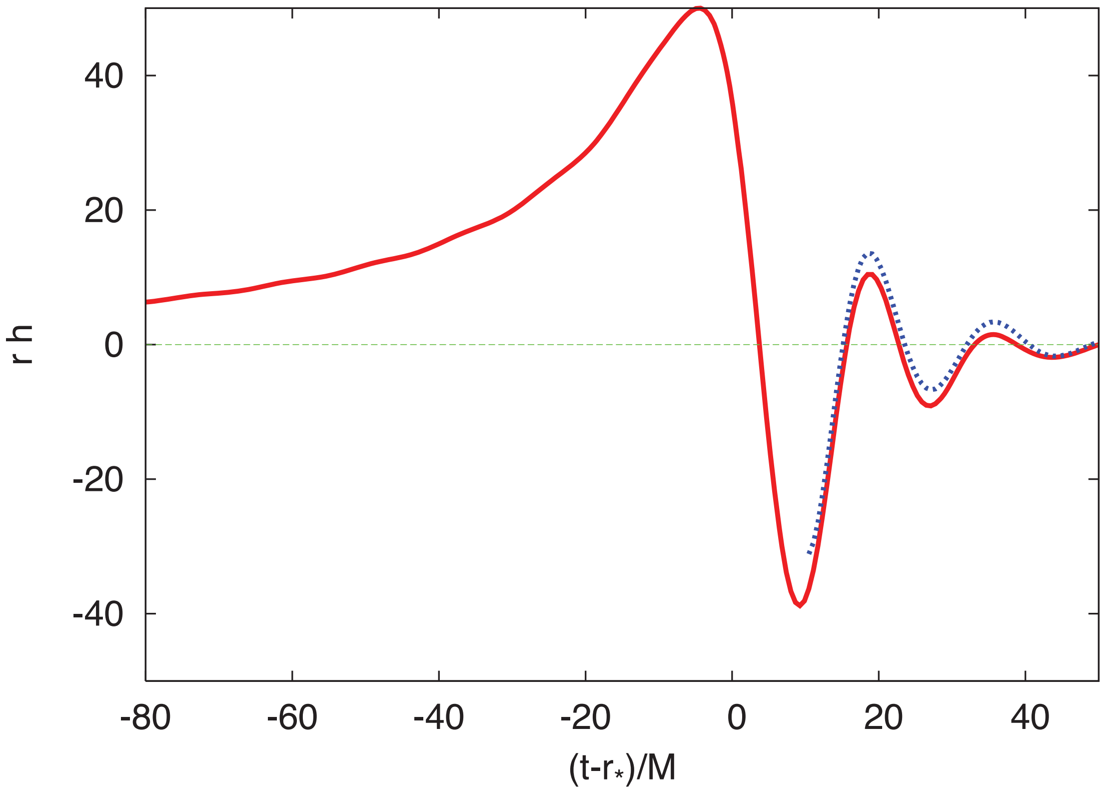
Are Kerr BH “marginally unstable”?

V. Ferrari, B Mashoon PRL 52, 1984,

Phys. Rev. D30 1984

Can the QNMs be excited in astrophysical phenomena?

First numerical experiment: radial capture of a small, massive particle



The "ringing tail" appears to be a superposition of the first few QNMs

These modes are excited when the infalling mass is at distances smaller than ~ 4 M_{BH}

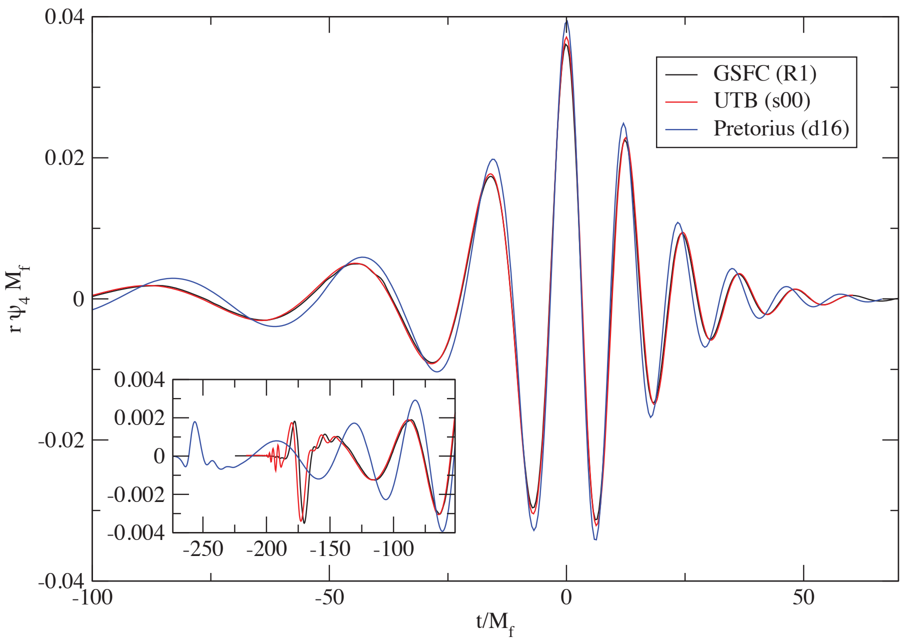
M.Davis, R.Ruffini, J.Tiomno Phys. Rev., D5, 2932 (1972)

V.Ferrari, R.Ruffini Phys. Lett. B98, 381 (1984)

The QNMs frequencies provide information on the BH spacetime in the strong field regime

NOWADAYS: Quasi-Normal Modes excitation is seen in numerical simulations of gravitational collapse to a black hole and of black hole coalescence.

Black hole coalescence



Gravitational radiation waveform emitted by two coalescing black holes

Baker J.G., Campanelli M., Pretorius F. and Zlochower Y.,
Class. Quant. Grav., 24, S25 (2007)

Recent developments of the theory of BH oscillations

In 2000 Horowitz and Hubeny proposed that the study of the black hole QNM's in anti-de Sitter spacetime could be useful to determine some properties of conformal field theories.

Horowitz G.T. and Hubeny V.E. 2000 Phys. Rev.D 62, 024027

Stimulated by this work, many authors computed the QNM eigenfrequencies in anti-de Sitter spacetime.

HoWang B., Lin C.Y. and Abdalla E. 2000 Phys. Lett. B 481, 79;
Wang B., Molina C. and Abdalla E. 2001 Phys. Rev. D 63, 084001;
Cardoso V. and Lemos J.P.S. 2001 Phys. Rev. D 63, 124015;
Cardoso V. and Lemos J.P.S. 2001 Phys. Rev.D 64, 084017;
Berti E. and Kokkotas K.D. 2003 Phys. Rev. D 67, 064020;
Cardoso V., Konoplya R. and Lemos J.P.S. 2003 Phys. Rev.D 68, 044024

It is worth reminding that the anti-de Sitter solution of Einstein's equations describes a universe with a negative cosmological constant; therefore these black holes should not be considered as astrophysical objects.

In 2003 Dreyer and Motl suggested that, in the asymptotic limit $n \rightarrow \infty$, black hole quasi-normal modes would allow to fix the value of the Immirzi parameter, a key parameter in loop quantum gravity.

Dreyer O. 2003 PRL 90, 081301;
Motl L. 2003 Adv. Theor. Math. Phys. 6, 1135

Following this proposal, studies of the asymptotic limit of QNM have further been developed.

Nollert H.-P. 1993 Phys. Rev. D 47, 5253;
Andersson N. 1993 Class. Quantum Grav. 10, L61;
Barreto A.S. and Zworski M. 1997 Math. Res. Lett. 4, 103;
Padmanabhan T. 2004 Class. Quantum Grav.21, L1;
Motl L. and Neitzke A. 2003 Adv. Theor. Math. Phys. 7, 307;
Cardoso V., Natario J. and Schiappa R. 2004 J. Math. Phys. 45, 4698
Berti E., Cardoso V., Kokkotas K.D. And Onozawa H. 2003 Phys. Rev. D 68, 124018;
Berti E., Cardoso V. and Yoshida S. 2004 Phys. Rev. D 69, 124018

More generally, inspired by these consideration in the contexts of string theory and loop quantum gravity, in recent years many authors have computed the eigenfrequencies of black hole quasi-normal modes in various background spacetimes, both in four dimensions and for higher dimensional spacetimes

Cardoso V. and Lemos J.P.S. 2003 Phys. Rev. D 67, 084020;
Konoplya R.A. 2003 Phys. Rev. D 68, 024018;
Cardoso V., Lemos J.P.S. and Yoshida S. 2004 Phys. Rev. D 69, 044004

Stellar oscillations

As for black holes, they are studied in the framework of perturbation theory;

Einstein + hydrodynamics equations need to be perturbed

For non rotating stars the theory can be developed in analogy with
the theory of Schwarzschild perturbations:

All perturbed quantities (metric functions+hydro variables) are expanded in spherical harmonics (scalar, vector and tensor), and the resulting equations split in two sets, the axial and the polar; and these equations are separable

REMIND: for black hole, both the axial and the polar equations are reducible to a wave equation with a potential barrier, i.e. the Regge-Wheeler and the Zerilli equation.

Stellar oscillations

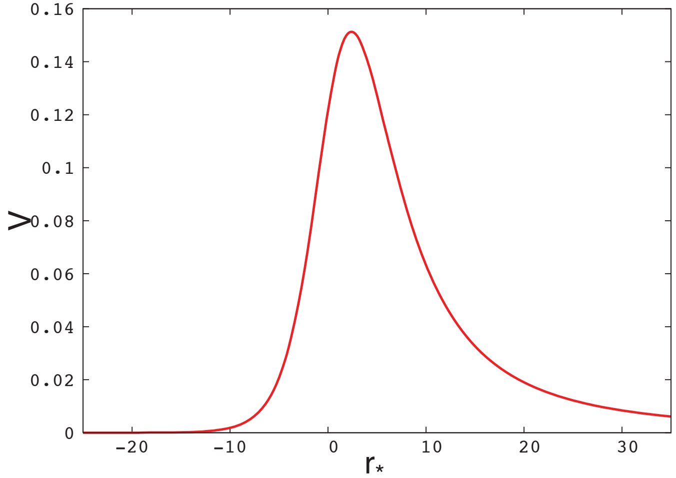
A wave equation for the axial perturbations: fluid motion is not excited

Chandrasekhar S. and Ferrari V. 1991, Proc. R. Soc. Lond. **434**, 449

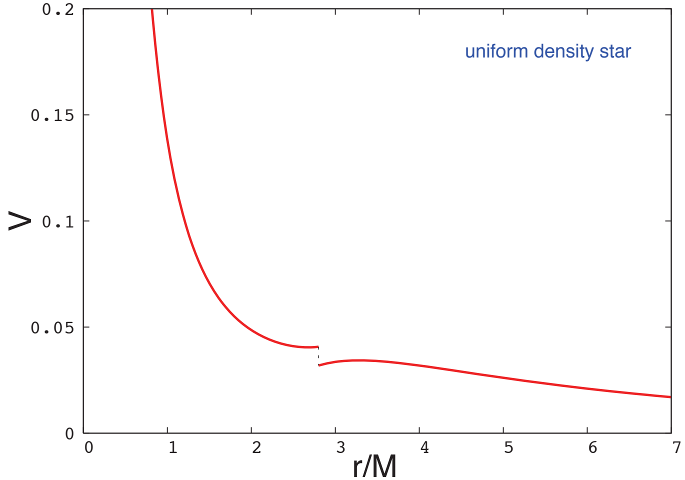
$$\frac{d^2 Z_\ell}{dr_*^2} + [\omega^2 - V_\ell(r)]Z_\ell = 0$$

$$V_\ell(r) = \frac{e^{2\nu}}{r^3}[\ell(\ell+1)r + r^3(\epsilon - p) - 6m(r)], \qquad \nu_{,r} = -\frac{p_{,r}}{\epsilon + p}$$

the potential barrier depends on how the energy-density and the pressure are distributed inside the star in its equilibrium configuration. For r > R it reduces to the Regge-Wheeler potential

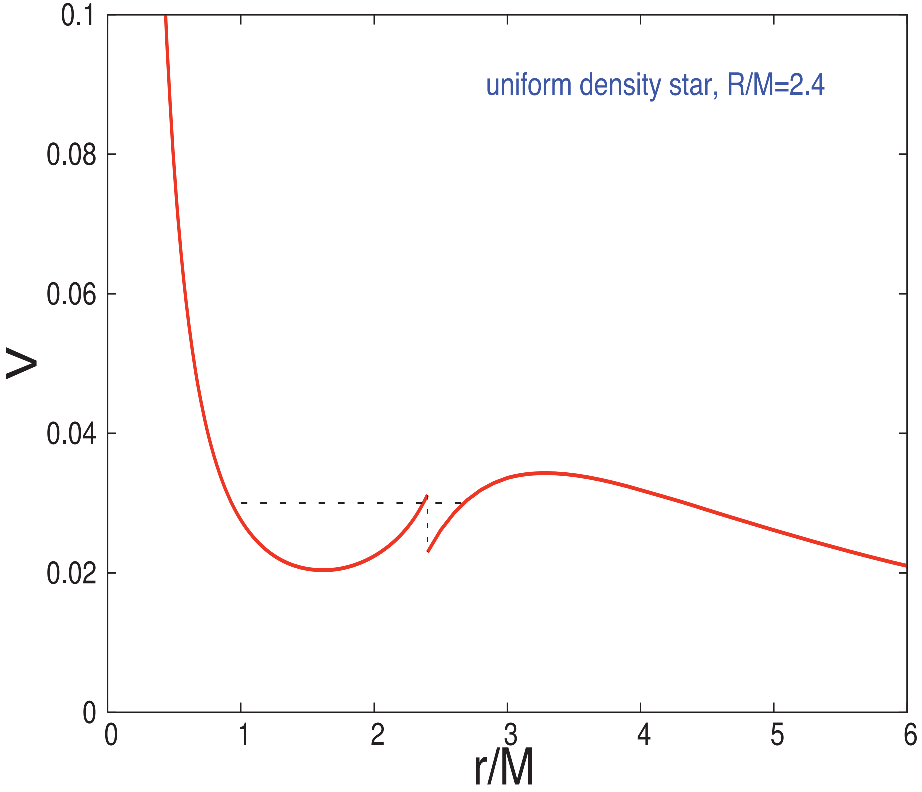


black holes: a one-dimensional potential barrier



stars: a central potential

Axial perturbations can be resonant! They are pure spacetime modes



if we look for solutions that are regular at r=0 and behave as pure outgoing waves at infinity

$$Z_\ell \longrightarrow e^{-i\omega r_*}, \qquad r_* \longrightarrow \infty$$

we find modes which do not exists in Newtonian theory

- if the star is extremely compact, the potential in the interior is a well, and if this well is deep enough there can exist one or more *slowly damped QNMs (or s-modes)*

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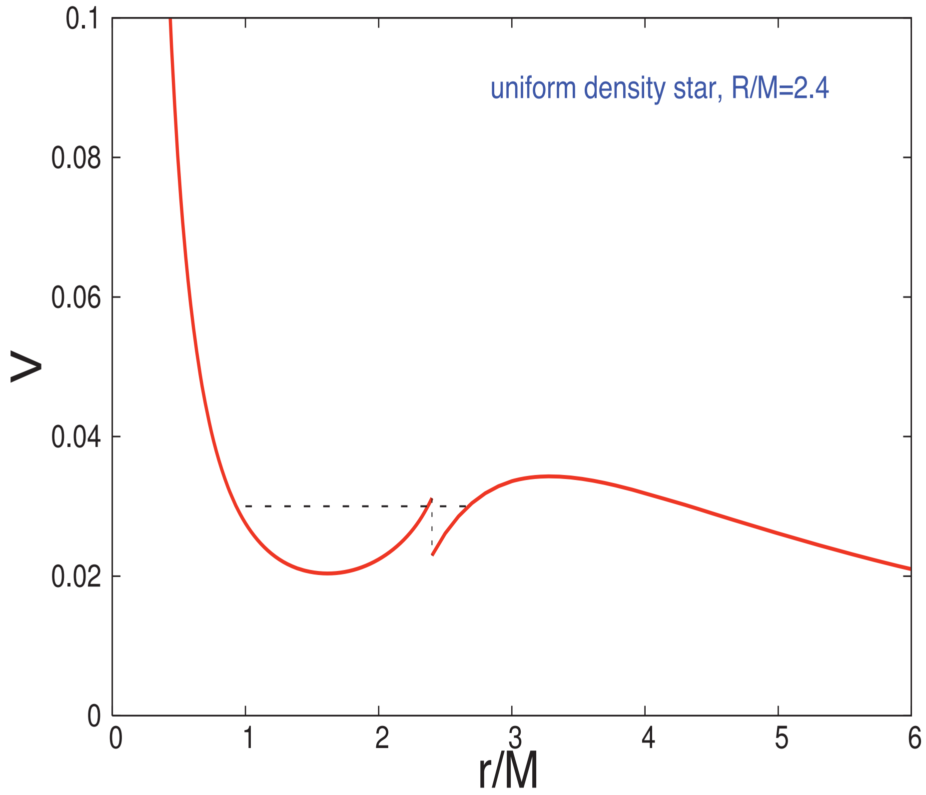

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Chandrasekhar S. and Ferrari V. 1991, Proc. R. Soc. Lond. **434**, 449

- another branch of modes are the *w- modes* they are associated to the scattering of GW- waves at the peaks of the barrier. They are *higly damped*

Chandrasekhar S. and Ferrari V. 1991, Proc. R. Soc. Lond. **434**, 449
Kokkotas K.S. 1994, MNRAS ***268***, 1015

	s-modes		w-modes		black hole	
$\frac{R}{M}$	ν_0 in kHz	τ in s	$\nu_0$in kHz	τ in s		
2.4	8.63	$1.5 \cdot 10^{-3}$	11.17	$1.7 \cdot 10^{-4}$	8.93	$7.5 \cdot 10^{-5}$
	–	–	14.28	$8.0 \cdot 10^{-5}$	8.28	$2.4 \cdot 10^{-5}$
	–	–	18.22	$5.7 \cdot 10^{-5}$	7.20	$1.4 \cdot 10^{-5}$
	–	–	22.67	$4.9 \cdot 10^{-5}$	6.01	$0.9 \cdot 10^{-5}$
2.3	5.62	0.54	11.11	$3.0 \cdot 10^{-4}$		
	7.56	$1.2 \cdot 10^{-2}$	13.04	$1.7 \cdot 10^{-4}$		
	9.33	$1.0 \cdot 10^{-3}$	15.15	$1.3 \cdot 10^{-4}$		
	–	–	17.44	$1.1 \cdot 10^{-4}$		
2.28	4.43	10.8	10.41	$5.5 \cdot 10^{-4}$		
	6.02	$2.5 \cdot 10^{-1}$	11.91	$2.9 \cdot 10^{-4}$		
	7.55	$1.4 \cdot 10^{-2}$	13.48	$2.1 \cdot 10^{-4}$		
	8.99	$1.8 \cdot 10^{-3}$	15.14	$1.7 \cdot 10^{-4}$		
2.26	2.60	$5.4 \cdot 10^{3}$	10.79	$7.6 \cdot 10^{-4}$		
	3.54	$1.7 \cdot 10^{2}$	11.69	$5.3 \cdot 10^{-4}$		
	4.48	$1.2 \cdot 10^{1}$	12.61	$4.2 \cdot 10^{-4}$		
	5.41	$1.4 \cdot 10^{-1}$	13.55	$3.6 \cdot 10^{-4}$		

Note that: $M = 1.35\, M_\odot = 1.35 \times 1.477 = 1.99$ km

$R = 2.4 \times 1.99 = 4.78$ km

extremely small!

The characteristic frequencies and damping times of the first four $\ell = 2$, s and w axial modes of homogenoeus stars, with $M = 1.35 M_\odot$, and different values of R/M. The data are compared with the eigenfrequencies of a black hole with the same mass.

Until very recently, the common belief was that *w- modes* are unlikely to be excited in astrophysical processes. However in 2005 it has been shown that, they are excited in the collapse of a neutron star to a black hole, just before the black hole forms

Baiotti L., Hawke I., Rezzolla L. and Schnetter E. 2005, *Phys. Rev. Lett.* **94**, 131101

We shall later see that the frequencies of the w-modes carry interesting information on the internal structure of the star.

POLAR MODES: fluid motion is excited.

different families of modes can be directly associated with different core physics.

f (fundamental)- **p** (pressure)- and **g** (gravity)- **r** –modes survive in the relativistic regime, but in GR they belong to complex eigenfrequencies since they are associated to GW emission

$$\boxed{..\omega_{g_n} < .. < \omega_{g_1} < \omega_f < \omega_{p_1} < .. < \omega_{p_n}..}$$

A mature neutron star also has elastic shear modes in the crust and superfluid modes. Magnetic stars may have complex dynamics due to the internal magnetic field.

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In GR there exists new families of modes for which fluid motion is negligible (spacetime modes)

w-modes are spacetime oscillations (high frequency very rapid damping). They exist both for polar and axial perturbations

s (trapped)-modes (axial modes ; they exist only for ultradense stars)

There is a lot of physics to explore!

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The modes frequency depends on the equation of state (EOS) of matter in the interior, on the rotation rate, on the phase of life the star is going through, namely on whether it is old and cold, or young and hot.

Gravitational wave asteroseismology

Suppose that a gravitational signal emitted by a perturbed neutron star is detected and, by an appropriate data analysis, we are able to determine the frequency of one or more mode:
will this information allow to constraints the equation of state of matter in the stellar core?

The EOS in the inner core of a neutron star is unknown and the energies prevailing in the inner core of a NS are unaccessible to high energy experiments on Earth

NEUTRON STARS ARE COSMIC LABORATORIES FOR EXTREME PHYSICS

The equation of state (EOS) in the interior of a neutron star is largely unknown

At densities lager than $\rho_0 = 2.67 \times 10^{14}\ \mathrm{g/cm^3}$ (equilibrium density of nuclear matter) the fluid is a gas of interacting nucleons

Available EOS have been obtained within models of strongly interacting matter, based on the theoretical knowledge of the underlying dynamics and constrained, as much as possible, by empirical data.

Two main, different approaches:

- nonrelativistic nuclear many-body theory **NMBT**

- relativistic mean field theory **RMFT**

Non Relativistic Nuclear Many-Body Theory **NMBT**

nuclear matter is viewed as a collection of pointlike protons and neutrons, whose dynamics is described by the nonrelativistic Hamiltonian:

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk}$$

-The two- and three-nucleon interaction potentials are obtained from fits of existing scattering data.
- ground state energy is calculated using either variational techniques or G-matrix perturbation theory

Relativistic mean field theory **RMFT**

- based on the formalism of relativistic quantum field theory, nucleons are described as Dirac particles interacting through meson exchange. In the simplest implementation of this approach the dynamics is modeled in terms of a scalar and a vector field.
- equations of motion are solved in the mean field approximation, i.e. replacing the meson fields with their vacuum expectation values
- the parameters of the Lagrangian density, i.e. the meson masses and coupling constants, can be determined by fitting the empirical properties of nuclear matter, i.e. binding energy, equilibrium density and compressibility

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NMBT and RMFT can be both generalized to account for the appearance of hyperons

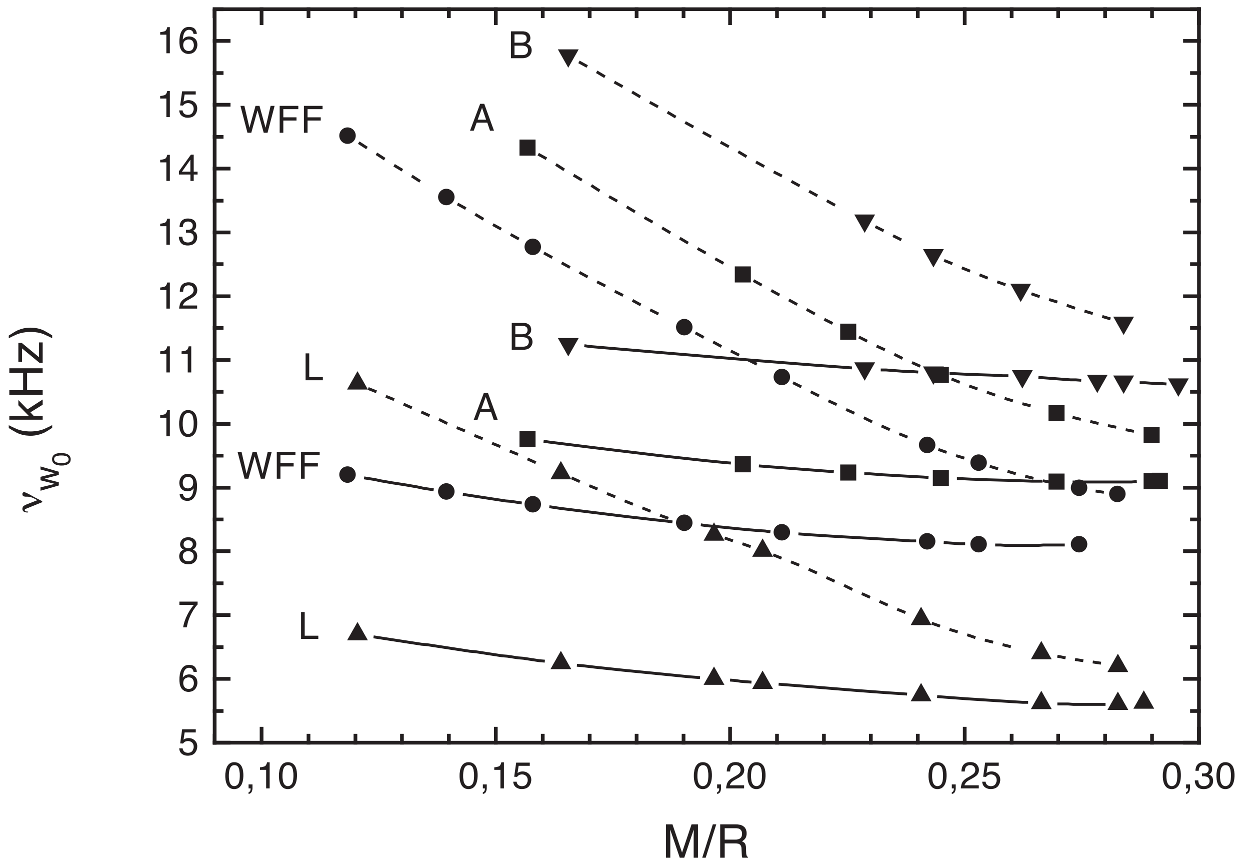
A useful way of classifying EOS's is through their stiffness, which can be quantified in terms of the speed of sound v_s :

stiffer EOS's correspond to higher v_s .

stiffer EOS's correspond to less compressible matter.

Pure spacetime modes

w-modes have high frequency and very rapid damping.
They exist both for polar and axial perturbations



*Andersson N., Kokkotas K.D, 1998 MNRAS **299***
*Benhar O., Berti E., Ferrari V. 1999 MNRAS **310***

dashed lines: polar w-modes
continuous lines: axial w-modes

EOS A . Pandaripande 1971. Pure neutron matter, with dynamics governed by a nonrelativistic Hamiltonian containing a semi-phenomenological interaction potential.

EOS B. 1971 Generalization of EOS A, including protons, electrons and muons in β-equilibrium, as well as heavier barions (hyperons and nucleon resonances) at sufficiently high densities.

EOS WWF. Wiringa, Fiks, Fabrocini 1988. Neutrons, protons, electrons and muons in β-equilibrium. The Hamiltonian includes two- and three-body interaction potentials. The ground state energy is computed using a more sophisticated and accurate many body technique.

EOS L. Pandaripande & Smith1975. Neutrons interact through exchange of mesons (ω,ρ, σ). The exchange of heavy particles (ω,ρ) is described in terms of nonrelativistic potentials, the effect of σ-meson is described using relativistic field theory and the mean-field approximation.

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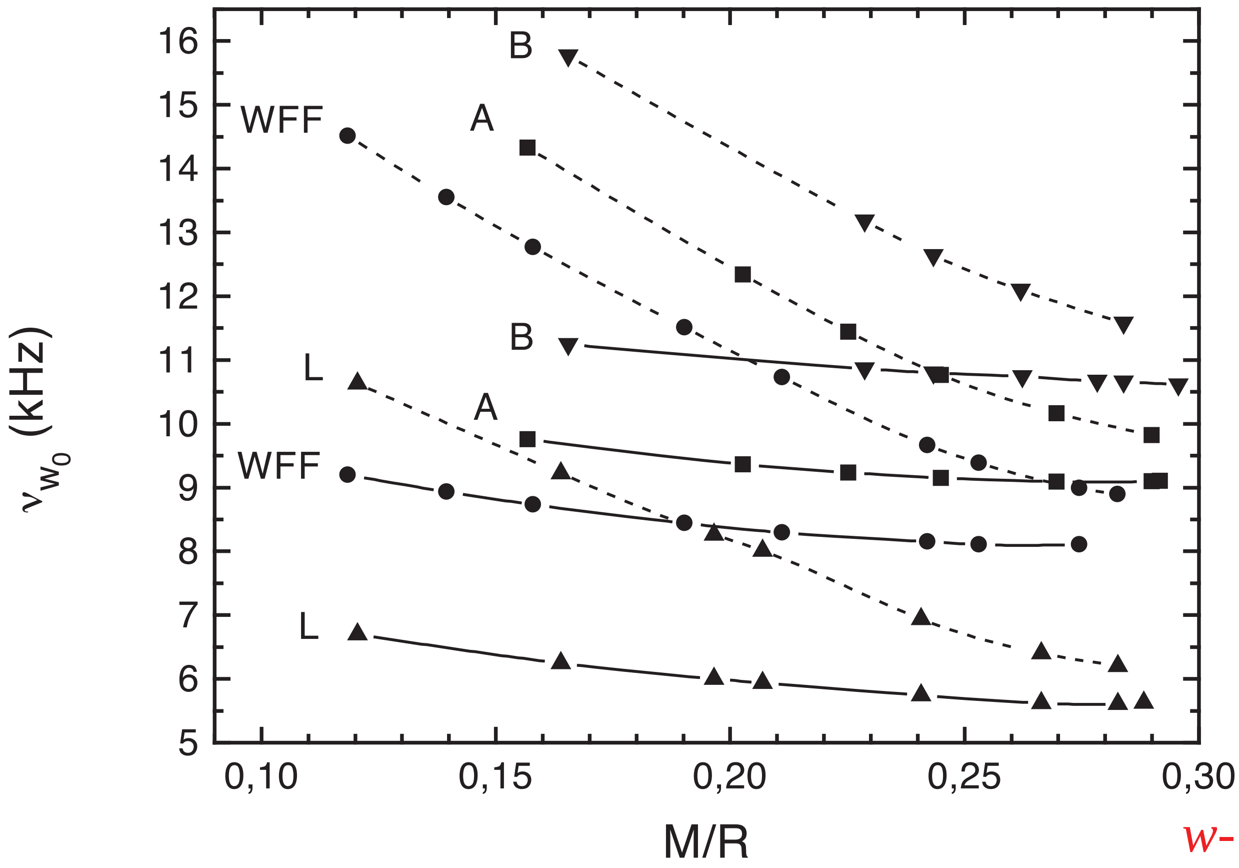

Andersson N., Kokkotas K.D, 1998 MNRAS ***299***
Benhar O., Berti E., Ferrari V. 1999 MNRAS ***310***

dashed lines: polar w-modes
continuous lines: axial w-modes

about the EOS compressibility:
- B softest (more compressible)
- L stiffest

-axial and polar w-modes depends essentially on the stiffness of the equation of state.

-axial w-mode frequencies range within intervals that are separated;
for each EOS
is nearly independent of M/R

w- modes are excited in the collapse of a neutron star to a black hole, just before the black hole forms
Baiotti et al 2005, *Phys. Rev. Lett.* **94**, 131101

Pure spacetime modes

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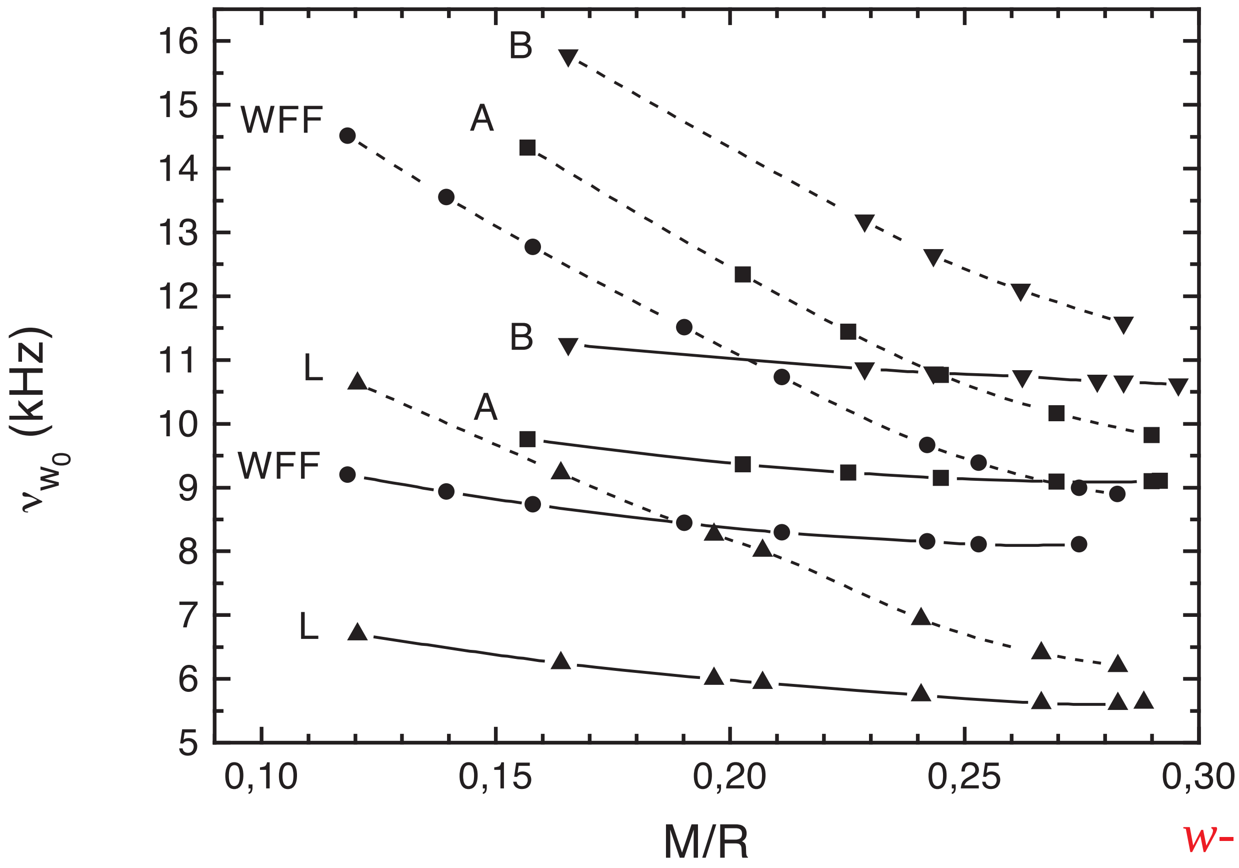

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unfortunately w-mode frequencies are too high to be detected by interferometric GW detectors

Equations of state considered

Inner core $\rho > \rho_0$ $\rho_0 = 2.67 \times 10^{14}$ g/cm^3

APR2:: n p e^- μ^- 3-body interaction phenomenological Hamiltonian
2-body potential= Argonne v18, 3-body potential=Urbana IX
Schroedinger equation solved using variational methods
including relativistic corrections
Akmal A., Pandharipande V.R, Ravenhall D.G., Phys. Rev C58, 1998

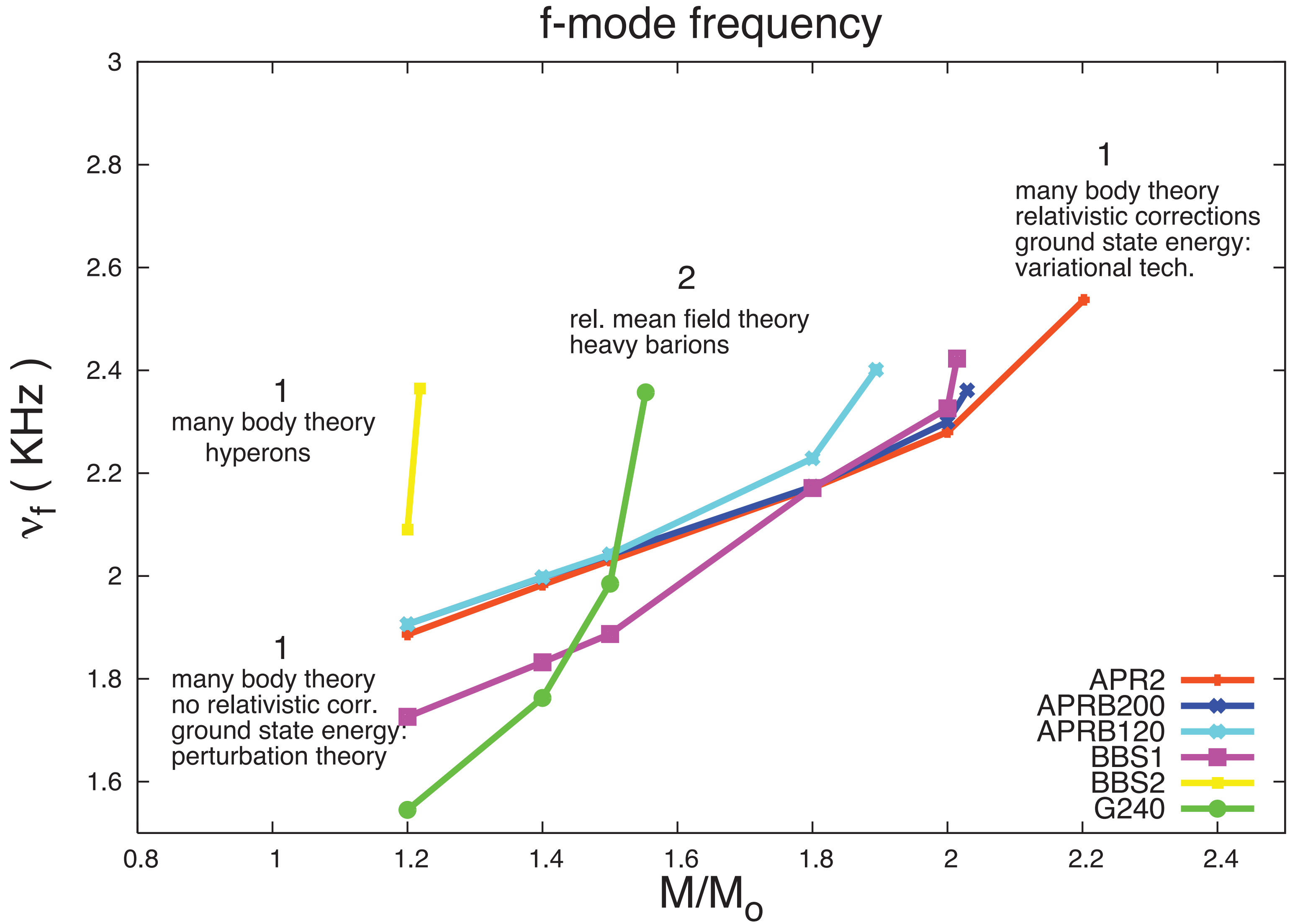
APRB120/200: APR2+ interacting quarks confined to a finite region (the bag) whose volume is limited by a pressure **B** said the bag constant
(**B**=120 or 200 MeV/fm^3, α_s=0.5 m_s=150 MeV)

BBS1: n p e^- μ^- 3-body interaction phenomenological Hamiltonian
2-body potential= Argonne v18, 3-body potential= Urbana VII
(no relativistic corrections); Schroedinger equation solved using perturbative methods *Baldo M., Bombaci I., Burgio G.F.*, A&A 328*, 1997*

BBS2: same as BBS1+ **heavy barions** Σ^{--} and Λ^0 (no relativistic corrections)
Baldo M., Burgio G.F., Schulze H,J, *Phys. Rev. C61, 2000*

G240: e^- μ^- and the complete octet of baryons; mean field approximation is used to to derive the equations of the fields *Glendenning N.K.*

Polar Quasi Normal Modes which are coupled to fluid motion: frequencies are smaller than for those of the w-modes.



Previous works:
Lindblom, Detweiler
ApJ Suppl, 1983

Andersson, Kokkotas,
MNRAS, 1998

Benhar, Ferrari, Gualtieri, Phys. Rev. D (2004)

Different ways of modeling hadronic interactions affect the pulsation properties of neutron stars

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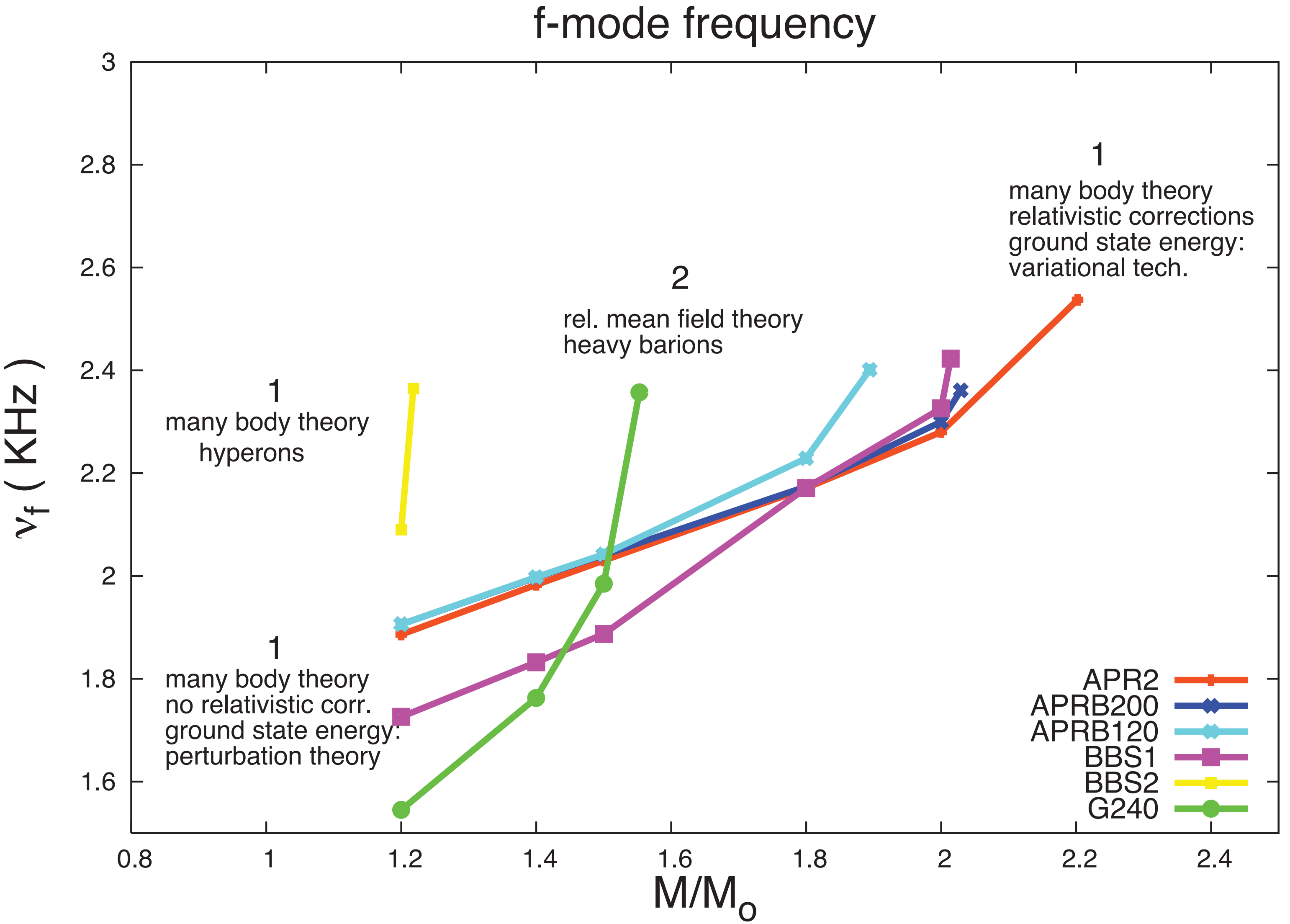

Benhar, Ferrari, Gualtieri, Phys. Rev. D (2004)

The presence of a quark core does not affect the f-mode frequency (except for higher massess)

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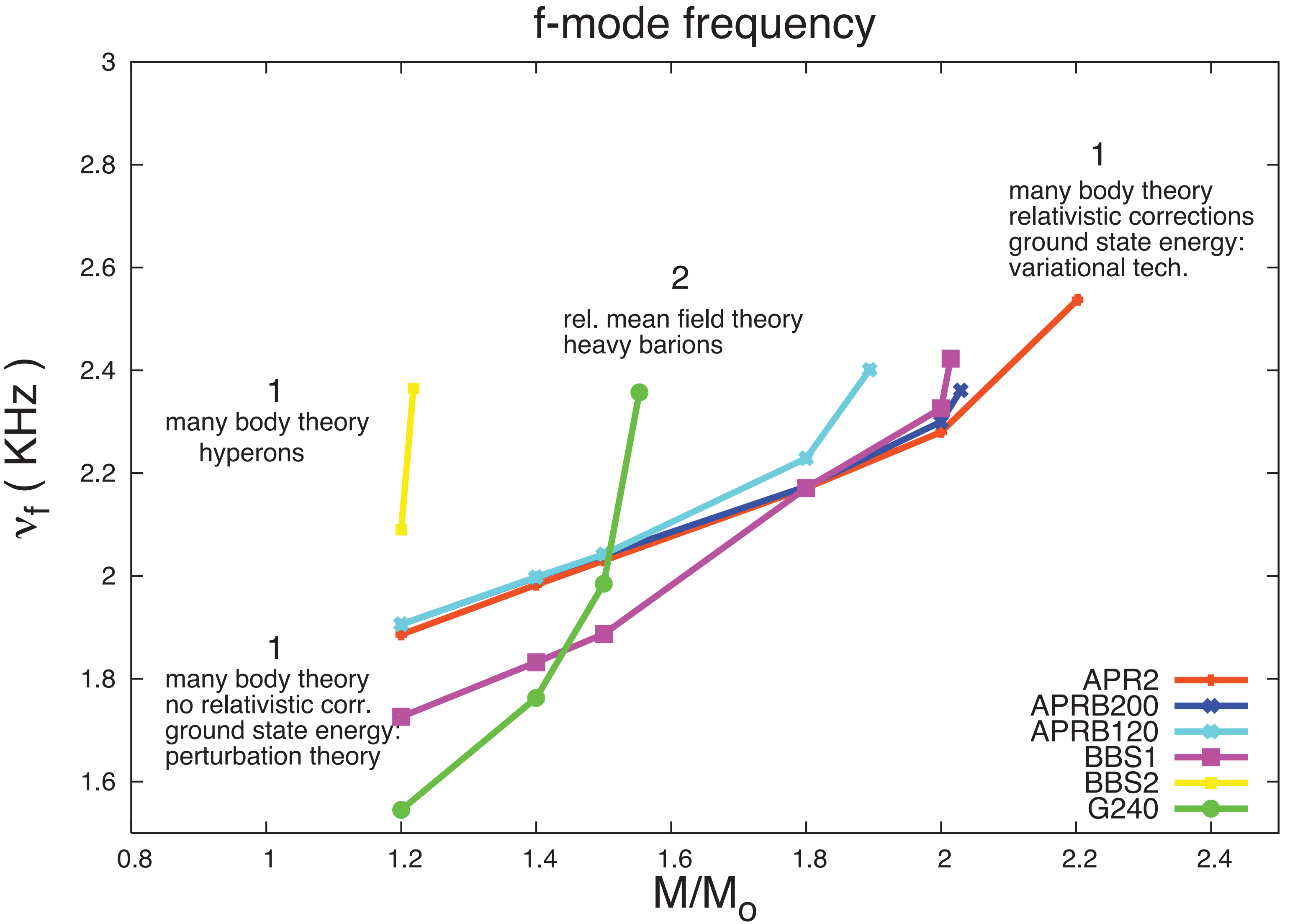

Benhar, Ferrari, Gualtieri, Phys. Rev. D (2004)

Compare purple and jellow for M=1.2 solar mass: hyperons soften the EOS average density increases, ν_f increases

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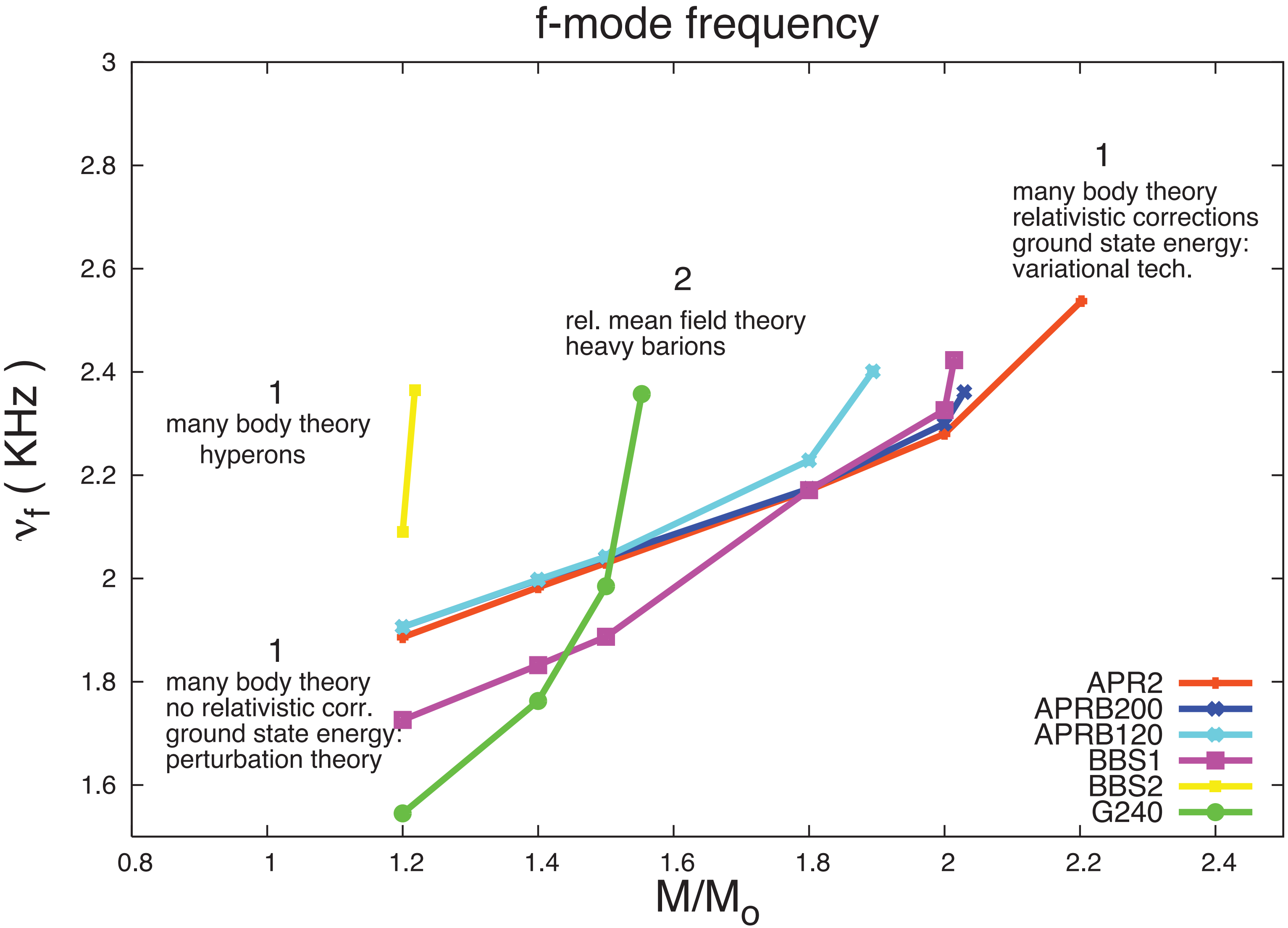

Benhar, Ferrari, Gualtieri, Phys. Rev. D (2004)

Mean field EOS (green curve) has a completely different behaviour which reflects a different relation between mass and central density

Can stars be entirely made of quarks ? **(Bodmer 1971,Witten 1984)**

MIT Bag model :

Fermi gas of up, down, strange quarks confined in a region with volume determined by pressure= **Bag constant B**. The interactions between quarks are treated perturbatively at first order in the coupling constant α_s

From Particle Data Book

$$m_u \sim m_d \sim \text{few MeV} \qquad m_s = (80\text{-}155)\ \text{MeV}$$

3 parameters α_s , m_s , B

- **Hadron collision experiments** $0.4 \leq \alpha_s \leq 0.6$

- **High energy experiments** $57 \leq B \leq 350$ MeV/fm3
  (hadron mass, magnetic moments, charge radii measurements)

- **The requirement that SQM is stable at zero temperature implies that** $B \leq 95$ MeV/fm3

We choose $57 \leq B \leq 95\ \text{MeV/fm}^3$

The fundamental mode frequency of old, cold neutron stars is plotted for different EOS versus the mass of the star.

strange stars (yellow region)
modeled using MIT bag model

$$m_s \in (80 - 155)\ \mathrm{MeV}$$
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Benhar, Ferrari ,Gualtieri Phys. Rev. D ***70****, 2004*
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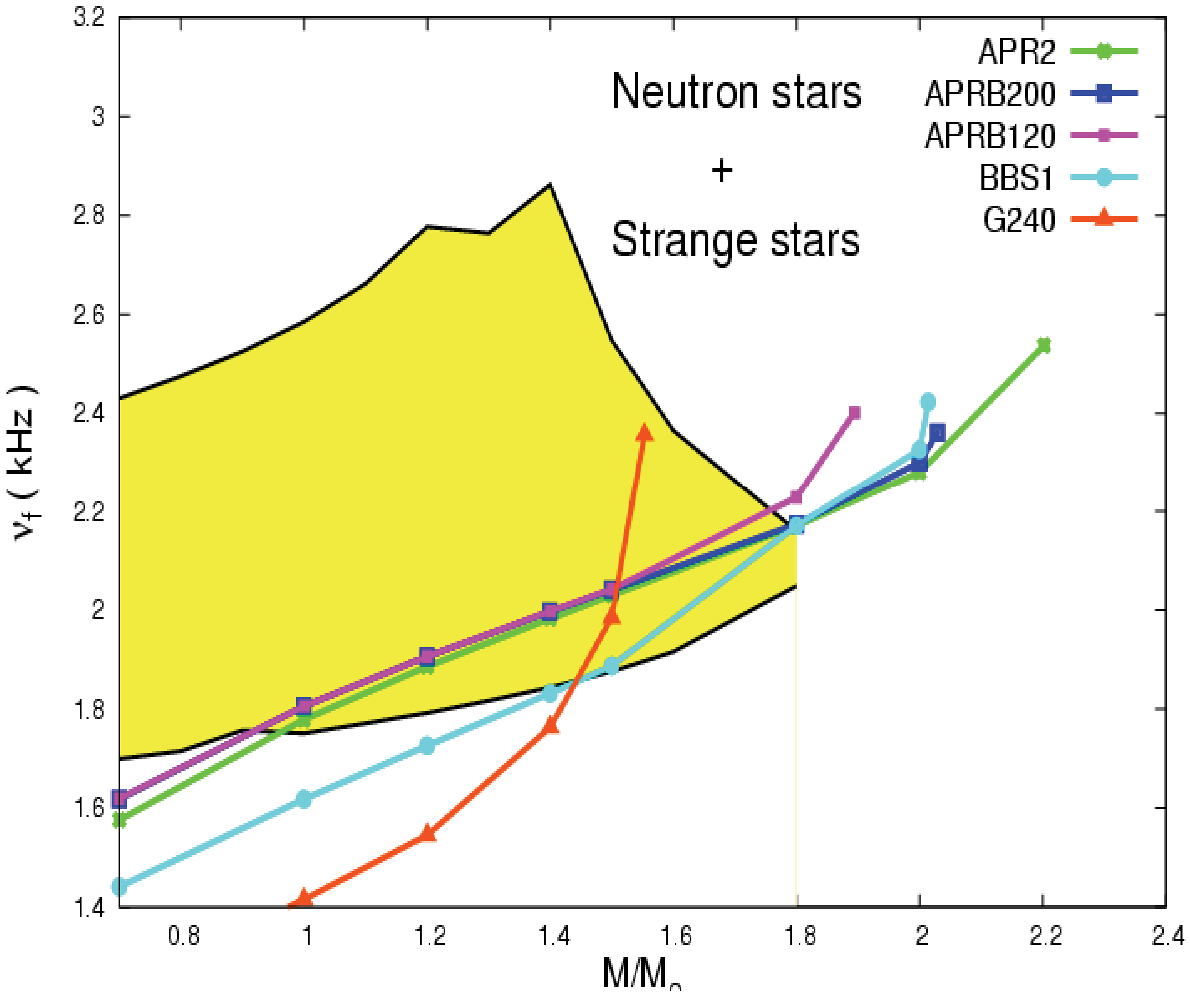

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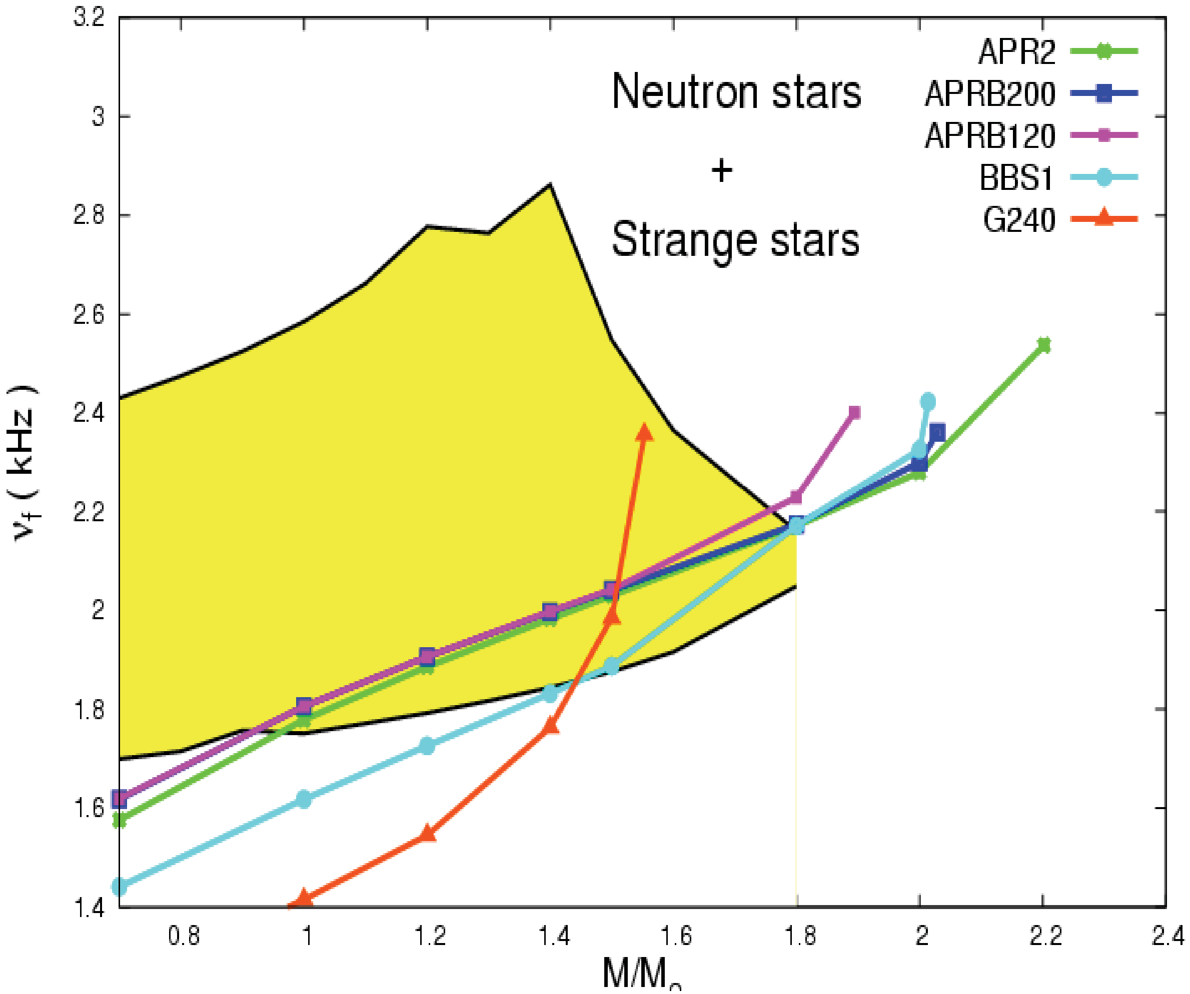

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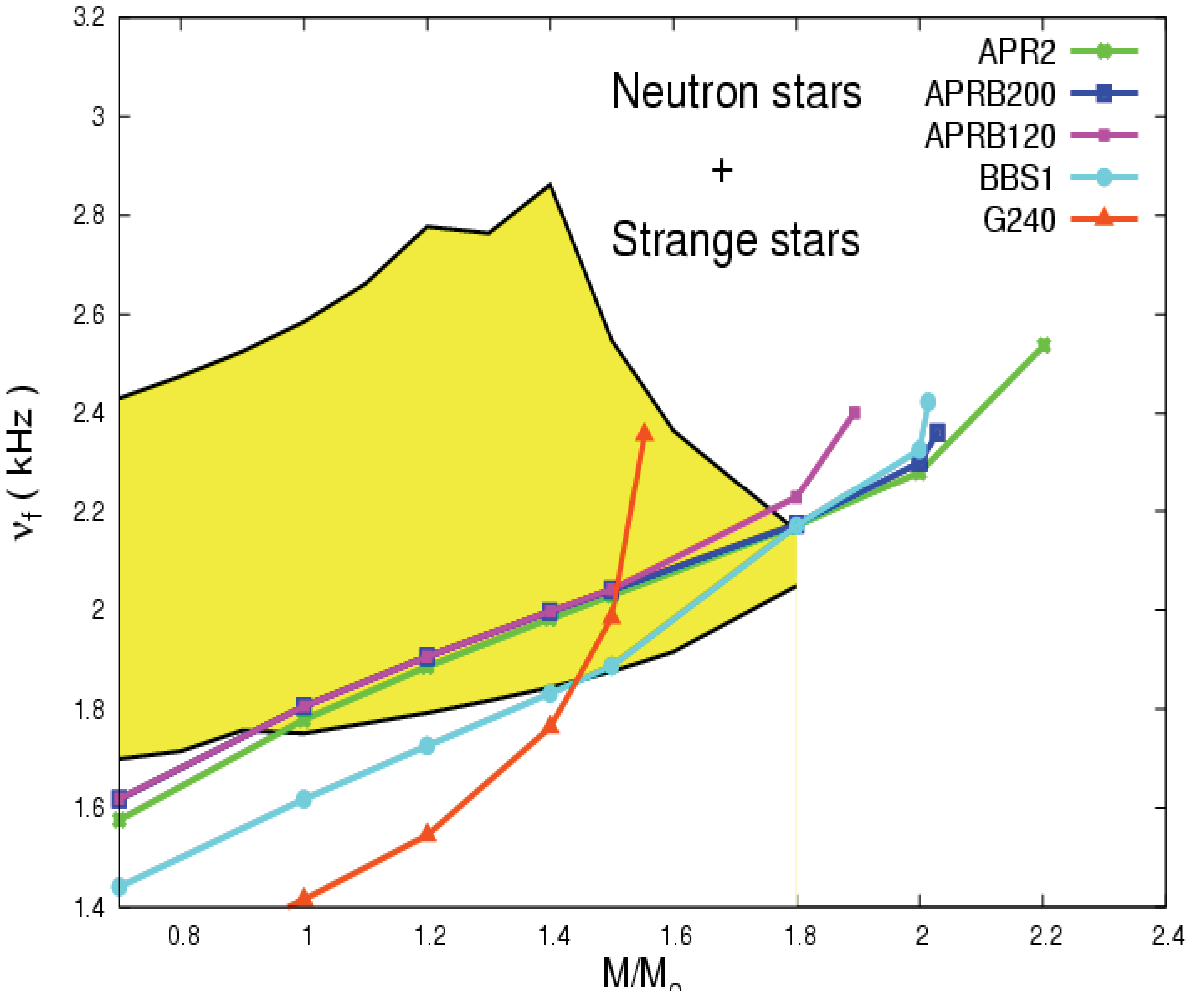

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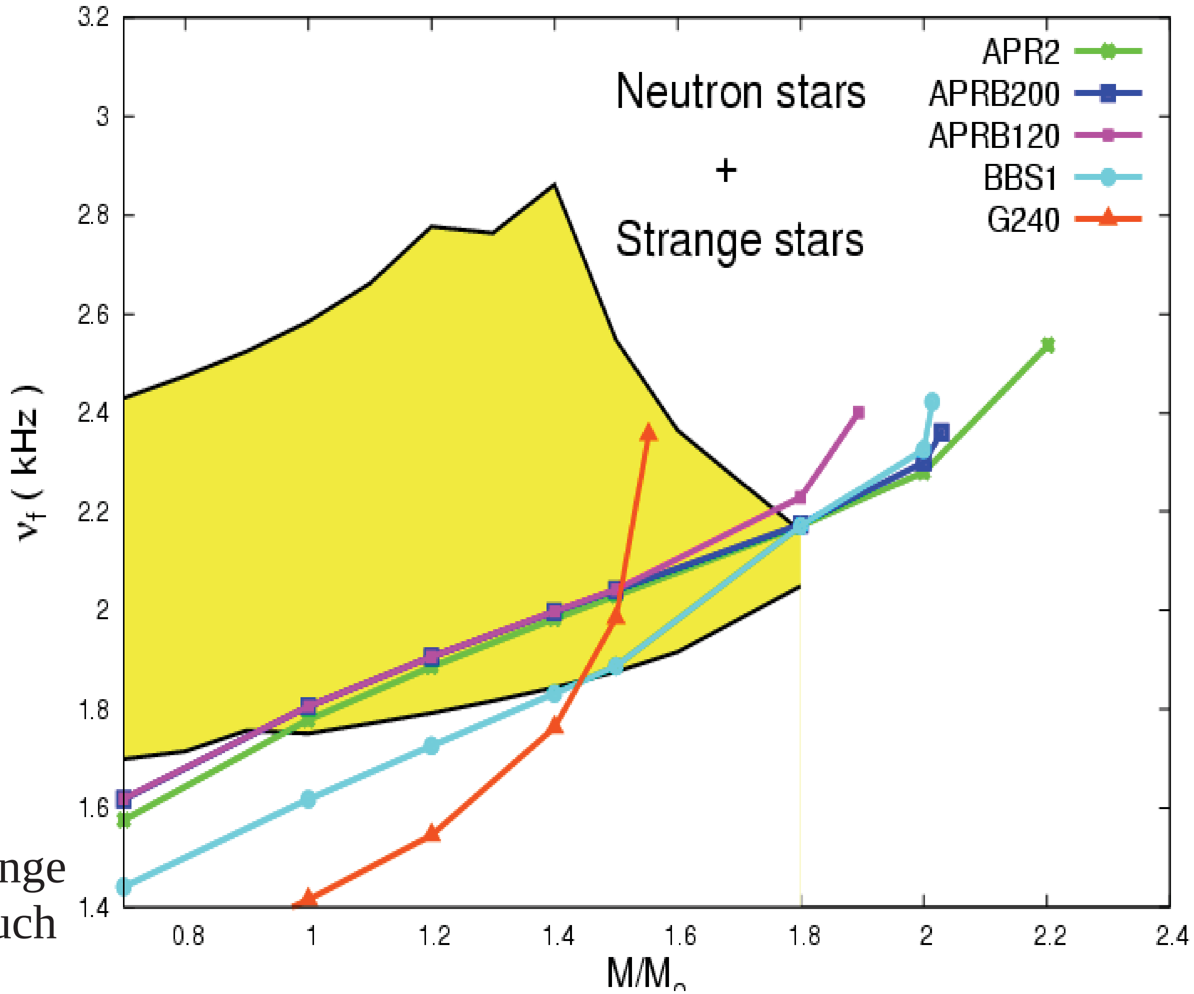

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- There is a small range of frequency where neutron/hybrid stars are indistinguishable from strange stars. However, there is a large frequency region where only strange stars can emit.
- Since ν_f is an increasing function of the bag constant B, detecting a GW from a strange star would allow to set constraints on B much more stringent than those provided by the available experimental data

do we have a chance to detect a signal from an old, cold neutron star oscillating in its fundamental mode?

A typical GW signal from a neutron star pulsation mode has the form of a damped sinusoid

$$h(t) = \boxed{\mathcal{A}}\, e^{-(t-t_0)/\boxed{\tau_d}} \sin[2\pi \boxed{f} (t-t_0)] \quad \text{for } t > t_0$$

$$\boxed{\mathcal{A}} \approx 7.6 \times 10^{-24} \sqrt{\frac{\Delta E_\odot}{10^{-12}} \frac{1\ \mathrm{s}}{\tau_d}} \left(\frac{1\ \mathrm{kpc}}{d}\right) \left(\frac{1\ \mathrm{kHz}}{f}\right) .$$

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How much energy would need to be channeled into a mode?

For mature neutron stars we can take as a bench-mark the energy involved in a typical pulsar glitch, in which case

$$\Delta E_{GW} = 10^{-13} M_\odot c^2$$

Assuming $\boxed{f} \sim 1500$ Hz, $\boxed{\tau_d} \sim 0.1$ s, $d = 1$ kpc, $\boxed{\mathcal{A}} \approx 5 \times 10^{-24}$

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3rd generation detectors are needed to detect signals from old neutron stars

Oscillations from newly-born neutron stars are more promising : more energy can be stored into the modes.

Gravitational waves from newly born, hot neutron stars

V. Ferrari, G. Miniutti, J. Pons, MNRAS 2004

F. Burgio, V. Ferrari, L.Gualtieri, H-J, Schultze, Phys. Rev D 2011

We studied how the frequencies and damping times of the QNMs change for 0.2 s < t < 50 s

after the bounce the star is composed of

- **low entropy, lepton rich core with trapped υ_s**
- **high entropy, low density, accreting mantle**
- **after Sn explosion, in a few tens of seconds extensive neutrino losses reduce lepton pressure and the mantle contracts: the radius of the PNS is now ~ 20-30 km**

To describe this part of the evolution ($t \lesssim 0.2$ s) dynamical simulations are needed (*Muller,Dimmelmeier,Zwerger, Font*…)

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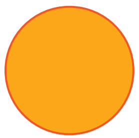
♣ Neutrino processes dominate the first minute of the evolution: Neutrino diffusion deleptonizes the core on a time scale of 10-15 s: the star heats up and the core entropy increases, reaching a maximum at the end of the deleptonization epoch.

♦ For $t \gtrsim 15$ s, the PNS becomes lepton poor, but it is still hot: the average neutrino energy decreases, and the neutrino mean free path increases. After approximately 50 s, the mean free path becomes comparable to R, and the star is finally transparent to neutrinos.

By this time, $T \sim 10^{10}$ K.



A NEUTRON STAR IS BORN

In *Ferrari, Miniutti, Pons, MNRAS 2004*

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The evolution is treated as a sequence of quasi-stationary states:
the thermodynamical variables and the lepton fractions are determined by solving evolution equations (for instance Boltzmann's equation to model neutrino transport), whereas at each time-step the stellar structure is found by solving the equations of stellar equilibrium. The equation is obtained within the **mean field approach**.

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In both cases we solve the equations of stellar perturbations for each quasi-stationary configuration and compute **the frequencies and the damping times of the quasi-normal modes** during the evolution

The oscillation spectrum evolves in the early phases of a PNS life

Remember that for a cold Neutron Star:

$\nu_f \approx 2$ kHz scales as $\propto (M/R^3)^{1/2}$

$\nu_{p1} \approx 7$ kHz

$\nu_{w1} \approx 11$ kHz

NO g-modes

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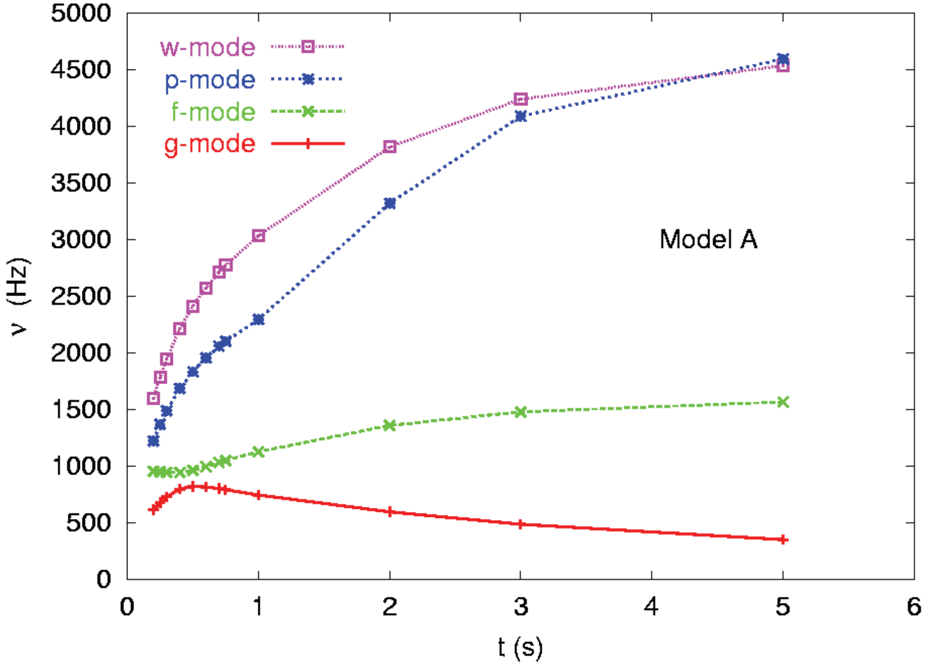
HOT proto-neutron star



*Ferrari ,Miniutti, Pons MNRAS, **342** 2004*

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For a HOT Proto-Neutron Star:

At very early time the frequencies of the f- p1- and w1- modes **are much lower** than those of a cold Neutron Star.

The oscillation spectrum evolves in the early phases of a PNS life

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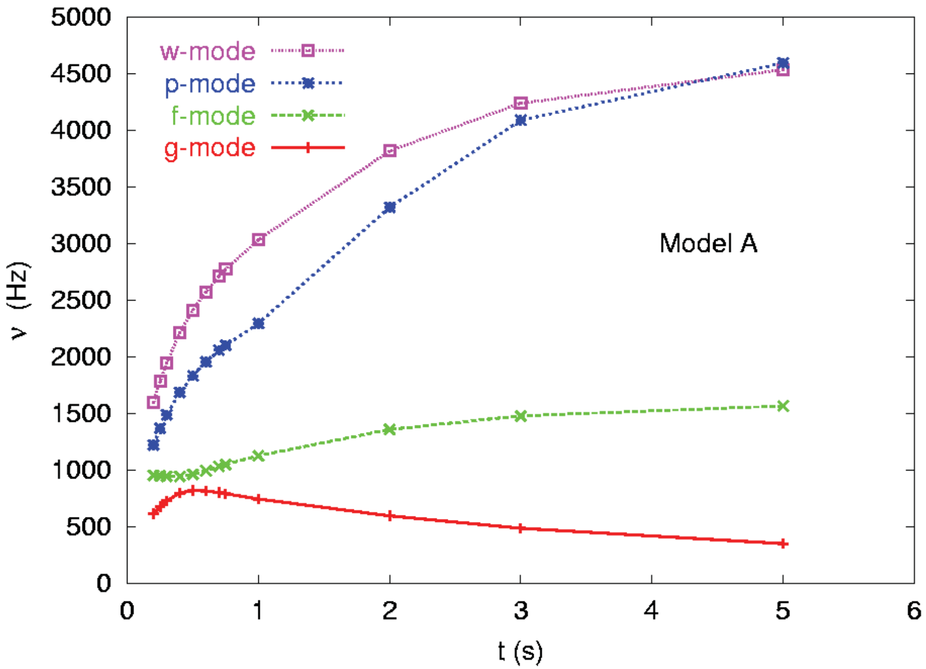

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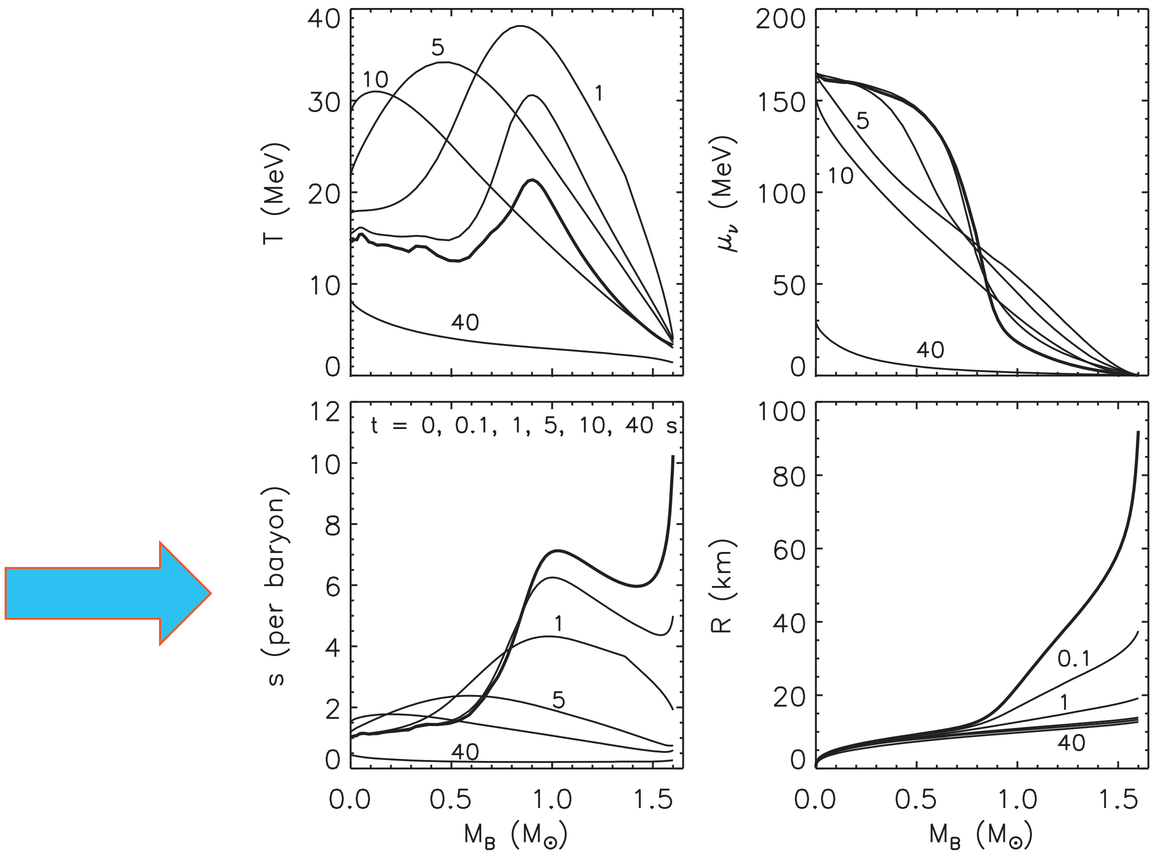
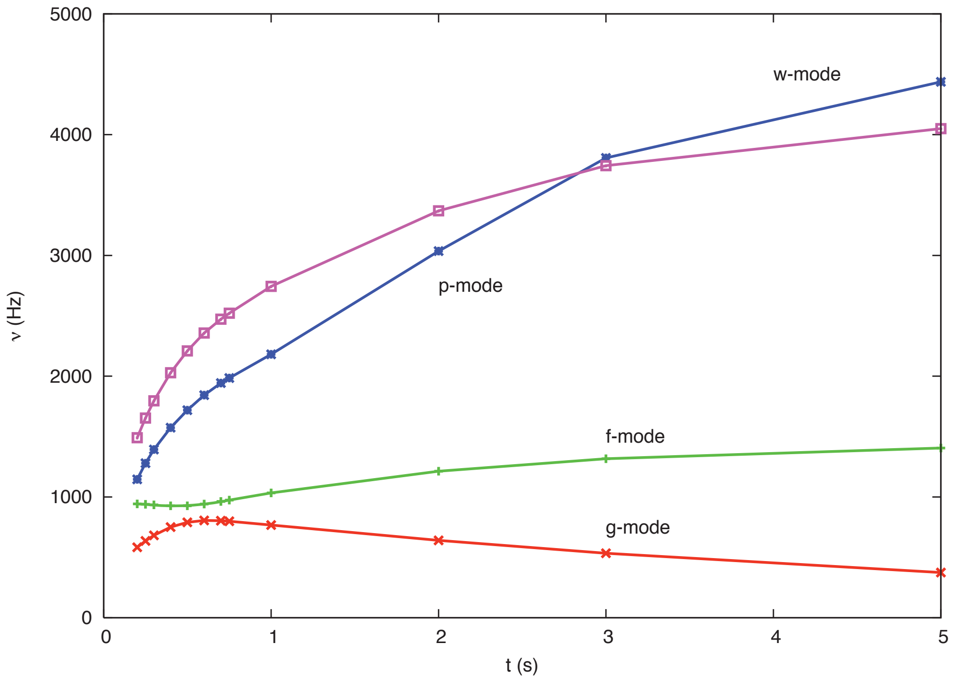
In a hot, proto-neutron star   g-modes appear: WHY?

During the first minute of a PNS life, strong entropy gradients characterize the theromdynamical evolution : this is why g-modes appear



Pons, *Reddy*, *Prakash*, *Lattimer*, *Miralles*, Ap.J. 1999

The oscillation spectrum evolves during the observation:

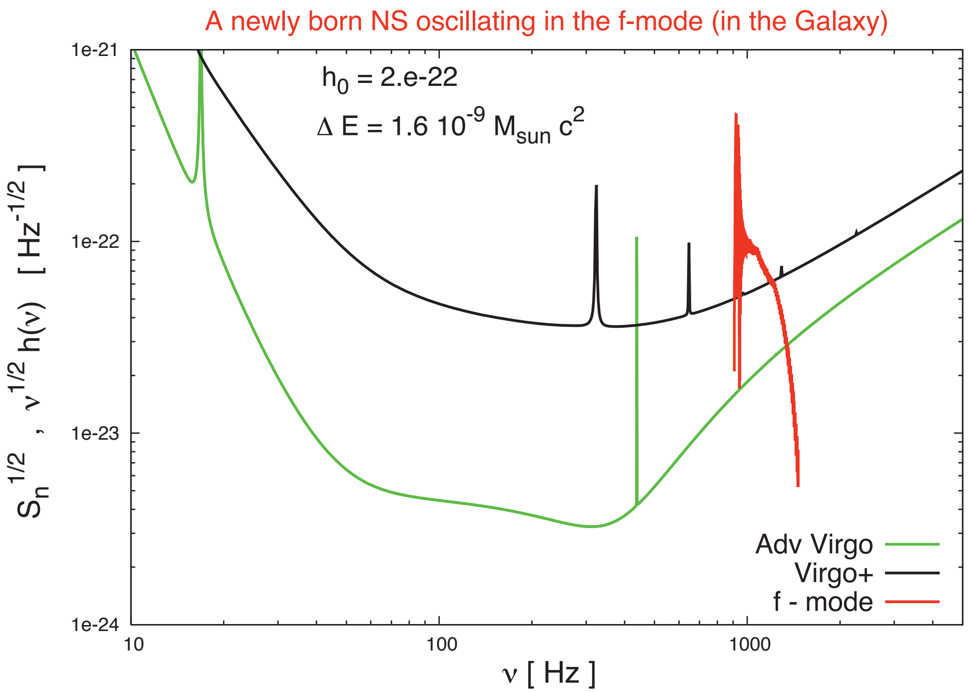


Having $\nu(t)$ and $\tau(t)$, we can estimate the amount of energy ΔE_{GW} that should be stored in a given mode for the signal to be detectable with an assigned SNR by a given detector.

The frequencies of the fundamental mode, and of the first g-, p- and w- modes of an evolving proto--neutron star are plotted as functions of the time elapsed from the gravitational collapse, during the first 5 seconds.

*Ferrari ,Miniutti, Pons MNRAS, **342** 2004*

If the newly born star is oscillating in the f-mode:



If we assume that

$$\Delta E_{GW} = 1.6 \cdot 10^{-9}\ M_{\odot} c^2$$

is stored into the f-mode

$$SNR = 2 \left[\int_0^{\infty} d\nu \, \frac{|\tilde{h}(\nu)|^2}{S_n(\nu)} \right]^{1/2},$$

SNR= 8 by Advanced Virgo/LIGO

CONCLUSIONS

- GWs emitted by a black hole oscillating in its QNMs would probe the strong field region near the horizon

- GWs emitted by a neutron star oscillating in its QNMs provide information on the EOS prevailing in the inner core, on which no much is known