Higher-dimensional Black Holes

Roberto Emparan
ICREA & U. Barcelona

Motivation: GR as a tool

- Most basic set up: vacuum GR

$$R_{\mu\nu}=0$$

  - $\exists$ only one parameter for tuning: D

Motivation: GR as a tool

- Most basic set up: vacuum GR

$$R_{\mu\nu}=0$$

  - $\exists$ only one parameter for tuning: D

- Most basic objects: Black Holes
- This talk:

  Stationary black holes of $R_{\mu\nu}=0$

GR as a tool

- Emphasis: instead of quick results with high-yield gain (applications), focus on developing fundamentals

  (learn from financial crises…)

- When first found, black hole solutions have always been "answers waiting for a question"

Stationary black holes of $R_{\mu\nu}=0$

- There are many in D>4!
- Complete classification?
  - Maybe. But might not be feasible or useful, esp. as D grows
- If not classification, understand main features of phase space
  - How phases evolve in solution space, limits...
  - How phases are related, connect/bifurcate...
- *Search for patterns*

How do we represent different phases?

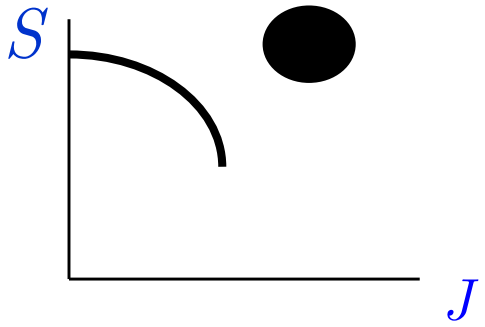
- Vacuum: $M,\ J_i \quad i{=}1,\ldots\ [(\mathrm{D}\text{-}1)/2]$
- Sometimes restrict to

  $J_1 > 0,\ J_{i>1} = 0$

  $J_1 = J_2 = \ldots$
- First law: $TdS{=}dM{-}\Sigma_i\ \Omega_i dJ_i \qquad S(M, J_i)$
- But $M,\ J_i$ do not fully specify a bh
  - discrete degeneracies (for single bh phases)
- Fix M, plot surfaces $S(J_i)$
- 1-diml curves if only one J

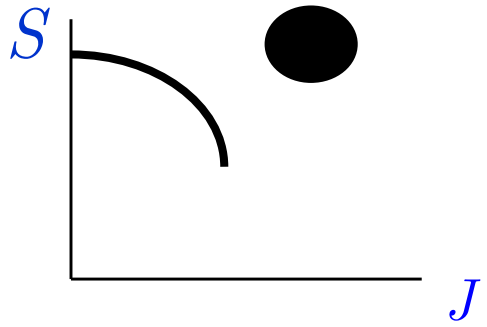
Known *exact* solutions w/ 1 spin

$D{=}4$

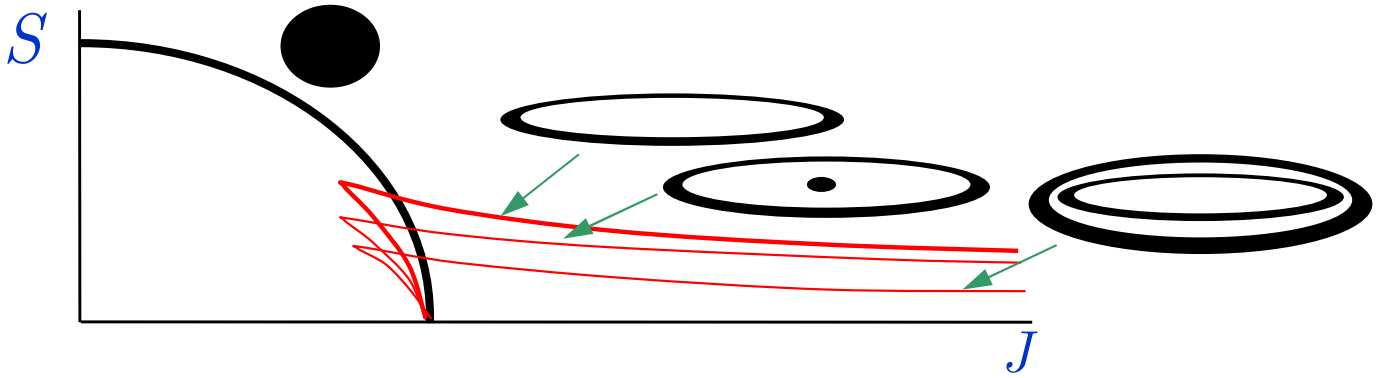


Known *exact* solutions w/ 1 spin

$D{=}4$

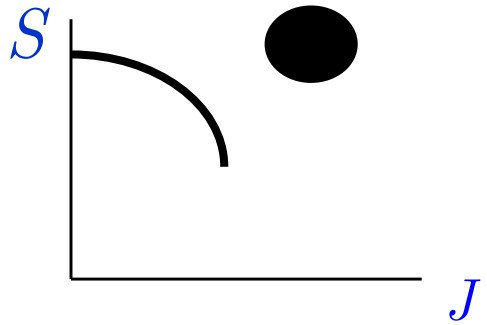


$D{=}5$

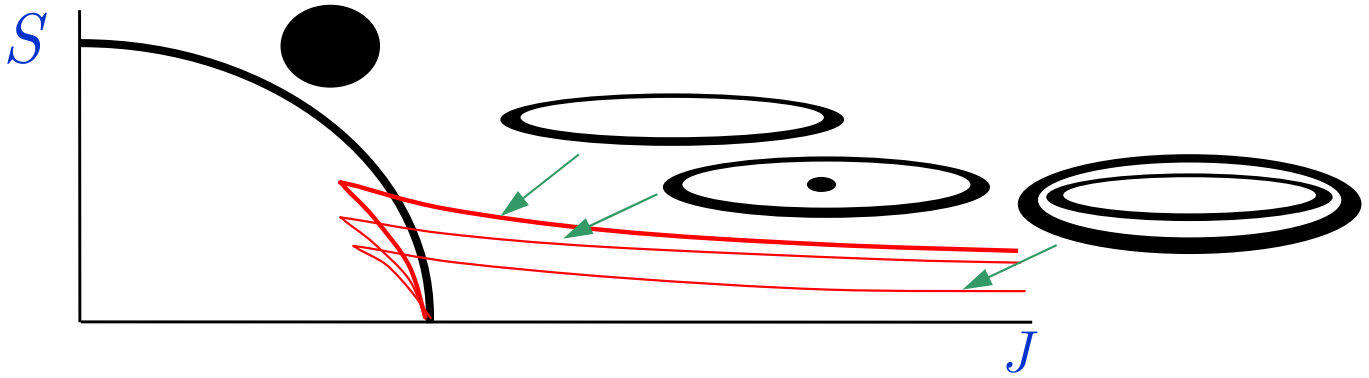


Known *exact* solutions w/ 1 spin

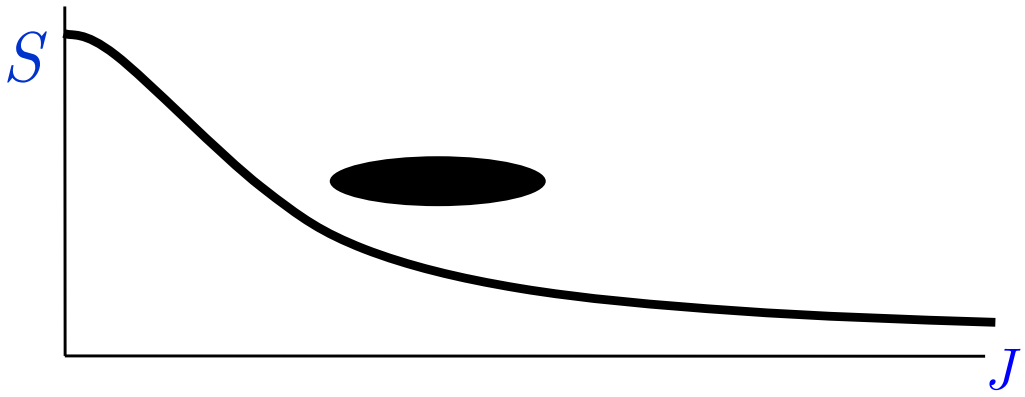
$D{=}4$



$D{=}5$

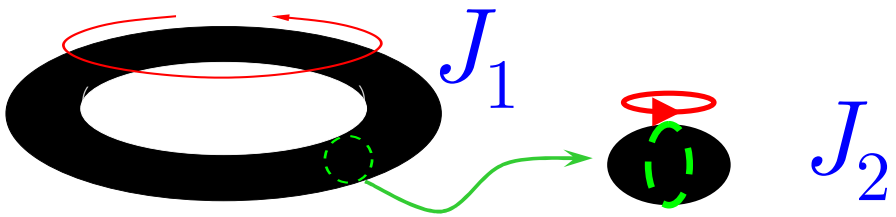


$D{\geqslant}6$

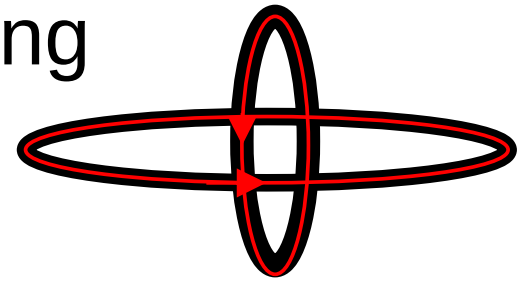


Known exact solutions >1 spin

- In all D: MP bhs
- In 5D:
  - Doubly-spinning black ring



  - Bi-ring



  - Only limited by willingness to include more rings: but we have the pattern

Beyond complete exact solutions

1. Large-J phases from blackfold approach
2. Zero-mode perturbations at branching points (soft+hard numerics)
3. Critical geometries at topology-changing merger transitions

These control overall features of phase space

– starting points for perturbations, or reference for numerics

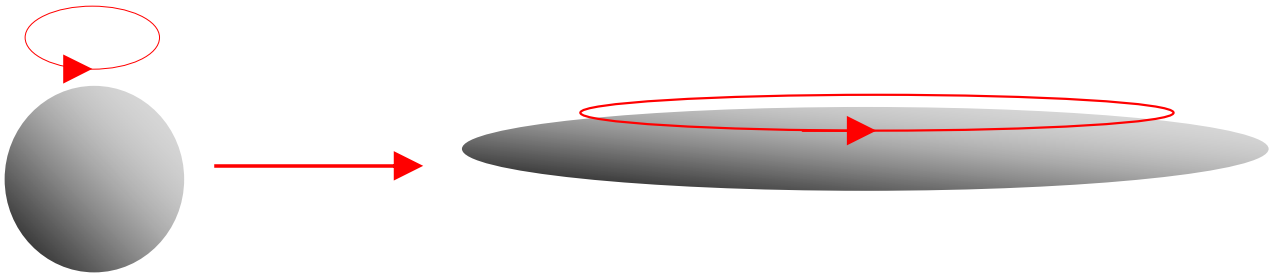
Beyond complete exact solutions

1. Large-J phases from blackfold approach
2. Zero-mode perturbations at branching points (soft+hard numerics) → OTHER TALKS
3. Critical geometries at topology-changing merger transitions

Large J regime of exact solns

- Doesn't exist in $D{=}4$
- In $D{=}5$: thin black rings (but not MP)

- In $D{\geqslant}6$: MP bhs



Large J regime

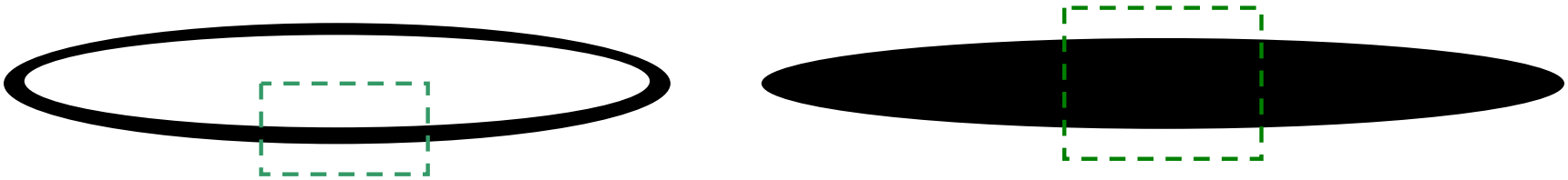
- Two length scales

$$\ell_M \sim (GM)^{1/(D-3)} \ll \ell_J \sim J/M$$

thickness << spread

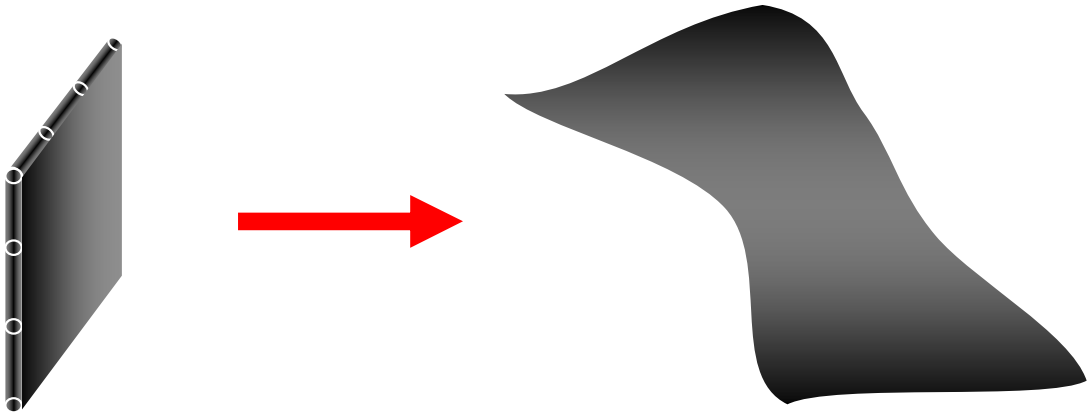
- BH spreads along rotation plane

- For $\ell_M / \ell_J \ll 1$, it becomes locally like black string/brane (cannot in D=4)

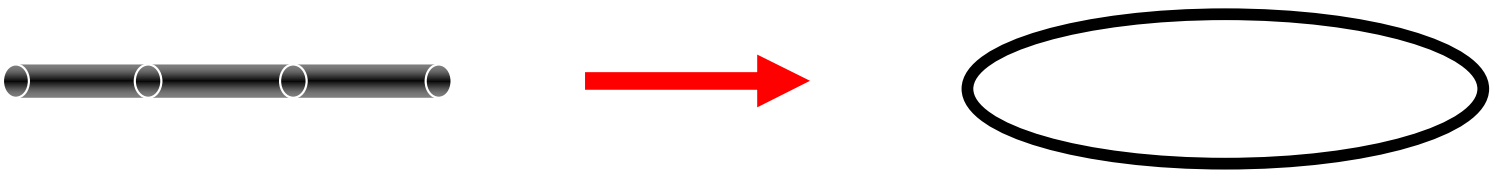


Black holes as *blackfolds*

- **Blackfold**: **Black** p-brane w/ worldvolume = curved submani**fold** of spacetime



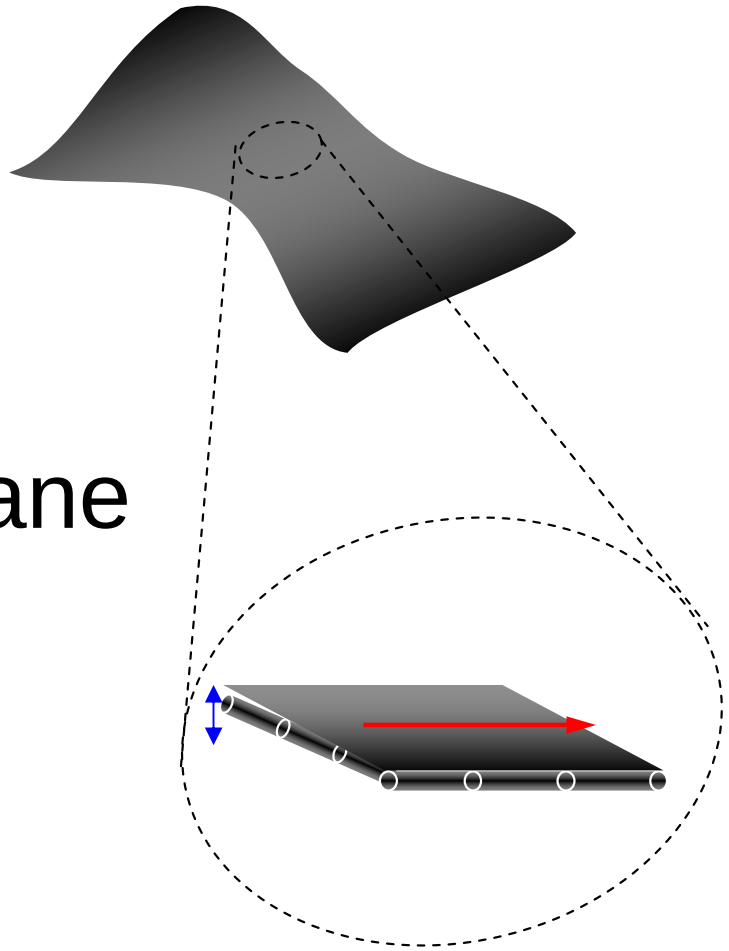
- Eg, black ring as circular black string:



Blackfolds: effective worldvolume theory of black branes

Bent brane

$X^\mu(\sigma^a)$ = embedding geometry



Locally equivalent to black brane

$u^\mu(\sigma^a)$ = wv velocity (local boost)

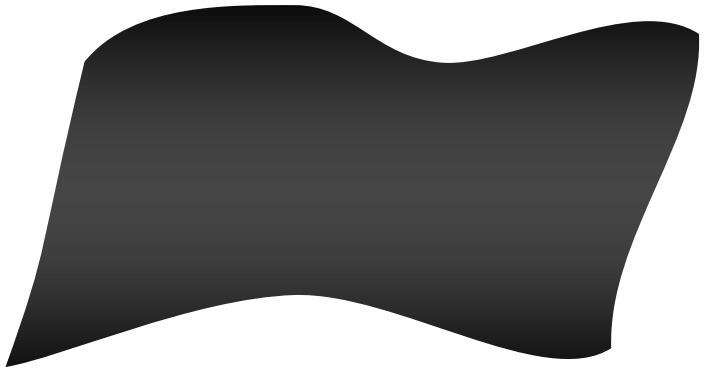
$r_0(\sigma^a)$ = horizon thickness

Dynamics

- In probe (test brane) approx

$T_{\mu\nu}$ localized on worldvolume

$$\nabla_\mu T^{\mu\nu} = 0$$



(from long-wavelength expansion of Einstein's equations)

Dynamics

- Along **worldvolume** directions:

$$\nabla_\alpha T^{\alpha\beta}=0$$

$$\Rightarrow \textbf{Worldvolume Fluid equations}$$

- Along **transverse** directions: *Carter*

$$\nabla_\mu T^{\mu\rho} = 0 \quad \Rightarrow \quad T^{\mu\nu} K_{\mu\nu}{}^{\rho} = 0$$

extrinsic curvature

$$\Rightarrow \textbf{Generalized geodesic equations}$$

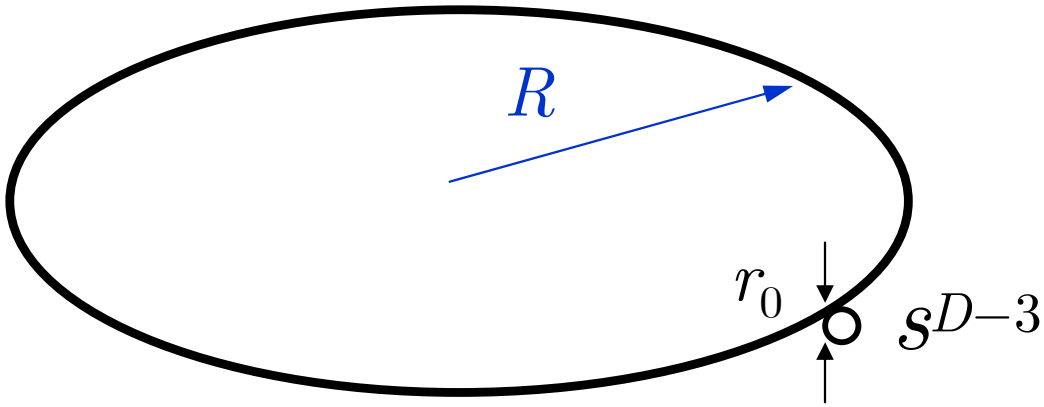
("mass x acceleration = 0")

Search for non-trivial blackfolds


Google
España
BLACKFOLD
Buscar con Google
Voy a tener suerte

Artist: M Rasmussen. "**Blackfolded form**", hand-built stoneware, height 22cm 20x25cm wide - **£325.00**

Simplest example: black rings in $D \geqslant 5$



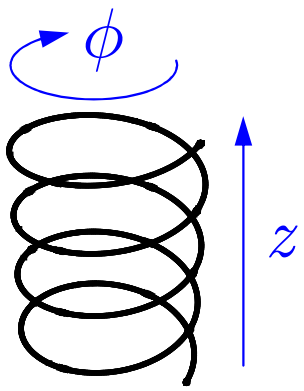
Horizon $S^1 \times s^{D-3}$

Helical black rings

- Black string along an isometry ζ of background

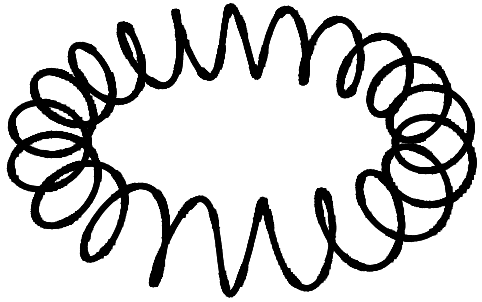
$\zeta = k\partial_z + \partial_\phi$

Helical
black string



$\zeta = n\partial_{\phi_1} + m\partial_{\phi_2}$

Helical
black ring
(*slinky*)



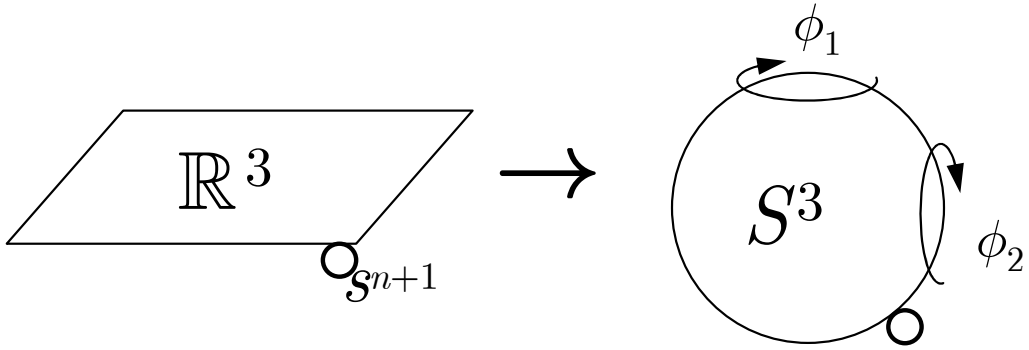
(n.b: profile is static!)

- The orthogonal isometry is broken:
  Horizon has *only one* spatial U(1)

Products of spheres

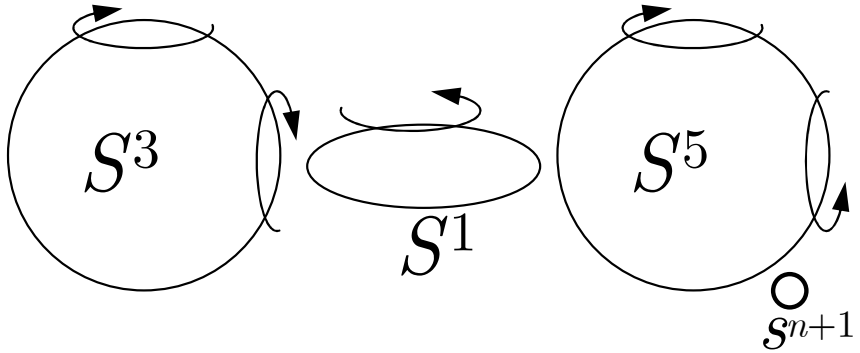
- $\mathbb{R}^3 \times s^{n+1} \rightarrow S^3 \times s^{n+1}$

$n = D - p - 3$

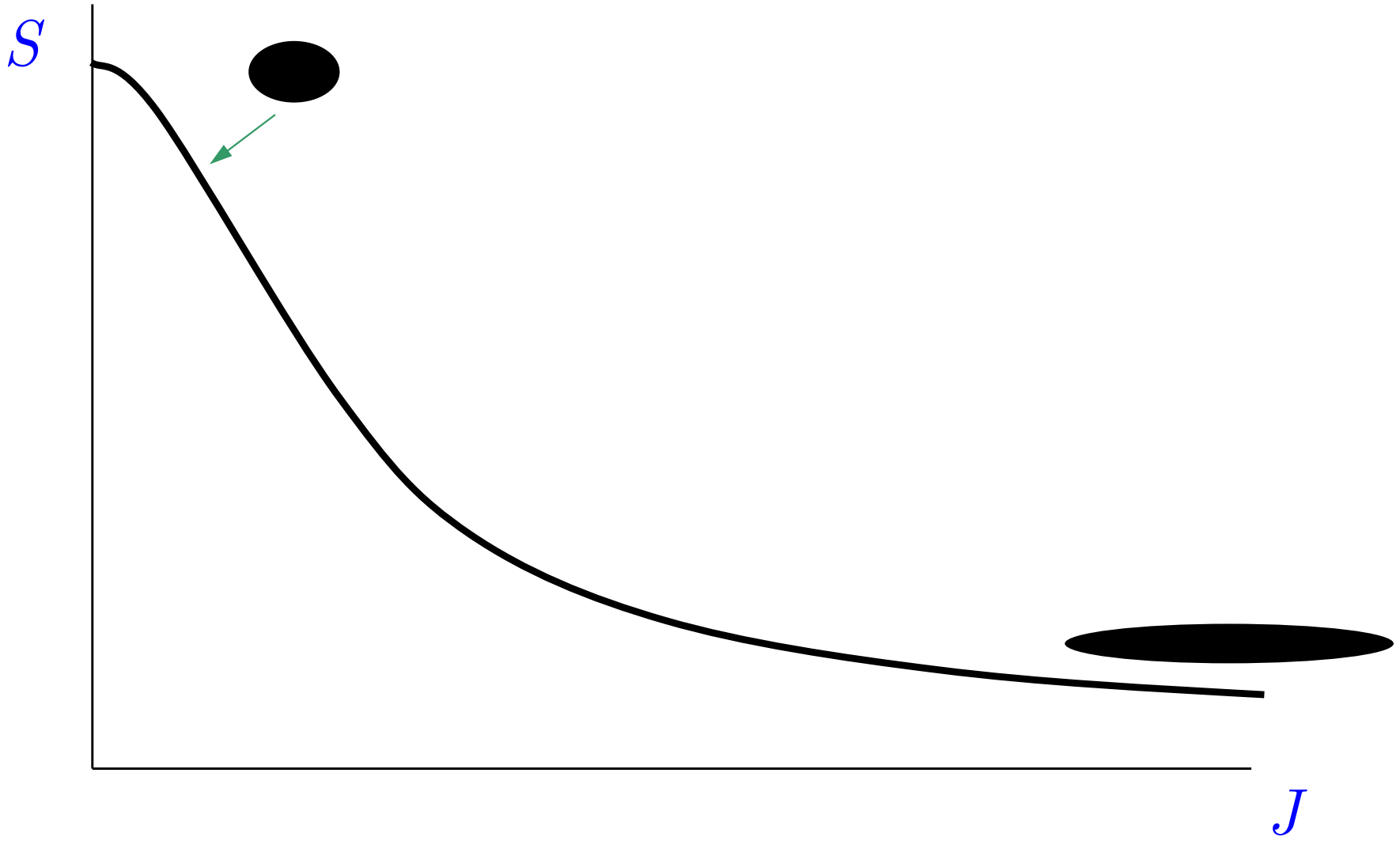


- Can do it for any product of odd-spheres

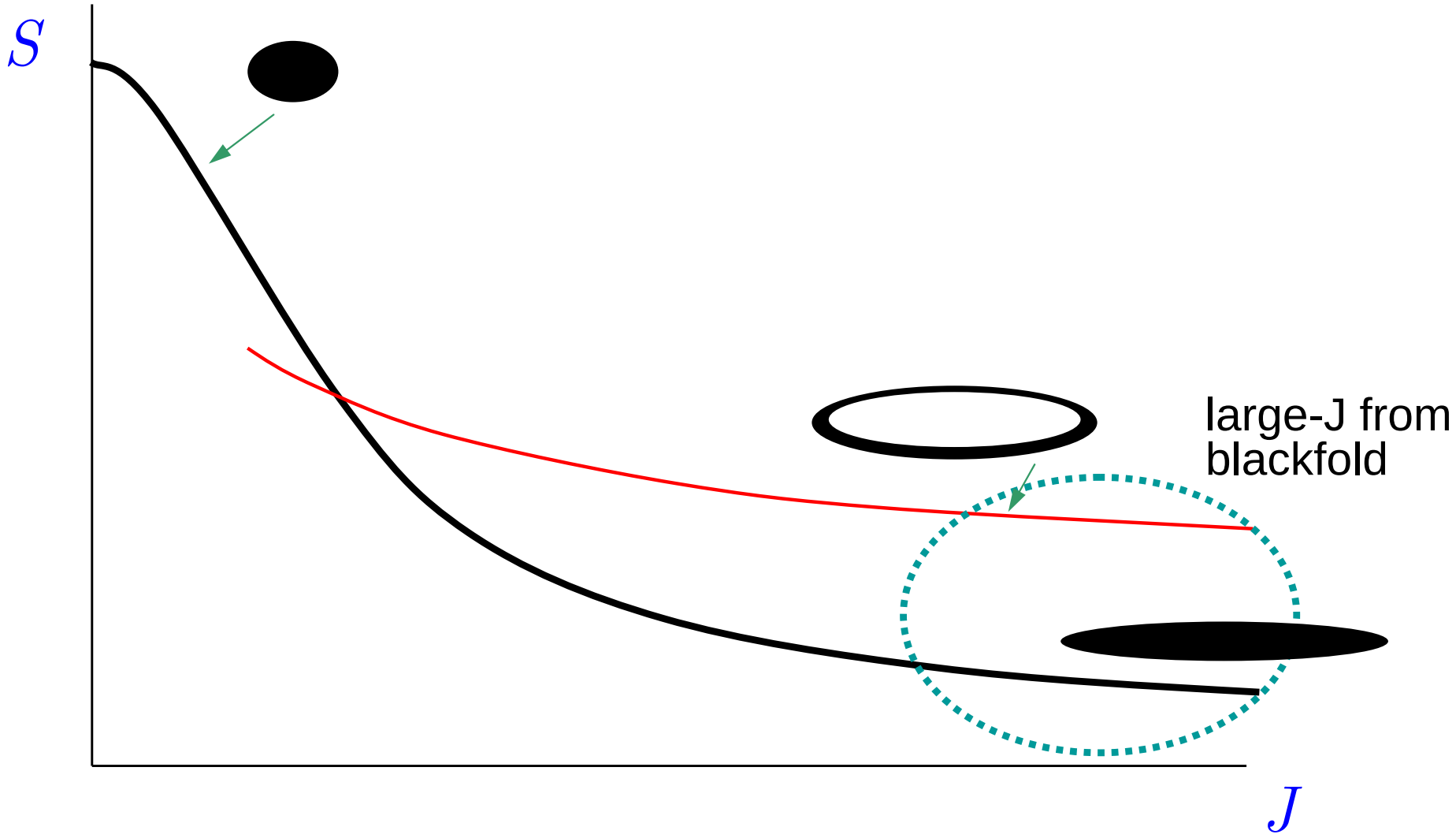
$$\prod_{p_a \in \text{odd}} S^{p_a} \times s^{n+1}$$



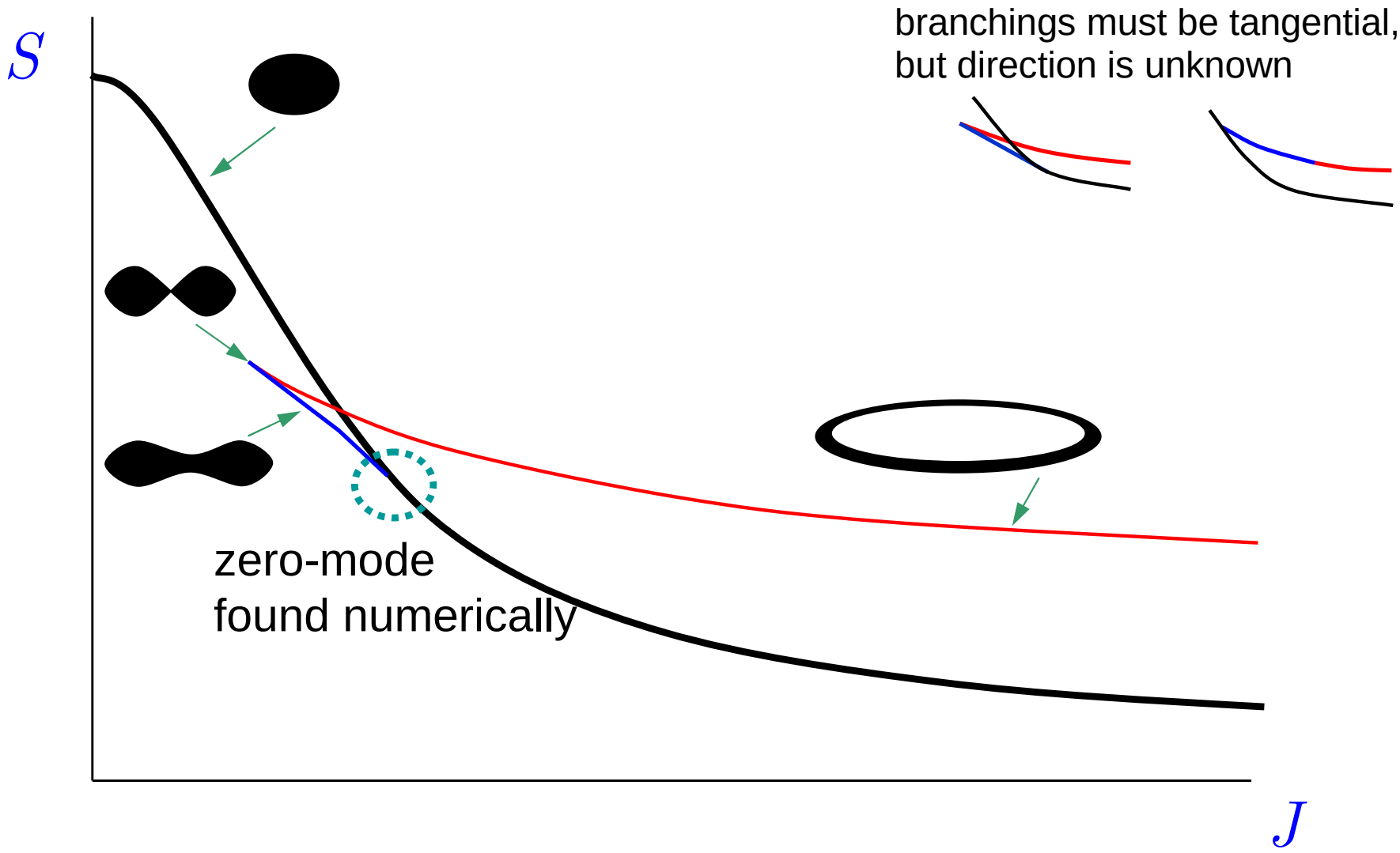
$D \geqslant 6$ phase diagram w/ 1 spin



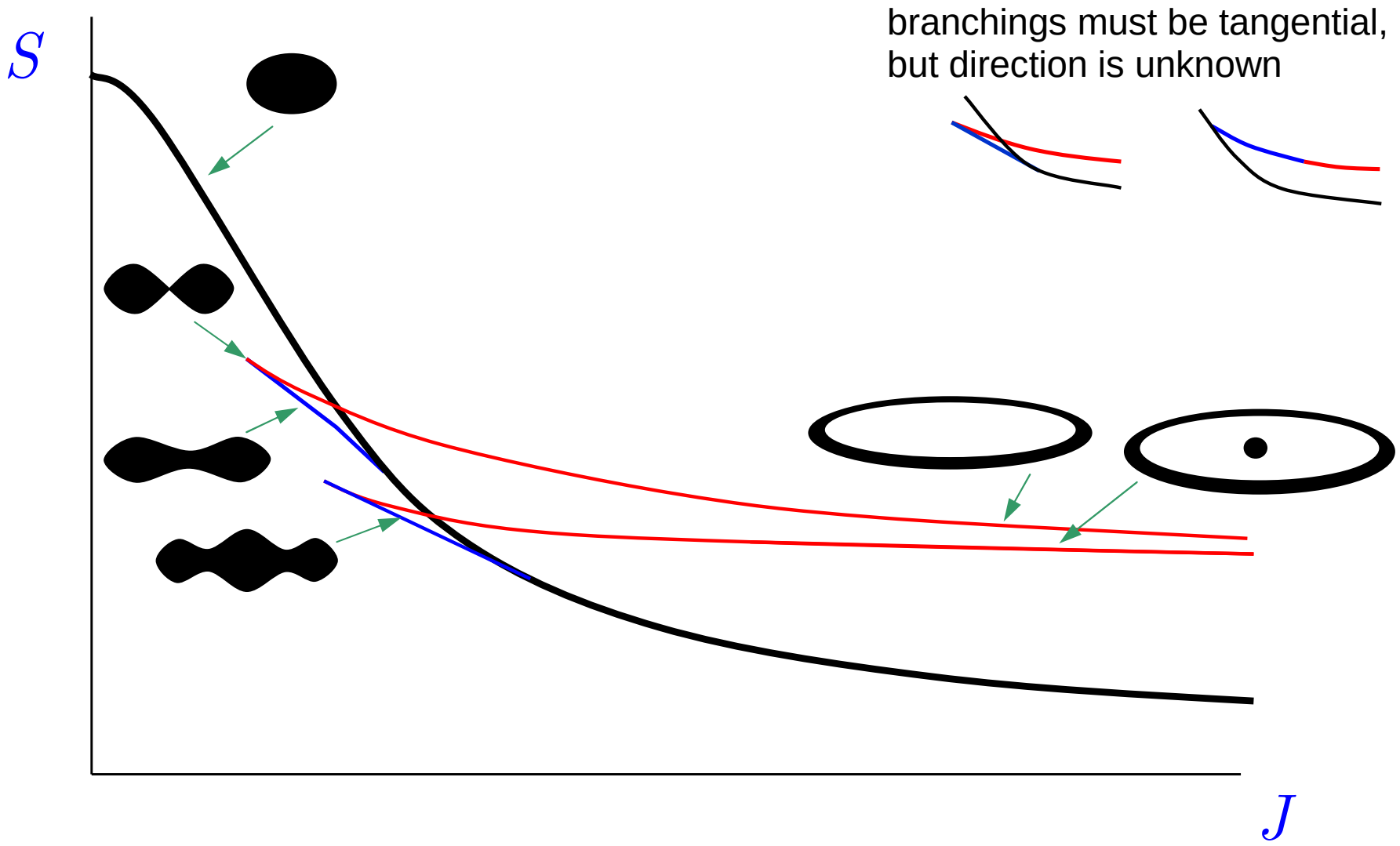
$D \geqslant 6$ phase diagram w/ 1 spin



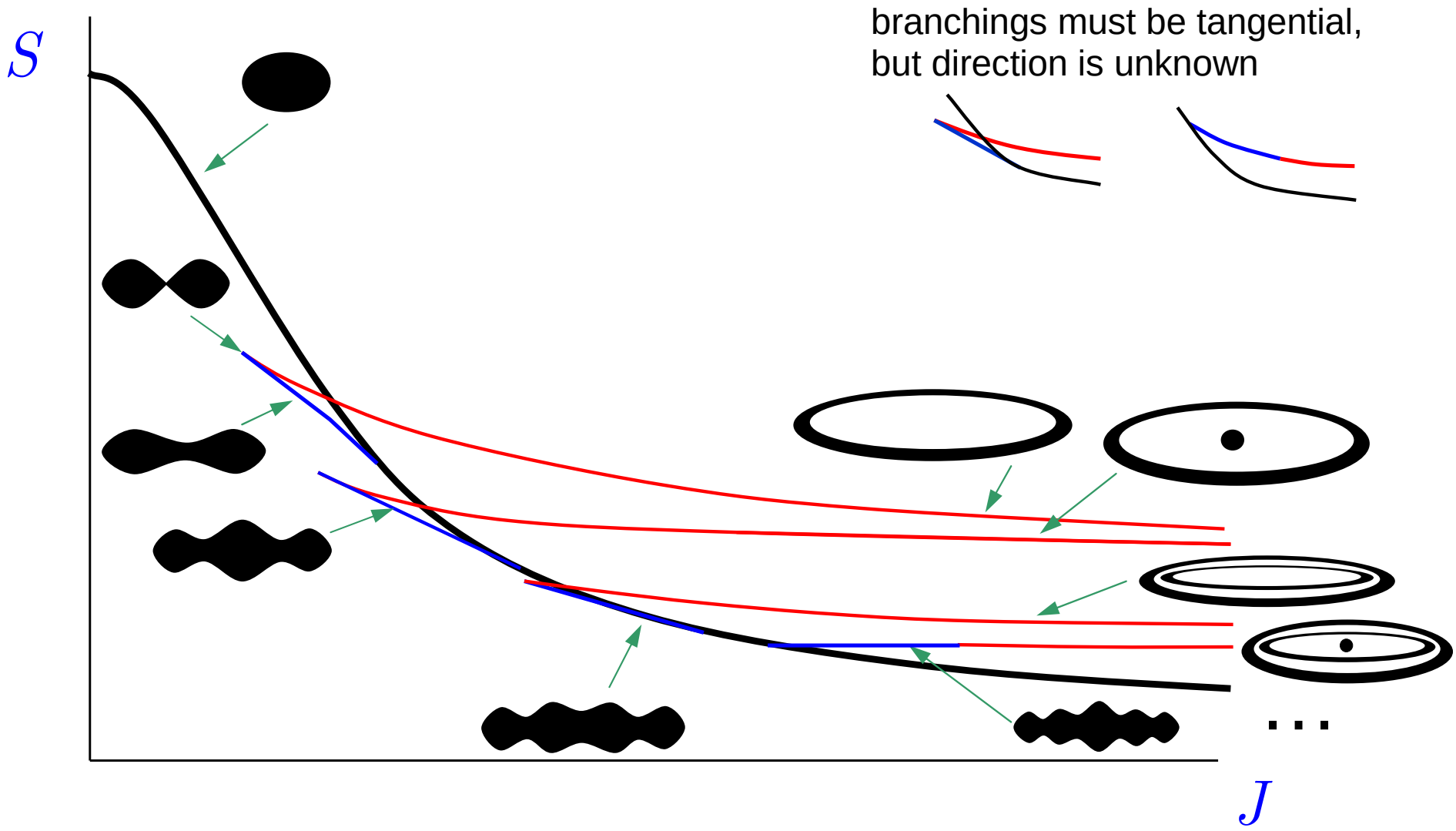
$D \geqslant 6$ phase diagram w/ 1 spin



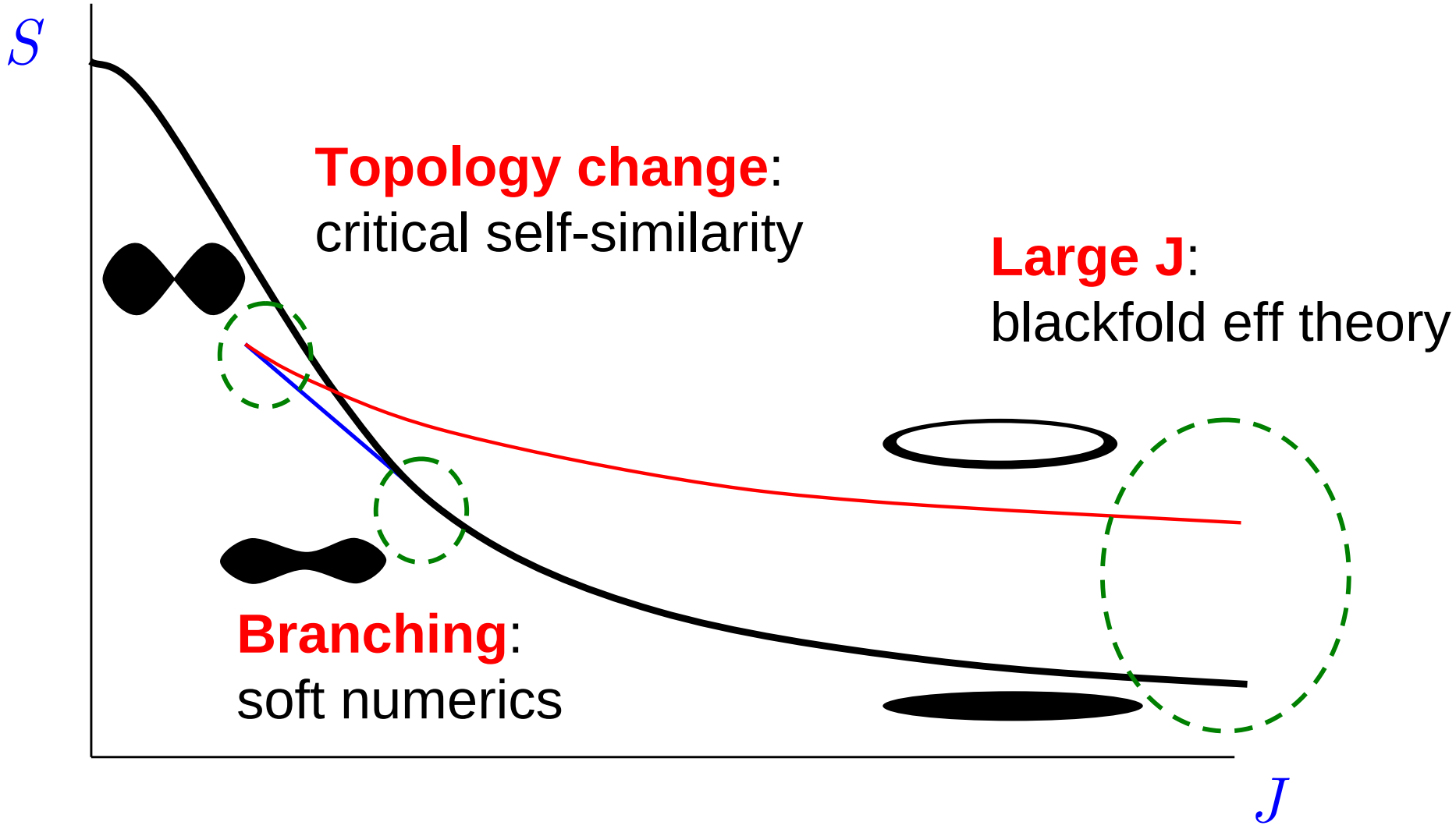
$D \geqslant 6$ phase diagram w/ 1 spin



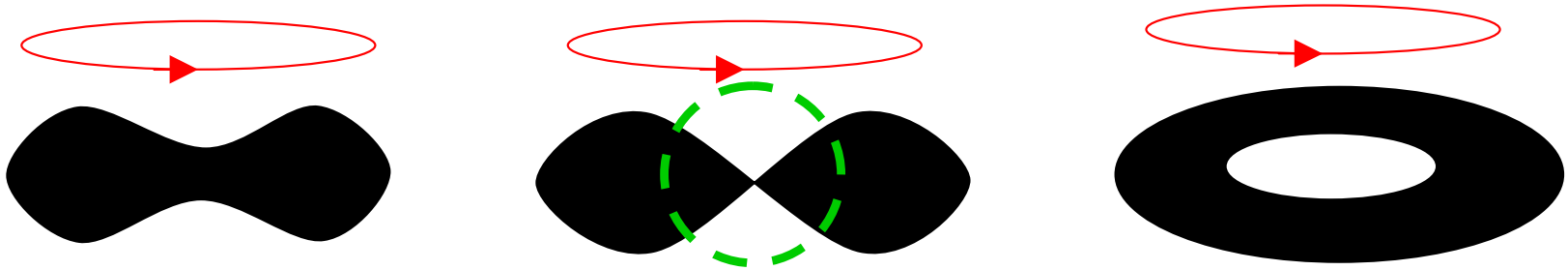
$D \geqslant 6$ phase diagram w/ 1 spin



Patterns in the phase diagram

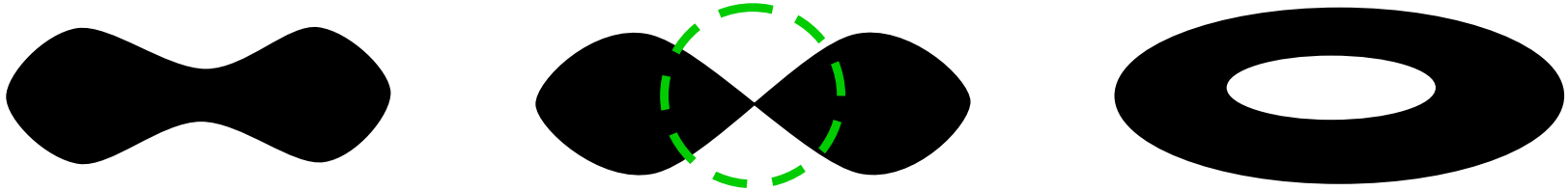


Self-similar criticality at topology-changing transitions

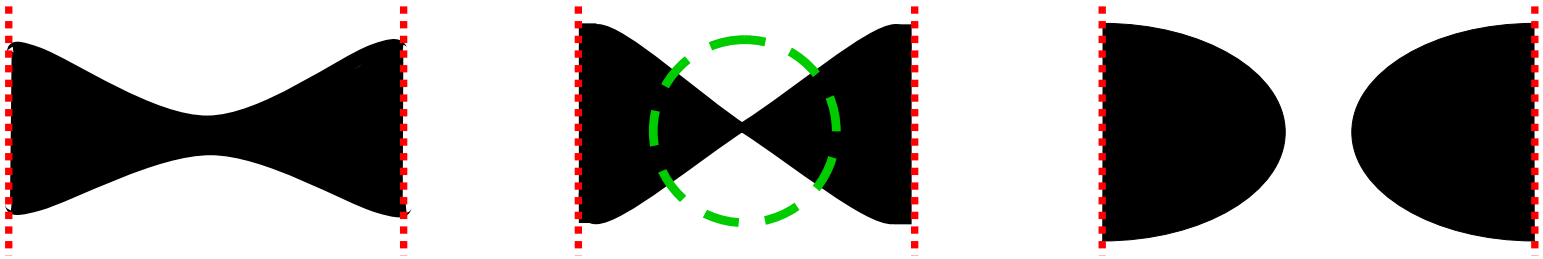


(NB: evolution in solution space, not dynamical)

Self-similar criticality at topology-changing transitions

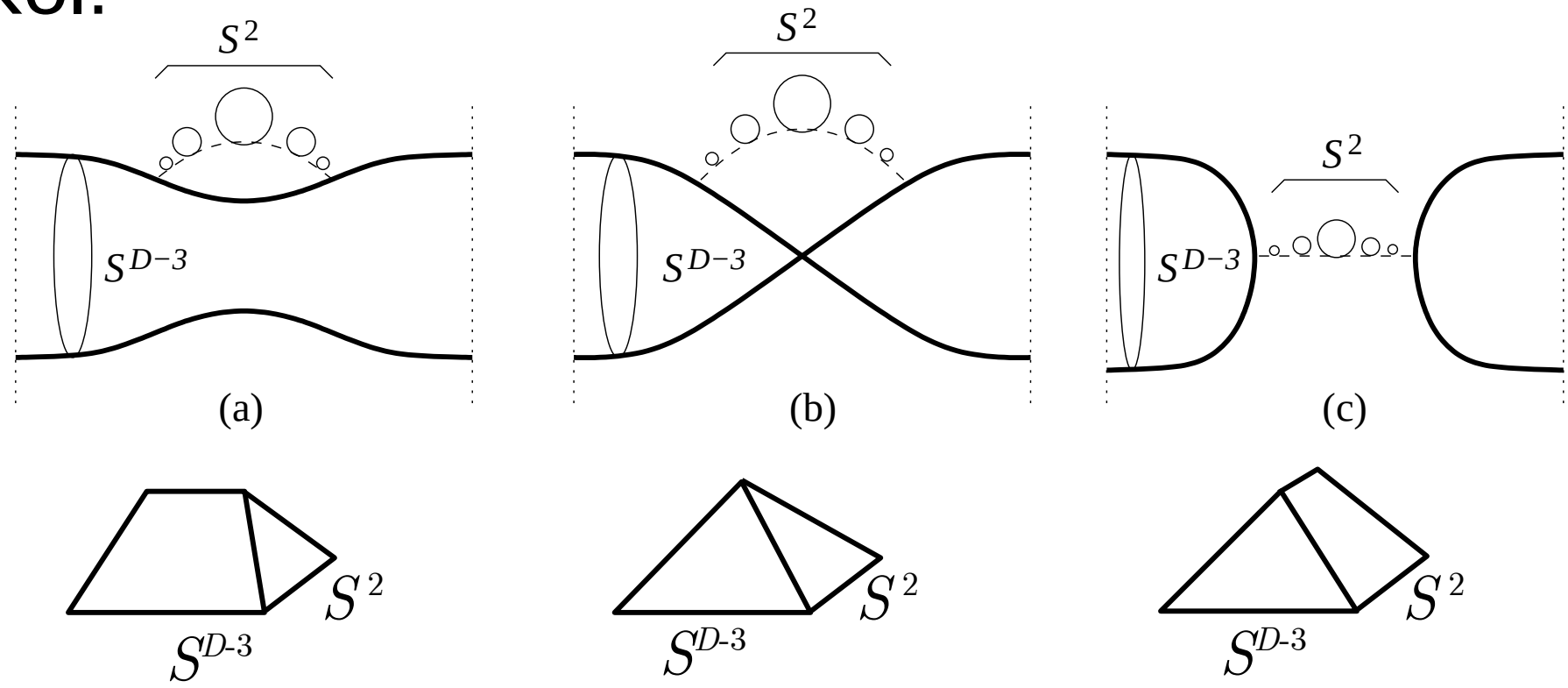


Paradigm: black hole/black string transition



Self-similar criticality at topology-changing transitions

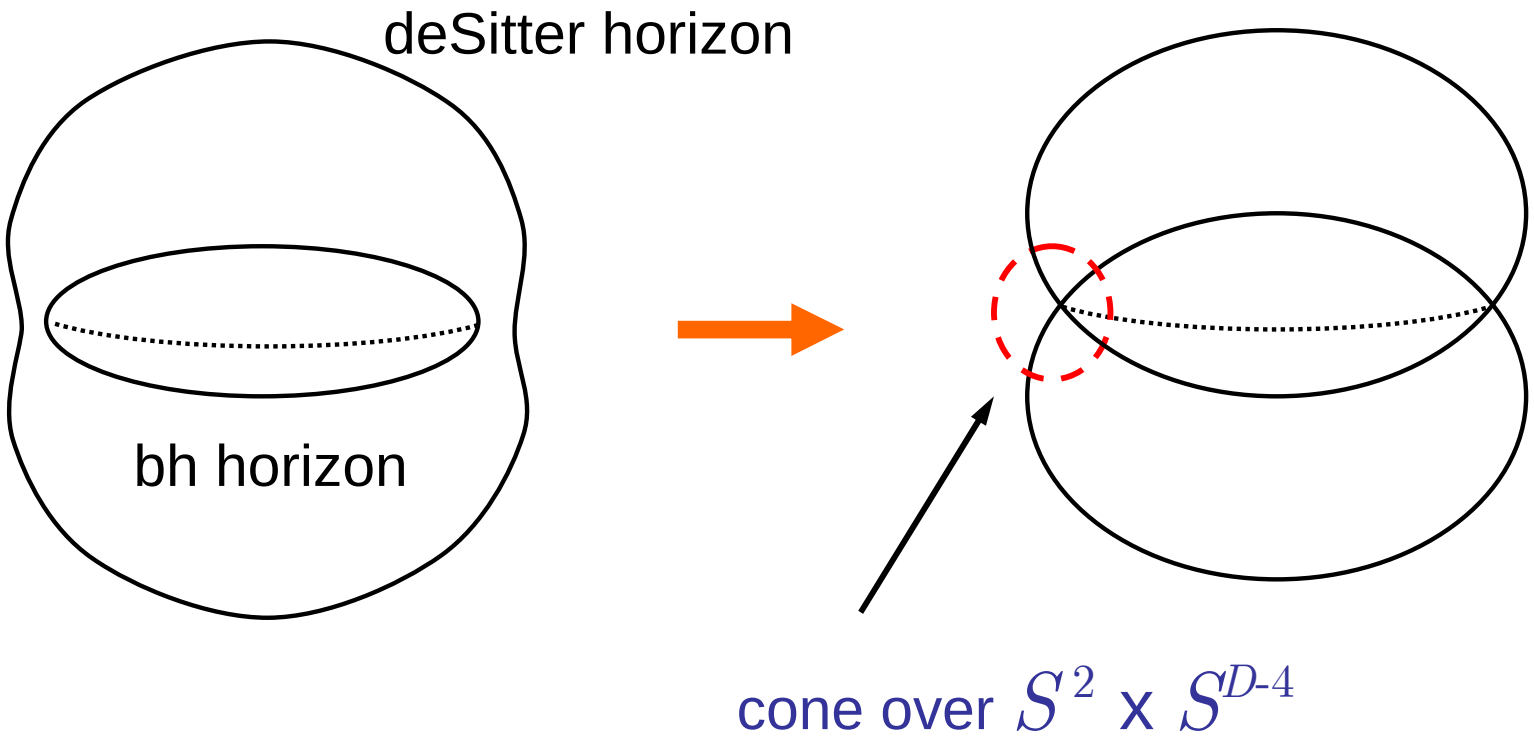
Kol:



Critical geometry:
self-similar cone over S^2 x S^{D-3}

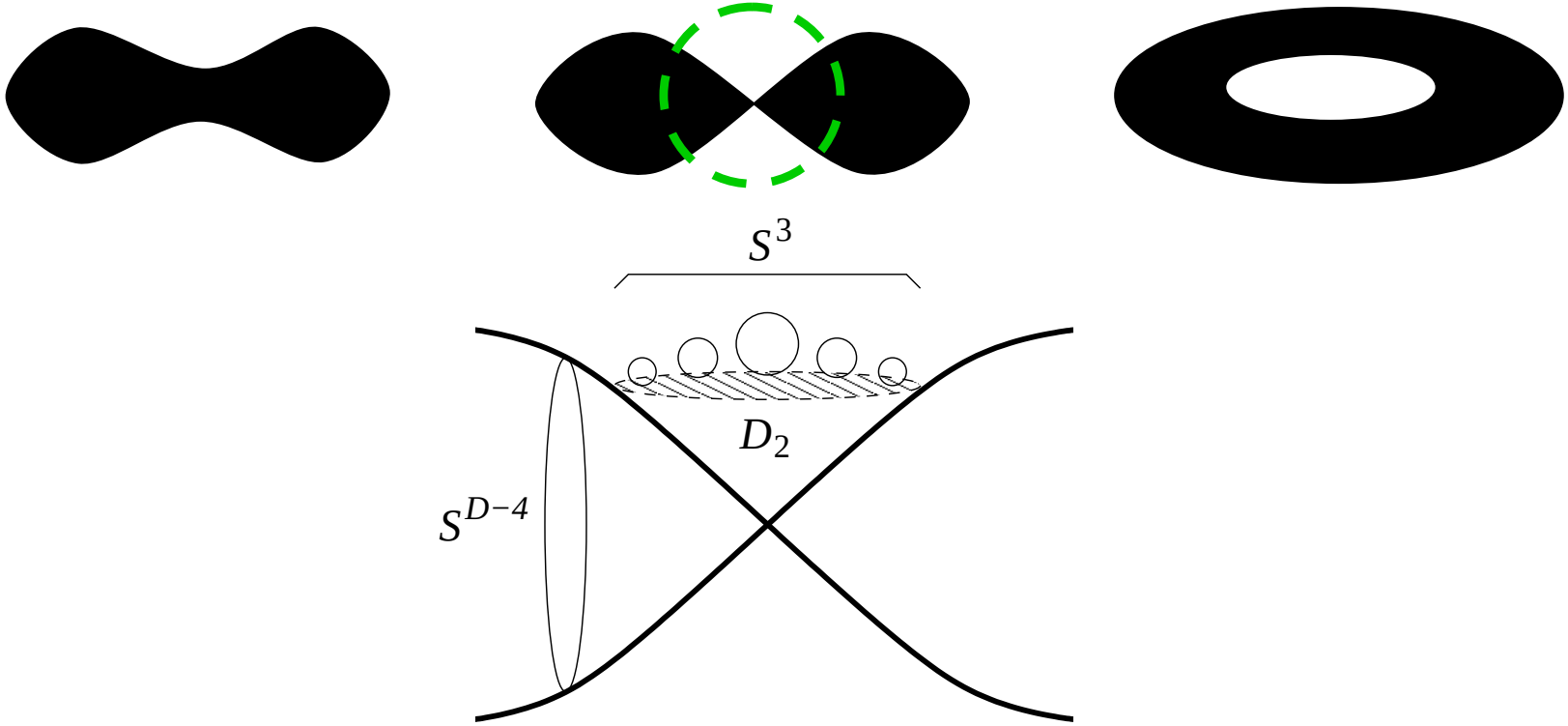
Self-similar criticality at topology-changing transitions

An exact example: Kerr-deSitter in D⩾6



Self-similar criticality at topology-changing transitions

Black ring pinch:



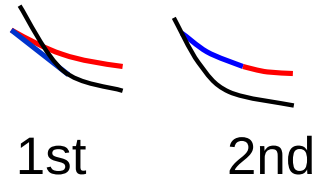
Critical geometry:
self-similar cone over S^3 x $S^{D\text{-}4}$

Where do we stand now?

- In 5D: close to a complete picture
  - We have identified (very likely) all relevant solutions:
    - MP bhs (exact)
    - planar black rings (exact)
    - helical black rings (approx)
    - combinations into black Saturns, multi-rings...
  - Possible to classify them
  - Few details missing, eg,
    - branching to helical rings
    - stability of black rings

Where do we stand now?

- In D⩾6:
  - single spin: general pattern
    - missing details: order of phase transitions



  - several spins: emerging overall patterns
    - **many** new large-J phases uncovered
    - zero-modes found, many-parameter families
    - criticality at topology change identified
    - but:

many phases still not identified

Hi-D BHs: opportunities for Numerical Relativity and HEP

- **NR**: clearly a lot of opportunities
  - search for new stationary solutions
  - investigate instability: onset & evolution
  - explore topology-changing transitions

- **HEP**: violations of cosmic censorship

  → quick route to Planck scale

HEP and cosmic censorship

- CC: from smooth initial data, GR evolution does not lead to naked singularities

- Believed to hold in D=4
- Violated in D$\geqslant$5

HEP and cosmic censorship

- Naked singularity: region of (trans)Planck-scale curvature visible for asymp observers
- CC: classical evolution won't take system to Planck-scale region
- Quantum evolution does take BH into Planck scale
- Classical evolution: **fast**: causality $t \sim R$
- Quantum evolution: **slow**: $t \sim R(R/L_{\rm Pl})^2$

HEP and cosmic censorship

- Violation of CC allows macroscopic (astrophysical?) system to get to Planck scale quickly
- But, to do so, system must probe extra dimensions
- eg, sub-mm-size black hole (primordial?) could do that in LXD
- other possibilities?