Ultraspinning Instability of Rotating Black Holes

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Based on: **0907.2248 ; 1001.4527 ; 1006.1904 ; 1011.0996 ; 1106.4554**

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Motivation to study higher dimensions

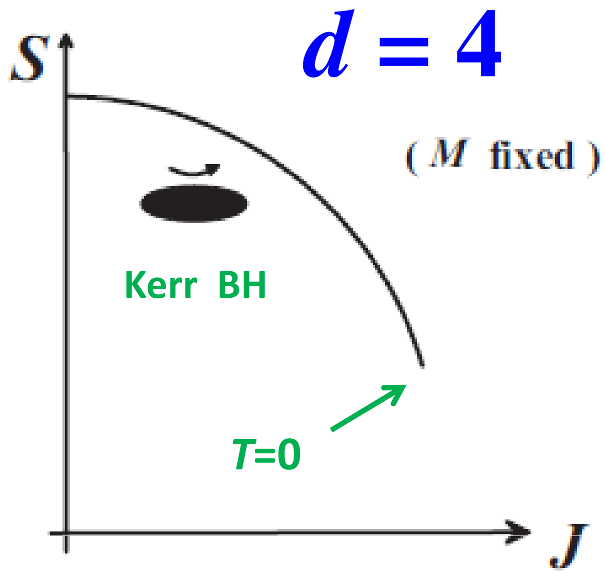
- Use dimension parameter d
  to find universal properties of Einstein's gravity
- String theory contains gravity in $d = 10$ dimensions
- Braneworld or TeV scenarios
  propose "large" extra dimensions
  to solve the Hierarchy problem.

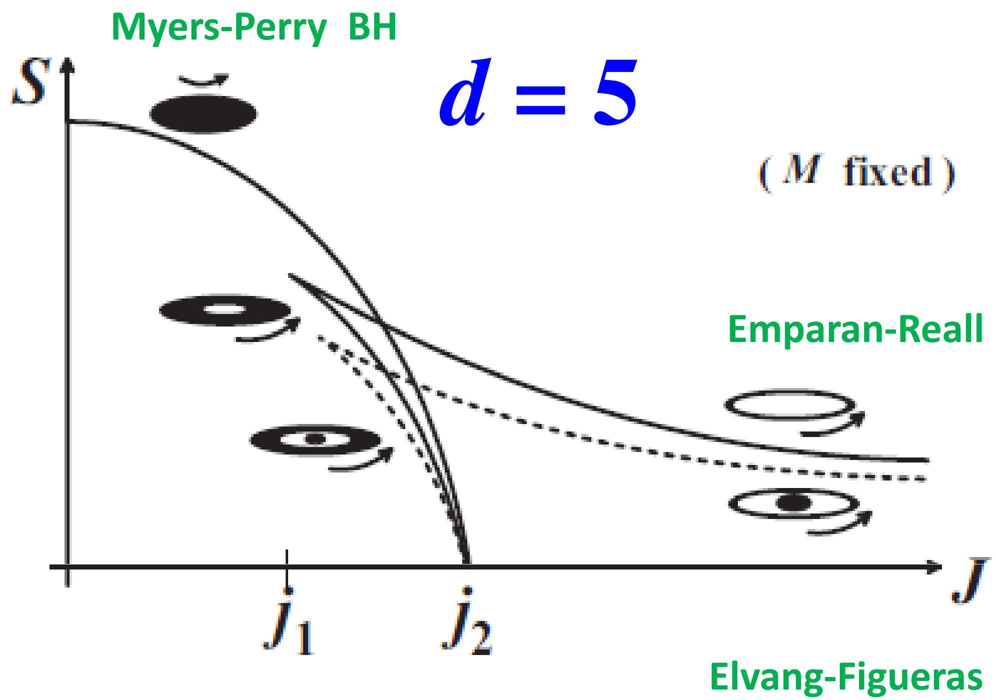
  $\longrightarrow$ Possible creation of micro black holes at the LHC ?
- Gravity in several dimensions is becoming a technical tool
  to understand several other branches of physics:
  - Gauge theory / gravity dualities
  - Fluid / gravity correspondence
  - Condensed matter / gravity correspondence

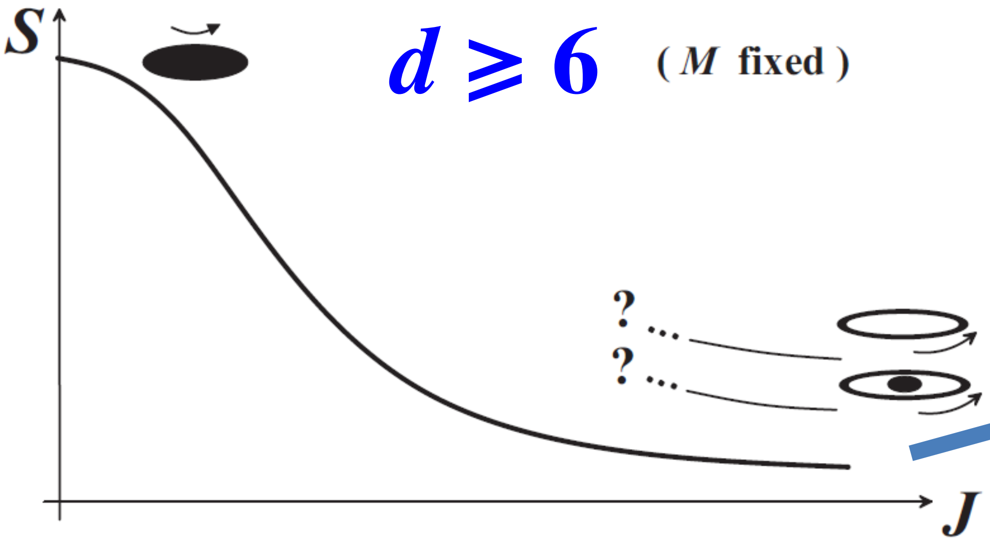
↳ Dualities between d-dim non-grav. theory & $(d+1)$–dim theory of gravity

In particular, BHs play a fundamental role in gravity

- Phase diagram of stationary BHs in $d = 4$, $d = 5$ & $d \geq 6$:





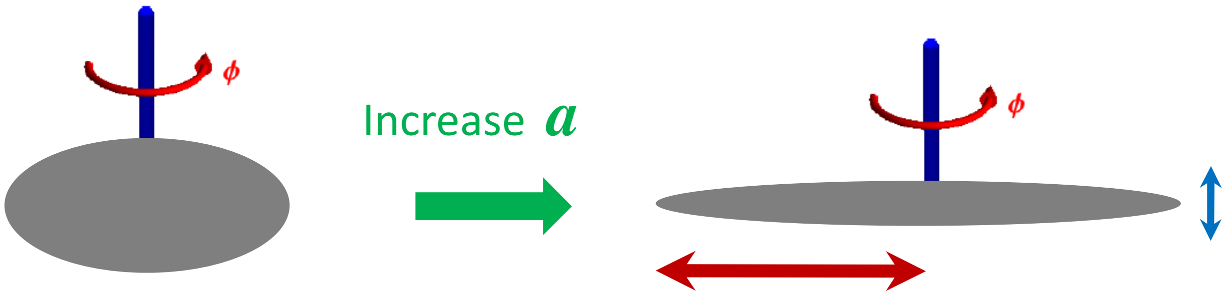


***J* can grow without bound !**

Ultraspinning Inst. : Original argument (single spin MP)

[Emparan, Myers 2003]

- **Take a singly spinning Myers-Perry (MP) black hole in $d \geqslant 6$:**



$\ell_{\perp} \sim r_+$

Lengthscale of $S^{d\text{-}4}$

$\ell_{||} \sim a$ Lengthscale parallel to (θ, ϕ) rotation plane

- **For $a >> r_+$: (θ, ϕ) rotation plane becomes a 2d plane**

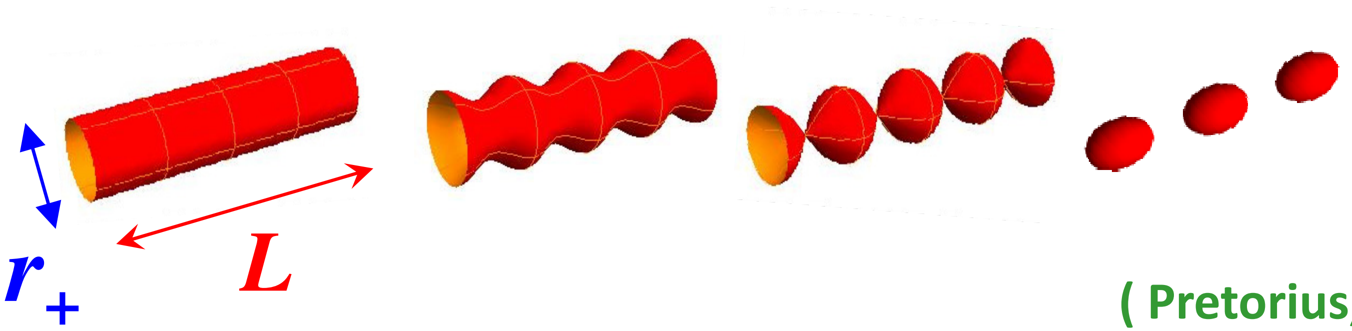
- **Zoom near the pole $\theta \sim 0$:** $a \to \infty \quad \hat{r}_m^{d-3} = r_m^{d-3}/a^2$ fixed

$$ds^2 \simeq -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-4} + \underline{d\sigma^2 + \sigma^2 d\phi^2}$$

We get the metric of a black membrane $(\sigma = a \sin\theta, \phi)$

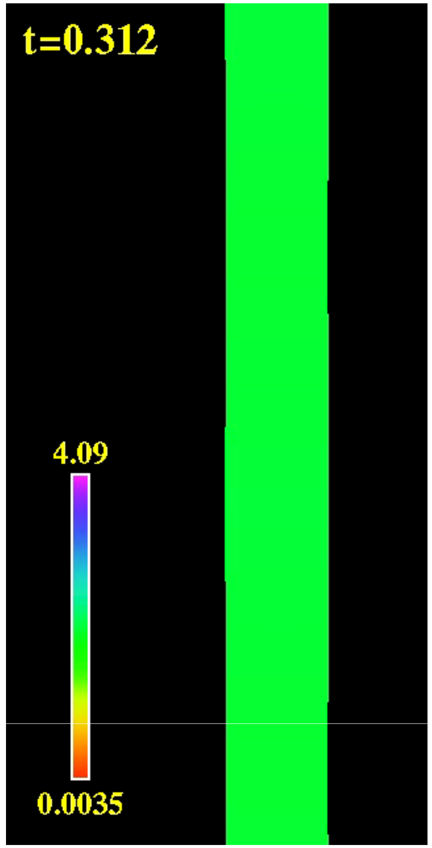
- **Far reaching consequence:**

black membrane (black string) **is**
Gregory-Laflamme unstable for $L > r_+$ **:**

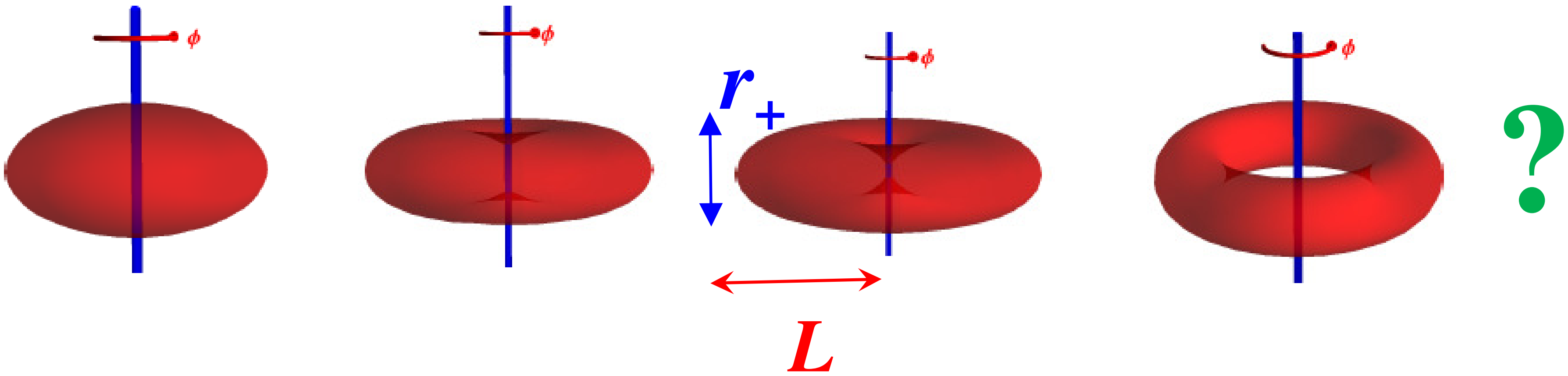


Time Evolution

(Pretorius, Lehner 2010)



- **Fastly singly spinning MP black holes should be unstable against an *axisymmetric* GL-*like* instability:**



- **Ultraspinning unstable modes** should **depend only on *r*, *θ*** and **preserve the *R* x *U*(1) x *SO*(d-3) symmetries** of the background MP bh

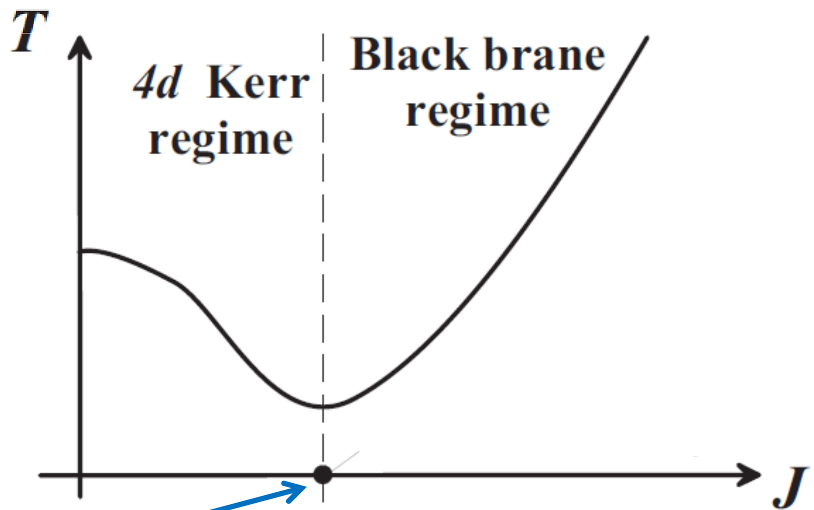
- As $a \to \infty$, the **GL nature** of instab implies perturbations with **Bessel form**:

$$h_{\mu\nu} \sim e^{\Gamma t} J_m(\kappa\sigma)\, e^{im\phi}\, \tilde{h}_{\mu\nu}(r)$$

$(\sigma = a\sin\theta, \phi)$

$m = 0$

- **Estimate for the critical rotation where the instability appears:**



- **Onset** of instability should occur around: $$\left(\frac{a}{r_+}\right)_{\rm mem} = \sqrt{\frac{d-3}{d-5}}$$

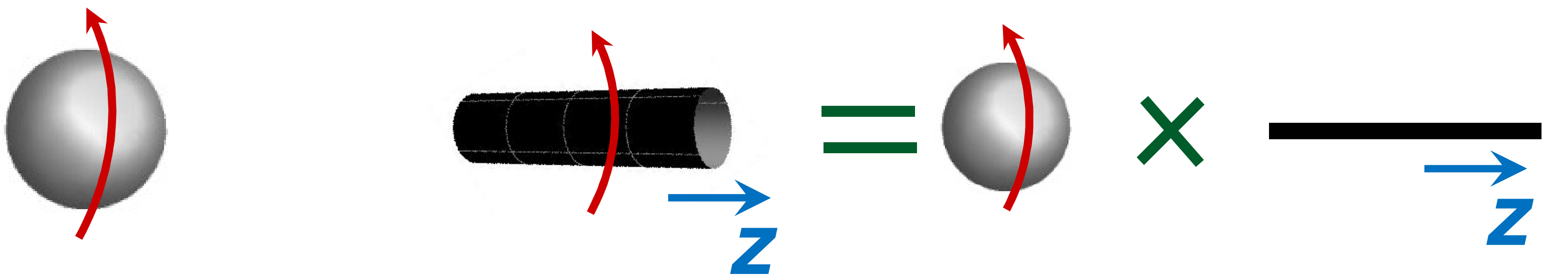
Ultraspinning instability: numerical search

- **Perturbed Einstein's eqs :** $(\Delta_L \delta g)_{ab} = 0$ Δ_L : Lichnerowicz Operator

We have to solve a **coupled** system of ***several*** **PDEs** !

Introduce **<u>Spectral</u>** **<u>numerical</u>** **<u>methods</u>**

- Interested also in **perturbations** of the **rotating black string:**



- Translationally invariant , so Fourier decompose along z: $\delta g_{ab} = e^{ikz} h_{ab}$

→ **Generalized** ***eigenvalue*** problem: $(\Delta_L h)_{ab} = - k^2 h_{ab}$

(BH case recovered when $k = 0$; black string otherwise)

Strategy to find the ultraspinning instability:

- Solve coupled system of several PDEs:

$$(\Delta_L h)_{ab} = - k^2 h_{ab} \quad \text{where} \quad \delta g_{ab} = e^{\Gamma t} e^{i k z} h_{ab}$$

- Fix gauge: Traceless-Transverse

preserve the T and Ω of the original BH

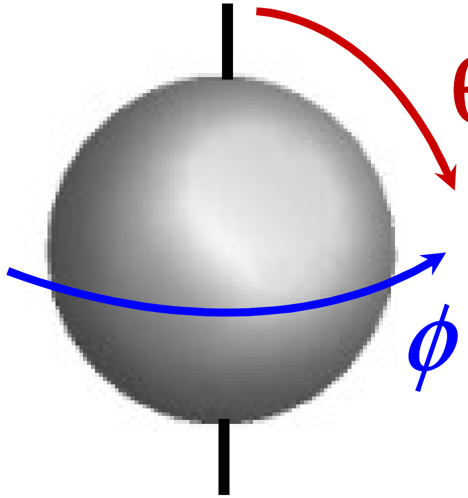
- **Boundary Conditions:**
  1) h_{ab} must be regular on the future event horizon $\mathcal{H}^+$ $(r = r_+)$
  2) h_{ab} must preserve asymptotic flatness

- This is an **eigenvalue problem** in k^2:

1) fix r_+, 2) give $\{J, \Gamma\}$ $\rightarrow$ 3) get $-k^2$.

- $\Gamma > 0$, $k \neq 0$ $\rightarrow$ instability in the (rotating) **black string**
- $\Gamma > 0$, $k = 0$ $\rightarrow$ instability in the rotating **black hole**

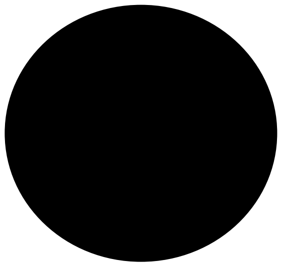
Perturbations have an **harmonic structure** labbelled by ℓ



$$\delta g_{ab} \sim e^{i m \phi} Y_{\ell m}(\theta)$$

- m deforms **equatorial** cross section

$m = 0$:

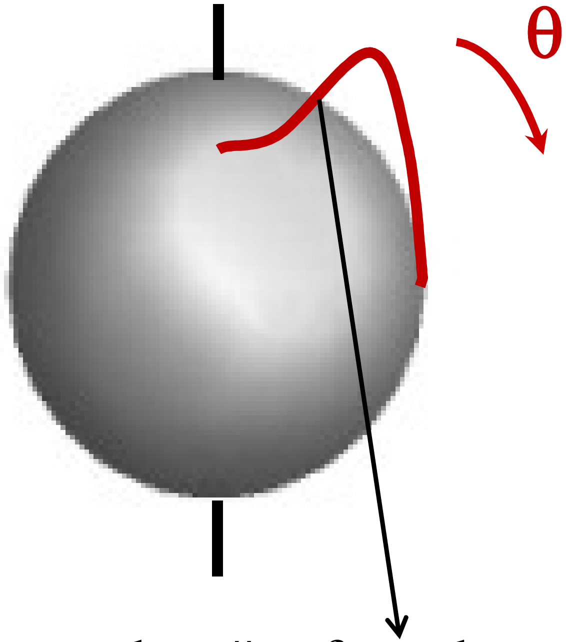


$m = 2$:

Ultraspin modes have $m = 0$

- ℓ deforms **polar** profile:

$\ell \neq 0$:



ℓ counts the # of nodes

Ultraspinning Inst. : numerical search **(single spin MP BH)**

- **Properties** of perturbations at the **onset** of the **ultraspinning instability:**

1) are **stationary ($\Gamma = 0$) axisymmetric ($m = 0$)** perturbations;

2) have an underlying **harmonic structure** labelled by ℓ ;

3) **preserve the T** and **Ω** of the original MP bh

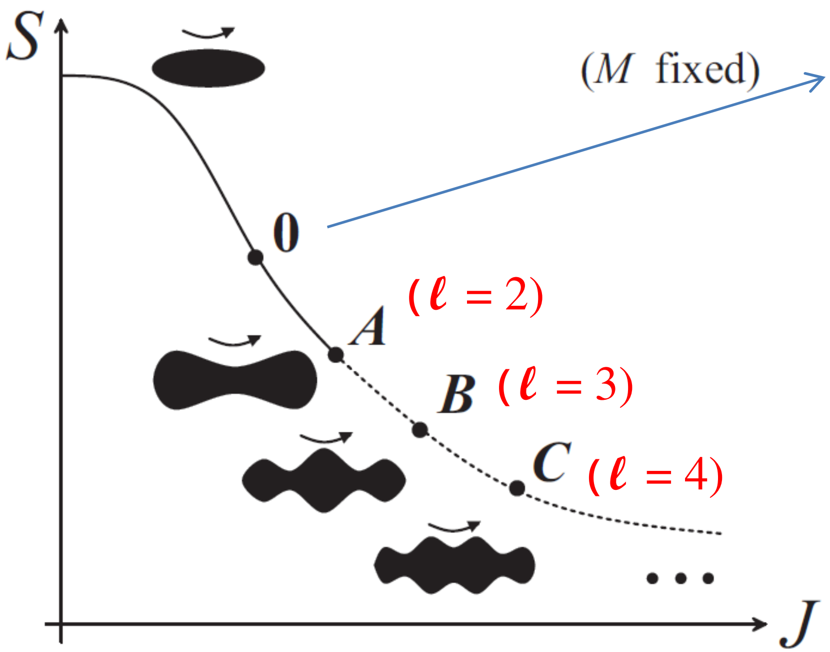
4) are **not pure gauge** modes

- **Critical rotations** for **onset** of the instability (for several d and ℓ):

d	$(a/r_m)\|_{\ell=1}$	$(a/r_m)\|_{\ell=2}$	$(a/r_m)\|_{\ell=3}$
6	1.097	1.572	1.849
7	1.075	1.714	2.141
8	1.061	1.770	2.275
9	1.051	1.792	2.337
10	1.042	1.795	2.361
11	1.035	1.798	2.373

Keypoint:

$$a / r_m \sim O(1)$$



• $\ell = 1$ mode: thermodynamic mode.

1) **Not** instability of **BH**

(**but** is **new** instability of **black string**).

2) Occurs at Emparan-Myers critical rotation

$$\left(\frac{a}{r_+}\right)_{\text{mem}} = \sqrt{\frac{d-3}{d-5}}$$

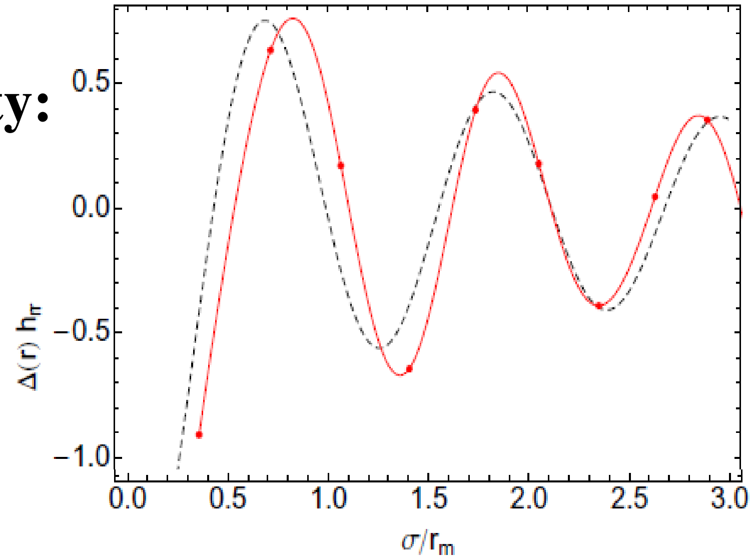
3) **BH:** Corresponds to **variations** of ***M*** & ***J*** **within MP family** that **preserves *T*** and **Ω**

• (***A, B, C, ...***): onset of **instabilities** of the MP **black hole**

(also describes **new instabilities** of the **black string**)

• **Confirming the GL nature of the instability:**

Our perturbations have a **Bessel function behavior** as the original Emparan-Myers argument for existence of a **GL-like** ultraspinning instability **predicts**



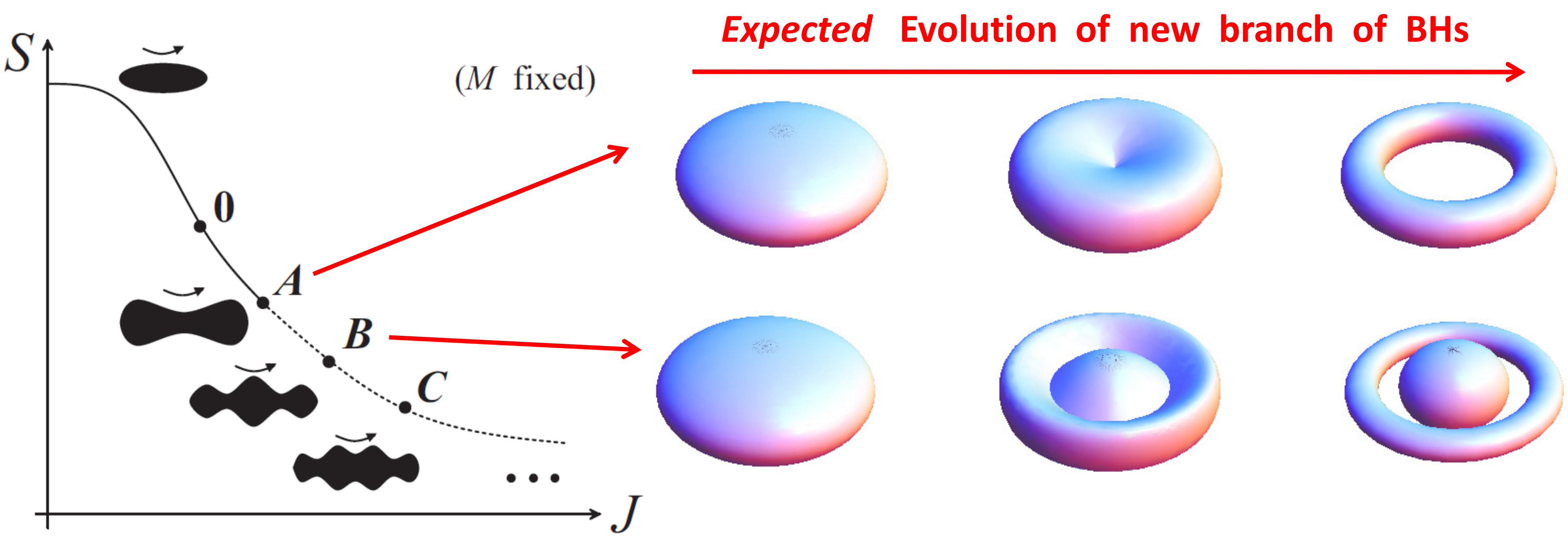
- **Instabilities** signal **bifurcations** in ***phase diagram*** of stationary solutions**:**

1) **Thresholds of the instability** expected to **signal bifurcations** to new branches of axisymmetric solutions with **pinched horizons**

2) These pinched BHs have the **same isometries** as the **original MP** BH, but their spherical **horizons are distorted by ripples** along polar direction

3) These new branches of solutions are **conjectured to connect to** the **black ring** and **black Saturn** families

[Emparan, Harmark,Niarchos,Obers, Rodriguez]



Thermodynamics & Ultraspinning instabilities

Follow strategy Gubser 2000, Reall 2001 , this time in the ultraspinning system **[1001.4527]**

$$\delta g_{ab} = e^{i\,k\,z}\, h_{ab}$$

- $\boldsymbol{(\Delta_L h)_{ab} = - k^2 h_{ab}}$ → threshold unstable mode of the black string maps to

a ***stationary*** negative mode of the Lorentzian & **Euclidean** $(\Delta_L h)_{ab}$ of the BH.

- Local thermodynamic stability → positivity of the thermodynamic Hessian:

$$-S_{\alpha\beta} \equiv -\frac{\partial^2 S(x_\gamma)}{\partial x_\alpha \partial x_\beta}\,, \qquad x_\alpha = (M, J_i)$$

- Take a family of off-shell Euclidean solutions (w/ same T and Ω as backg.).
  Use Hamiltonian formalism and 1st law to prove that:

$$\left(\frac{\partial^2 I}{\partial y^\alpha \partial y^\beta}\right)_{y=x} = -S_{\alpha\beta}(M, J)$$

I **: Euclidean action**

So, local thermodynamic instability ($-S_{\alpha\beta} \leq 0$) implies the existence of a negative mode (ie the euclidean action must decrease in some direction)

- $-S_{\alpha\beta}$ has at least one **negative eigenvalue** for any AF BH (Smarr, 1st law)

→ **all (any *a*) Asymp. Flat BH are locally thermodynamically unstable**
& associated black string is classically GL unstable ($\ell = 0$ mode)

- For $d \geq 6$, as a grows, the **BH** acquires an **additional** thermodynamic instability, & the associated black brane develops a new classical instability (in addition to GL)

Consequences:

1) this is a **refinement of the** *Gubser-Mitra conjecture*:

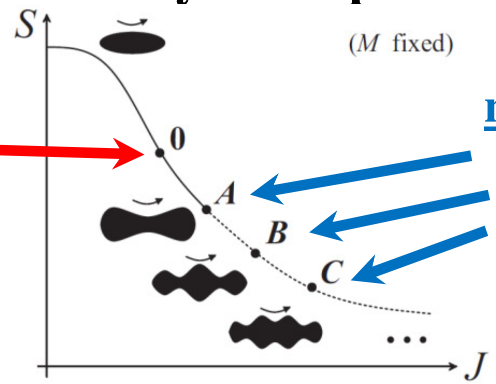
not only local thermodynamic instability implies the existence of a classical GL instability of the string, **but in addition** there is

a **distinct** classical instability associated to each **new** negative mode of $-S_{\alpha\beta}$

2) *Ultraspinning Surface*: surface in the parameter space $\{J_i /M\}$ where additional thermodynamic zero-mode appears

3) **In BH:** Such a **thermodynamic zero-mode** describes **variations of parameters within MP family** that **preserves** $\boldsymbol{T}$ and $\boldsymbol{\Omega}$ **.**

This is the **thermodinamic** $\ell = 1$ mode

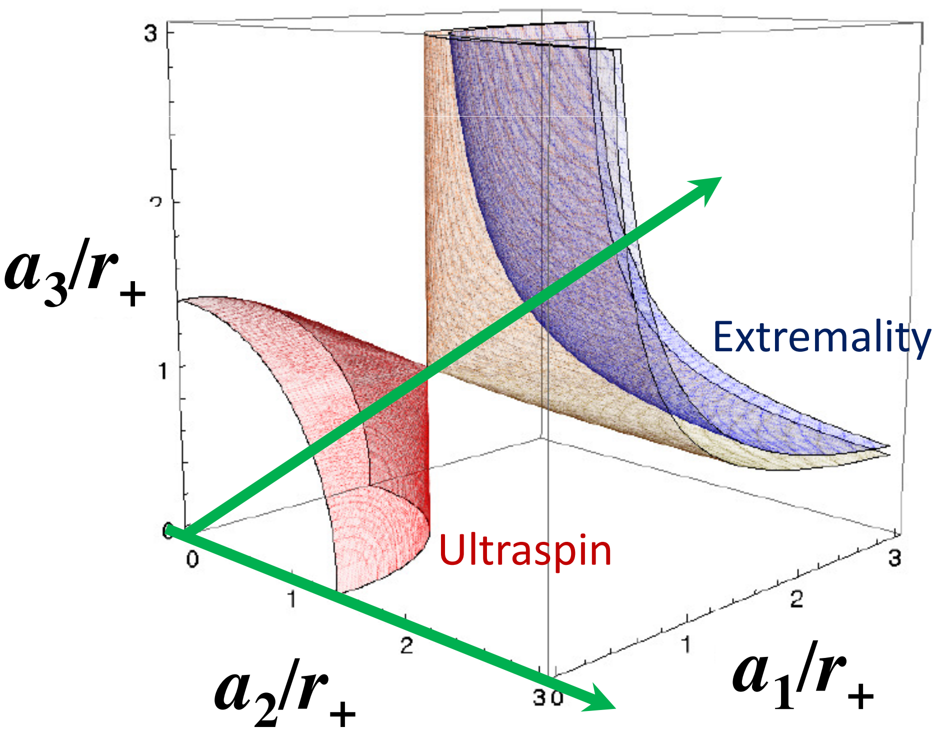


non-thermodynamic zero-modes (next slide)

• Ultraspinning Conjecture:

classical instabilities of **BHs** whose threshold is a $\omega = 0$ and $m = 0$ zero-mode are associated with **non-thermodynamic** zero-modes ($\ell = 2,3,4$) and can **occur only for rotations higher** than that of the **thermodynamic** zero-mode, that defines the **Ultraspinning Surface** ($\ell = 1$)

• Parameter space of MP bhs in *7d* with ultraspinning & extremality surfaces:



• Some MP BHs are special: moving along these families **intersect only** the **ultraspinning surface**

• In the **equal** angular momenta MP there is an ultraspinning regime **between** the **ultraspinning surface** and the **extremality surface**

• **Ultraspinning Conjecture:**

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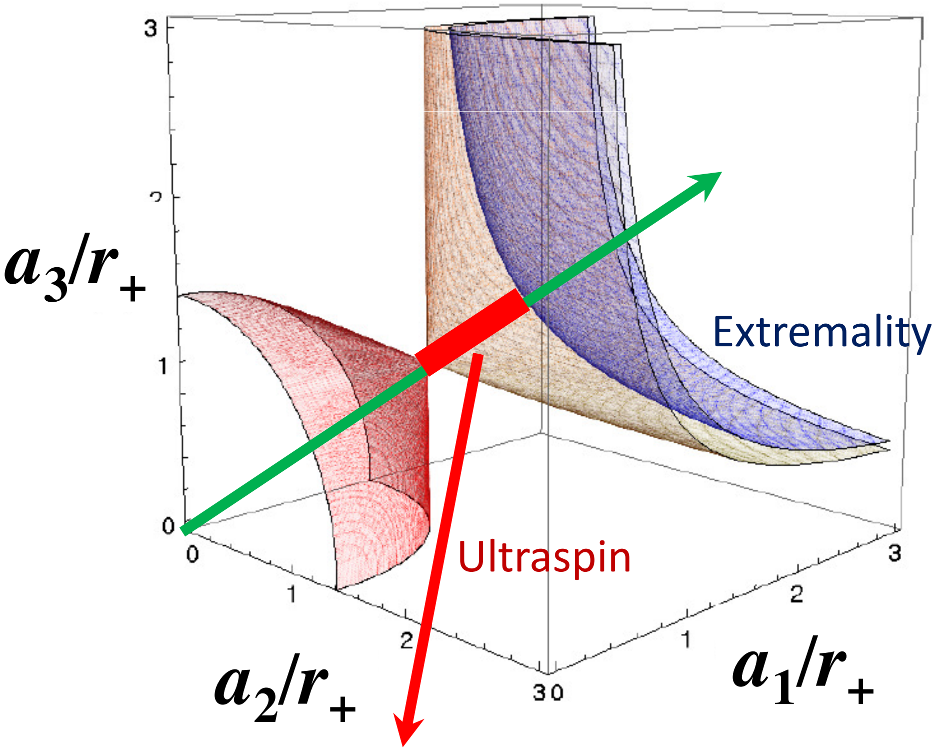


• Some MP BHs are special: moving along these families **intersect only** the **ultraspinning surface**

• In the **equal** angular momenta MP there is an ultraspinning regime **between** the **ultraspinning surface** and the **extremality surface**

• **There is room for an ultraspinning instability in equal *J* MP BHs!**

Two remarks:

- Ultraspinning conjecture: great **thermodynamic candle** since

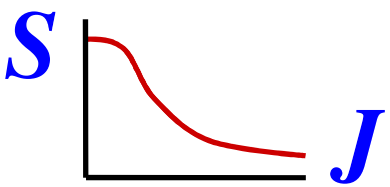
  it **predicts** region of **parameter space** where we ***might*** have **instability**

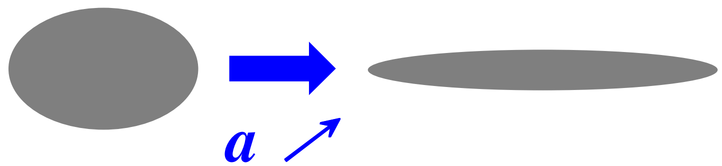
  ... but to prove its actual presence we really have to solve

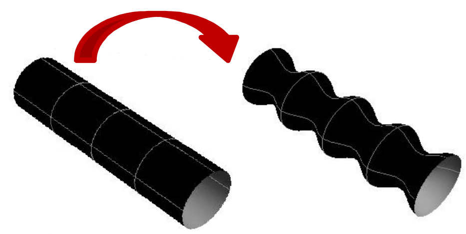
$$(\Delta_L h)_{ab} = - k^2 h_{ab}$$

- **Fundamental nature of ultraspinning instability:**

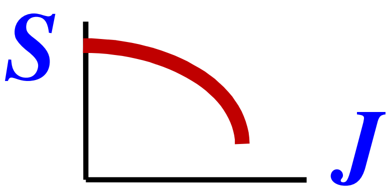
Ultraspin in **<u>Singly</u>** spinning BH**:** **Gregory-Laflamme <u>nature</u>**







Ultraspin in **<u>Equal</u>** ***<u>J</u>*** BH**:** **Gregory-Laflamme <u>nature</u>**



.... So let's study it !

Equal J Myers-Perry (MP) black hole (odd d)

- In $d = 2N + 3$, the d–dim **Schwarzschild** black hole is given by:

$$ds^2 = -\tilde{f}(r)dt^2 + \frac{dr^2}{\tilde{f}(r)} + r^2 ds^2_{S^{2N+1}}$$

- We can write S^{2N+1} as a **Hopf fibration** of S^1 over $\mathbb{C}\mathrm{P}^N$:

$$ds^2 = -\tilde{f}(r)dt^2 + \frac{dr^2}{\tilde{f}(r)} + r^2 \underbrace{\left\{[d\psi + \mathbb{A}_{\hat{a}}dx^{\hat{a}}]^2 + ds^2_{\mathbb{CP}^N}\right\}}_{ds^2_{S^{2N+1}}}$$

where $ds^2_{\mathbb{C}\mathrm{P}^N}$ and $\mathbb{J} = d\mathbb{A}/2$ are the metric and Kahler form on $\mathbb{C}\mathrm{P}^N$, $\hat{a}$ runs over the $\mathbb{C}\mathrm{P}^N$ coords.

- **Odd dimensional MP (equal J's)** is a "deformation" of the former:

$$ds^2 = -\frac{f(r)dt^2}{H(r)} + \frac{dr^2}{f(r)} + r^2\left\{H(r)[d\psi + \mathbb{A}_{\hat{a}}dx^{\hat{a}} - \Omega(r)dt]^2 + ds^2_{\mathbb{CP}^N}\right\}$$

$f(r) = H(r) - r_m^{2N}/r^{2N}$, $H(r) = 1 + a^2 r_m^{2N}/r^{2N+2}$

$\Omega(r) = a r_m^{2N}/[r^{2(N+1)}H(r)]$

Nice properties of Perturbations in equal J MP

- **Cohomogeneity-1** BHs: only depend on **radial coord** (but not polar coord)

- Perturbations in $\mathbb{C}P^N$ base space have **harmonic expansion** labelled by ℓ
  $\rightarrow$ allows to study which **symmetries** are **broken by the perturbations**

(Similar to Schwarzchild: expand h_{ab} as they transform under coord transf in S^{2N+1})

Lichnerowicz equations reduce to a system of **ODEs** in r

Yes, we can:

- study **not only** stationary modes ($\omega = 0$)
  (so far: limitation of single spin MP because we had PDEs)
- **but also** the **time dependence** (confirms that is a true instability)
- **and** find which **symmetries** are **broken by** the **perturbations**

Recall: Strategy to find the ultraspinning instability:

Use $\mathbb{C}\mathrm{P}^N$ **harmonic decomposition:** ℓ

- Solve coupled system of several ODEs:

$$(\Delta_L h)_{ab} = -k^2 h_{ab} \quad \text{where} \quad \delta g_{ab} = e^{\Gamma t} e^{ikz} h_{ab}$$

- Fix gauge: Traceless-Transverse

preserve the T and Ω of the original BH

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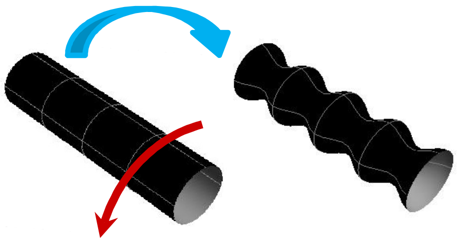
- This is an **eigenvalue problem** in k^2 **:**

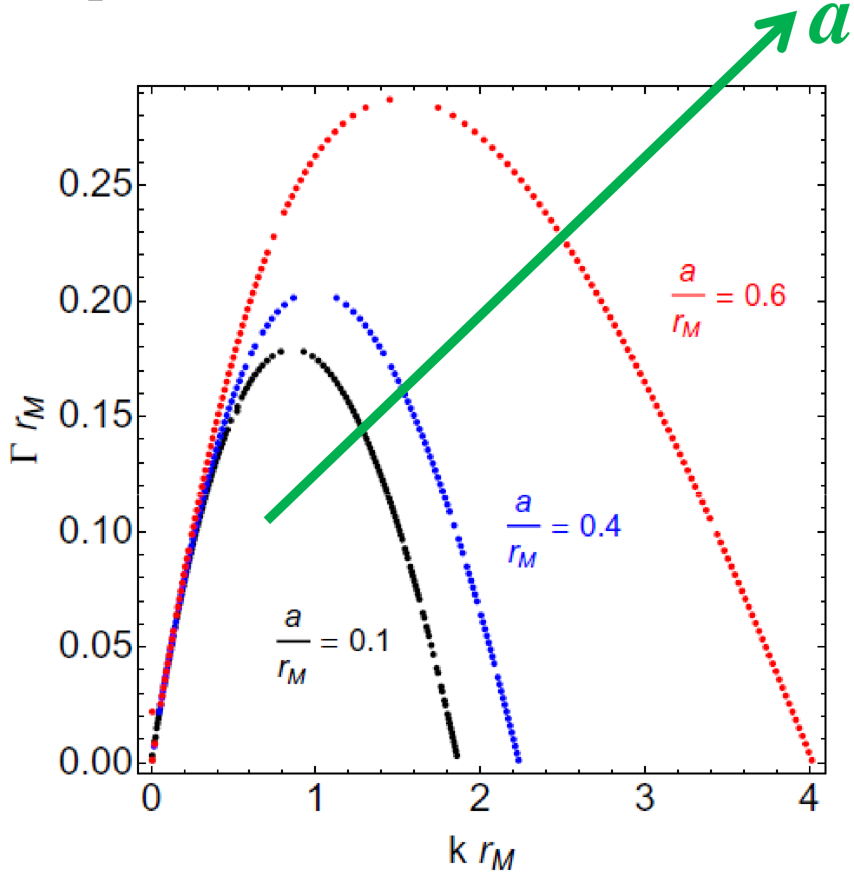
1) fix r_+, **2) give** $\{a, \Gamma, \ell\}$ $\rightarrow$ **3) get** $-k^2$.

- $\Gamma > 0,\ k \neq 0 \rightarrow$ instability in the (rotating) **black string**
- $\Gamma > 0,\ k = 0 \rightarrow$ instability in the rotating **black hole**

- **Recap**: 1) $(\Delta_L h)_{ab} = -k^2 h_{ab}$ 2) $\delta g_{ab} = e^{\Gamma t} e^{ikz} H_{ab}$ 3) $\mathbb{C}P^N$ Harmonics ℓ

- **Thermodynamic** $\boxed{\ell = 0}$ **mode:** present for $a \geq 0$



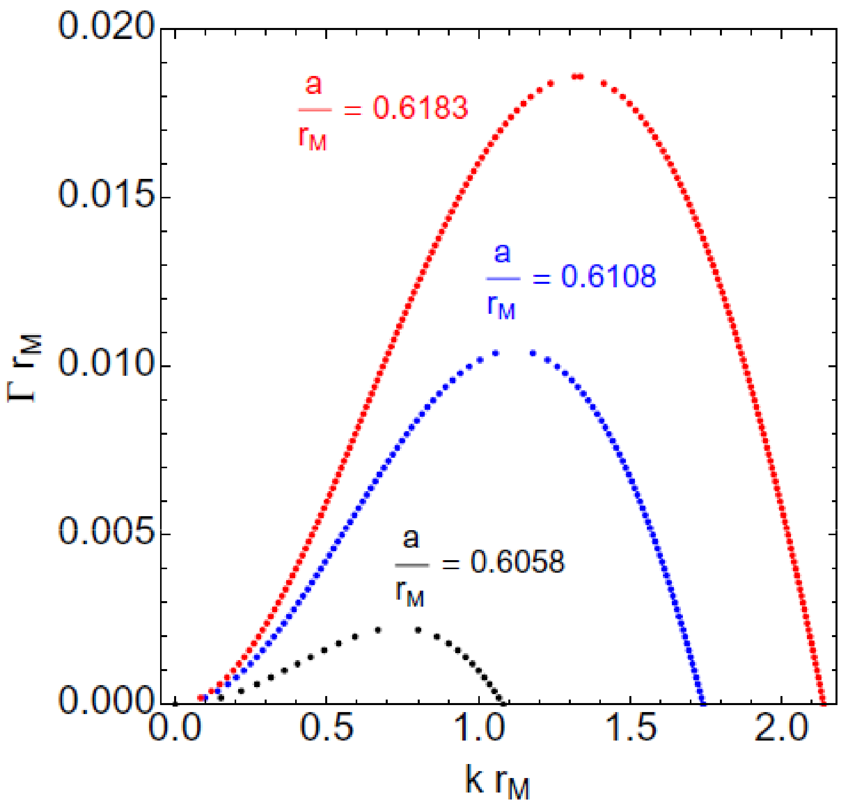


$\rightarrow$ It is the standard **GL instability** of the associated **black string.**

$\rightarrow$ Black string becomes **more GL unstable as rotation increases.**

$\longrightarrow$ **Not** an **instability** of the **BH:** $\Gamma = 0$ **at** $k = 0$ **[changes** M **but not** J **]**

• Thermodynamic $\boxed{\ell = 1}$ mode: Present for $\boxed{a \geq a_{\text{ultra surf}}}$.



→ **Not** an **instability** of the **BH: $\Gamma = 0$ at k=0**

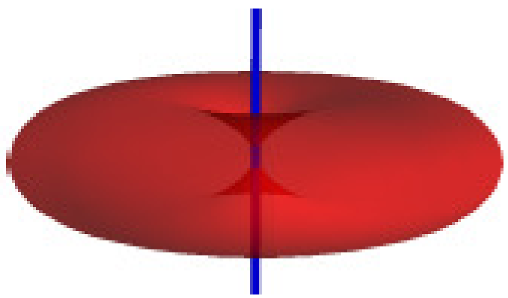
[Corresponds to variations within MP family that preserves T and Ω but **changes J**]

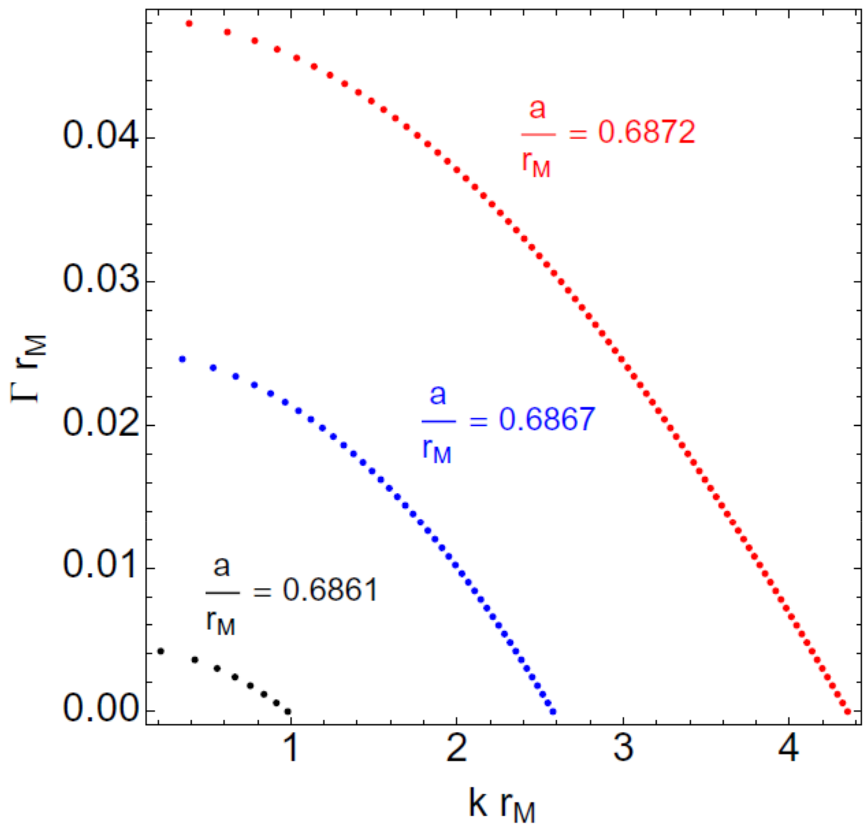
→ It is a **<u>new</u> instability** of the associated **black string** (in addition to GL)

→ Stationary (ω =0) zero-mode (k =0) **defines ultraspinning surface**

→ For higher a we might have <u>non</u>-thermodynamic modes that describe true BH instabilities

• Non - thermodynamic $\boxed{\ell = 2}$ mode: Present for $\boxed{a > a_{\text{ultra surf}}}$





→ **It is true ultraspinning instability of the BH: $\Gamma \neq 0$ at $k=0$**

First instability of a vacuum BH ! Requires $J > J_{\text{crit}}$

→ It is also **new instability** of the associated **black string.**

→ Explicitly checked that these modes are **not pure gauge**

Saturation of Hawking's rigidity theorem in $d > 4$

- Rigidity Theorem : stationary BHs have at least one $U(1)$ rotational symmetry (d>4) [Hollands, Ishibashi, Wald, 2006] [Isenberg, Moncrief 2008]

- The $\ell = 2$ **stationary zero-mode** of the BH is interpreted as
  a **bifurcation point** to new family of solutions
  that **preserves/breaks the same symmetries as** the perturbation **mode**
  [this was strategy suggested by **Reall 2001** to find BH with less symmetries]

- $\ell = 2$ mode can **break all** the ***CP***^{n} **symmetries** and **preserves only** ∂_t and ∂_ψ

$\rightarrow$ **new BH** solutions will possess just a **<u>single</u> rotational symmetry**

$\rightarrow$ saturate the Rigidity Theorem (also 'Helicoidal' black ring; **Emparan *et al***)

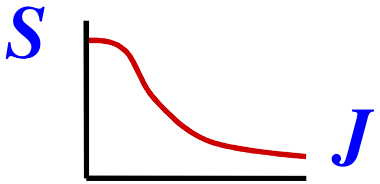
- **Most general $\ell = 2$ harmonic** is labelled by $(n+2)^2(n+1)^2/4-(n+1)^2$ **parameters**.
  We can **eliminate** $n\,(n+2)$ parameters **by rotations** of the background.

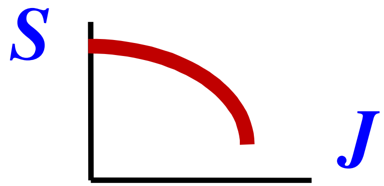
$\rightarrow$ So we are left with $(n+2)^2(n+1)^2/4-(n+1)^2 - n\,(n+2)$ parameters **plus** the **BH mass**:

7d: BH with **20 parameters !** **9d:** BH with **70 parameters!**

The nature / origin of the ultraspinning instability

Singly spinning BH: **GL nature** | **Equal *J*** BH: **NOT GL nature**

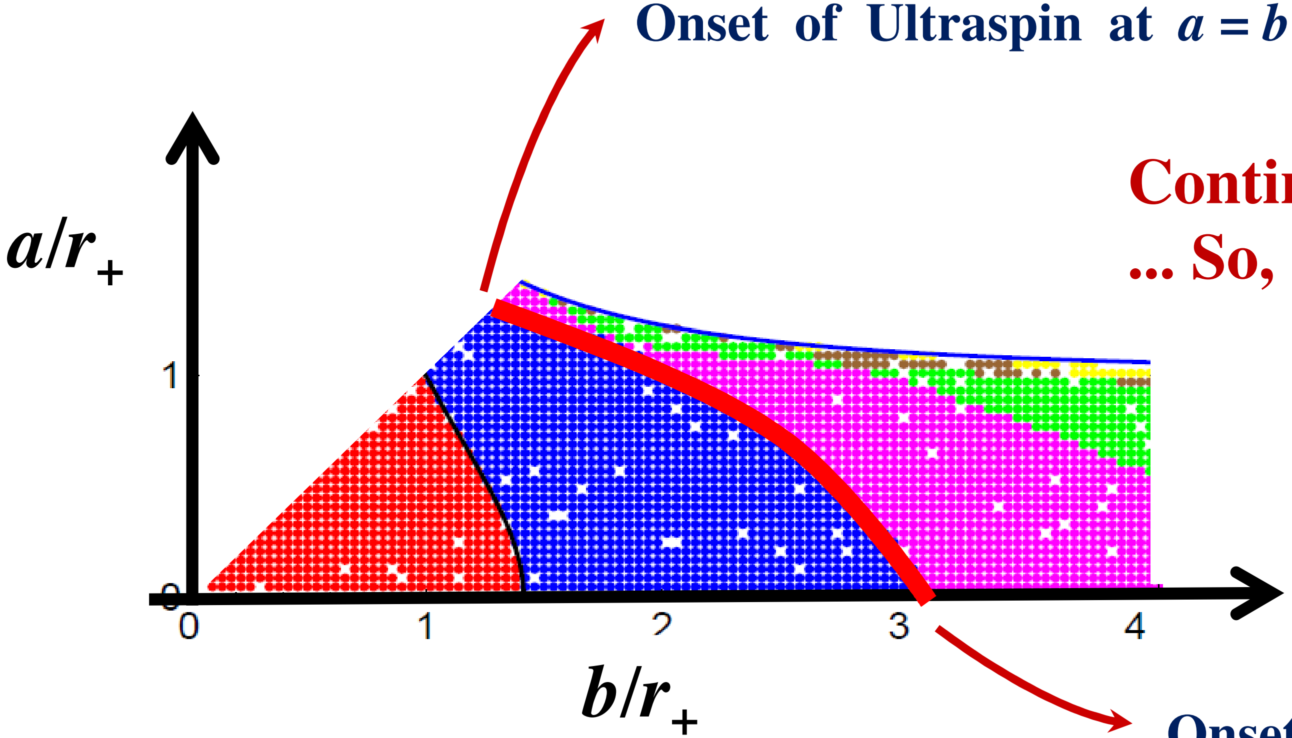




Follow Ultraspin instability

along family with TWO angular momenta, $\{a_1 = a_2 = a, b\}$:

Onset of Ultraspin at $a = b$



Continously connected,
... So, same nature / origin

Onset of Ultraspin at $a = 0$

"Analytical" methods to study instability: (extra - soft numerics)

- **Durkee, Reall 2010 + Mahdi's talk:**

Near -horizon geometry (of **extremal** rotating BHs) is Kundt solution

$\longrightarrow$ Gravitational perturbations decouple & separate

$\longrightarrow$ Master equation à la Teukosky using GHP formalism

If symmetries generated by $\boldsymbol{m}$ are preserved : (Killing horizon: $\partial_t + \Omega_H \boldsymbol{m}$)

$\longrightarrow$ **instability of NH geometry => instability full extreme BH.**

$\longrightarrow$ suggests that near-extreme BHs are also unstable.

- **Figueras, Murata, Reall 2011:**

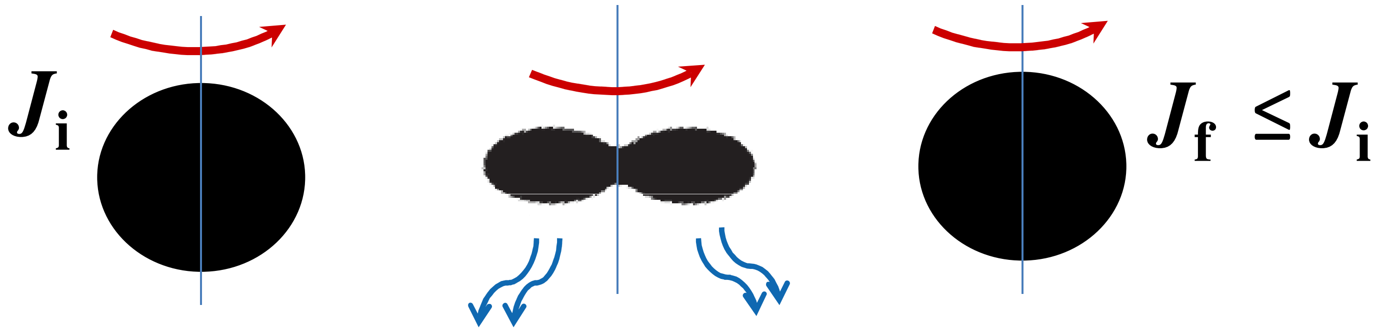
Demonstrate **BH instability** by finding **initial data** that describes a small perturbation of the BH and **violates** a **local Penrose inequality**:

$$A_{min} \leq A_{BH}(E_i)$$

greatest lower bound on area of any surface enclosing the apparent horizon of initial data

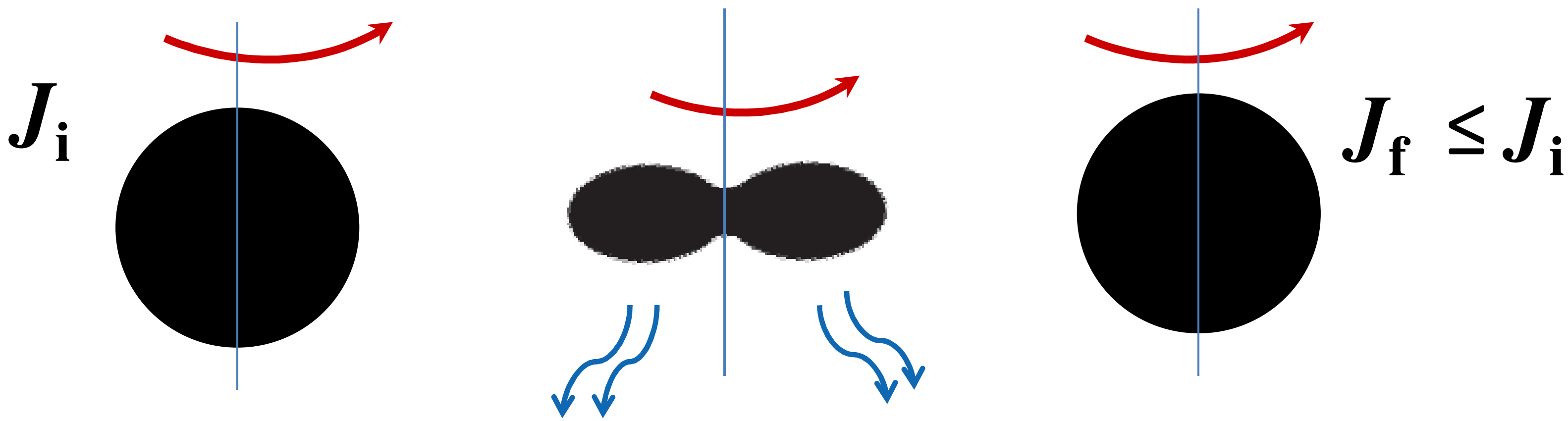
The *non*-axisymmetric bar - mode instability

- Quickly spinning rotating BHs in d>4 are also unstable
  against a ***non*-axisymmetric instability**
  predicted also by **Emparan-Myers 2003**
  and explicitly found
  in a full numerical time evolution analysis by **Shibata, Yoshino, 2010**



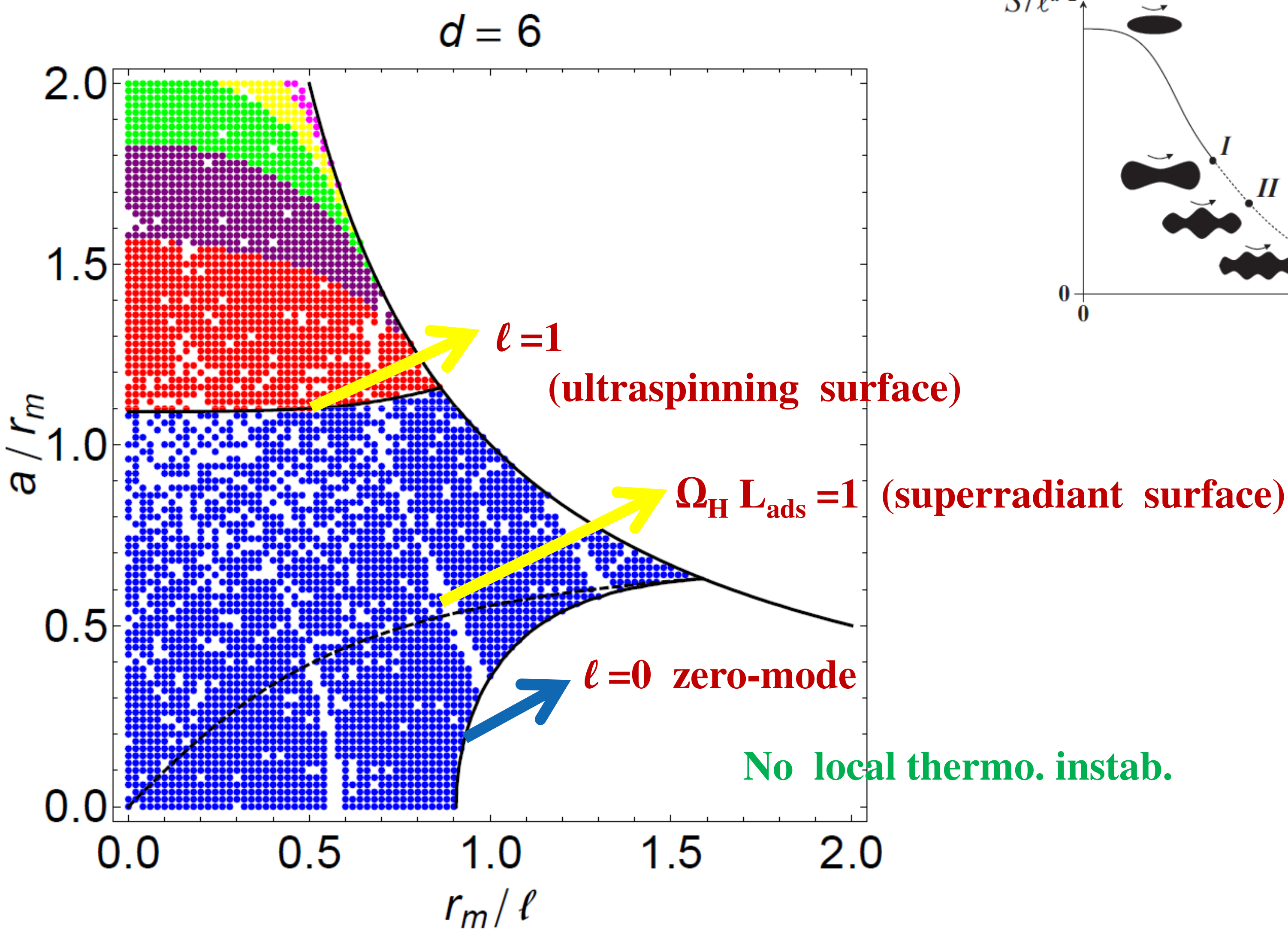
- This non-axisymmetric **bar -mode instability**
  quicks in at **smaller** values of the **rotation**,
  when compared with the axisymmetric **ultraspinning instability**.

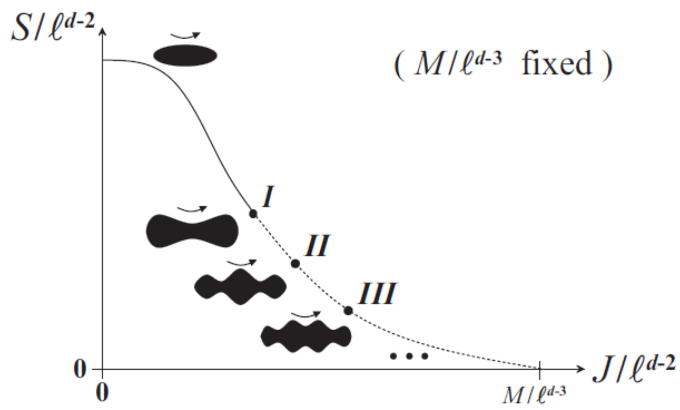
- The bar-mode instability however
  does **not** signal a **bifurcation** to new BH solutions
  in the phase diagram of stationary asymptotically flat BHs.



J_i
$J_f \leq J_i$

Ultraspinning instability persists in AdS MP-BHs (single J)





No local thermo. instab.

Ultraspinning instability **persists** in **AdS**

... *Interpretation within the Gravity / gauge duality* ?

BH with **Temperature** ***T*** ⟷ **QFT at finite** ***T***

BH quasinormal modes ⟷ **Thermalization** of **QFT**

Ultraspinning instability ⟷ **?**

Messages to take home :

- **Ultraspinning instability:** **First** **instability** of a **vacuum** **black hole**
- **All** (for any rotation) vacuum BHs are **thermodynamically unstable**
- Proved that **rotation increases** the standard **Gregory-Laflamme** instability
- Found **new (ultraspinning) instabilities** of the black string
- **Refinement of the Gubser-Mitra conjecture:**

  not only local thermodynamic instability implies the existence of a classical instability of the string, but in addition
  there is a distinct classical instability associated
  to **new** negative modes of thermodynamic Hessian
- Found **1st example** of a (perturbative) **BH with spherical topology**

  that **saturates** generalisation of **Hawking's rigidity theorem** to $d > 4$
- Ultraspinning instability **persists** in **AdS**

Thanks !