Nonperturbative Strong-Field Effects in Tensor-Scalar Theories of Gravitation

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It is shown that a wide class of tensor-scalar theories can pass the present weak-field gravitational tests and exhibit nonperturbative strong-field deviations away from general relativity in systems involving neutron stars. This is achieved without requiring either large dimensionless parameters, fine tuning, or the presence of negative-energy modes. This gives greater significance to tests of the strong gravitational field regime, notably binary pulsar experiments.

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Tensor-scalar theories, in which gravity is mediated by one or several long-range scalar fields in addition to the usual tensor field present in Einstein's theory, are the most natural alternatives to general relativity. Kaluza-Klein, supergravity, and superstring theories naturally give rise to massless scalar fields coupled to matter with gravitational strength. Recently, "extended" inflationary models [1] furnished a new motivation for considering tensor-scalar theories.

Solar-system experiments set tight constraints on possible post-Newtonian deviations from general relativity, namely (at the one sigma level) [2,3],

$$|\tilde{\gamma}-1| < 2\times 10^{-3}, \quad |\tilde{\beta}-1| < 2\times 10^{-3} , \tag{1}$$

where $\tilde{\beta}$ and $\tilde{\gamma}$ denote the usual post-Newtonian parameters (we add a tilde to the standard notation to distinguish them from the underlying theory parameters introduced below). Within tensor-scalar theories, the combination

$$\alpha_0^2 = (1-\tilde{\gamma})/(1+\tilde{\gamma}) \tag{2}$$

plays a basic role because it measures the ratio between the couplings to matter of scalar and tensor fields. The Jordan-Fierz-Brans-Dicke theory, which is the simplest tensor-scalar theory, has $\alpha_0^2 = (2\omega+3)^{-1}$ as a unique free parameter, and all its predictions (in both weak-field and strong-field conditions [4]) fractionally differ from the general relativistic ones by quantities of order α_0^2. More generally, a recent study of generic tensor-scalar theories [5] has formally shown that all the deviations from general relativity, observable at the present cosmological epoch, can be expanded in series of powers of α_0^2. These results seem to suggest that the limit $\alpha_0^2 < 10^{-3}$ deduced from post-Newtonian experiments, Eq. (1), *a priori* constrains the possible level of deviation from general relativity in all other gravitational experiments, including those involving strong gravitational fields [6].

The purpose of the present work is to prove that such a conclusion would be illegitimate. By studying neutron-star models within general tensor-scalar theories, we find that, when a certain inequality is satisfied $[(\tilde{\beta}-1)/(\tilde{\gamma}-1) \gtrsim +1]$, these models develop some nonperturbative strong gravitational field effects which induce order-of-unity deviations from general relativity, even when the linear coupling constant α_0^2 is very small.

The most general metric tensor-monoscalar theory (with one massless scalar field) contains one arbitrary "coupling function" $A(\varphi)$ [7]. Its action reads

$$S = (16\pi G_*)^{-1}\int d^4x\, g_*^{1/2}[R_* - 2\, g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi] + S_m[\psi_m,\, A^2(\varphi)g_{\mu\nu}^*] \; . \tag{3}$$

G_* denotes a bare gravitational coupling constant, and $R_* \equiv g_*^{\mu\nu}R_{\mu\nu}^*$ the curvature scalar of the "Einstein metric" $g_{\mu\nu}^*$. The last term in Eq. (3) denotes the action of the matter, which is a functional of some matter variables, collectively denoted by ψ_m, and of the ("Jordan-Fierz") metric $\tilde{g}_{\mu\nu} \equiv A^2(\varphi)g_{\mu\nu}^*$. See Ref. [5] for the extension to the multiscalar case, and a comprehensive discussion of the observable consequences of tensor-scalar theories.

The universal coupling of matter to $\tilde{g}_{\mu\nu}$ means that nongravitational clocks and laboratory rods measure this metric. However, the field equations of the theory are better formulated in terms of the variables $(g_{\mu\nu}^*, \varphi)$. The field equations derived from Eq. (3) read

$$R_{\mu\nu}^* = 2\partial_\mu\varphi\,\partial_\nu\varphi + 8\pi G_*\left(T_{\mu\nu}^* - \frac{1}{2}T^* g_{\mu\nu}^*\right) , \tag{4a}$$

$$\Box_{g_*}\varphi = -4\pi G_*\alpha(\varphi)T_* , \tag{4b}$$

with $T_*^{\mu\nu} \equiv 2(g_*)^{-1/2}\delta S_m/\delta g_{\mu\nu}^*$ denoting the stress-energy tensor in the g^* units, and where all tensorial operations are performed by using this metric. The quantity $\alpha(\varphi)$ in Eq. (4b) denotes the logarithmic derivative of the coupling function, $\alpha(\varphi) \equiv \partial \ln A(\varphi)/\partial\varphi$. It plays the role of the basic (field-dependent) coupling strength between the scalar field and matter. [The original Jordan-Fierz-Brans-Dicke theory is characterized by having a field-independent coupling strength, $\alpha(\varphi) = \alpha_0$, i.e., $A(\varphi) = \exp(\alpha_0\varphi)$.]

Post-Newtonian experiments probe just the low-order perturbative structure of tensor-scalar theories in that they depend only on the values of $\alpha(\varphi)$ and its field-gradient, $\beta(\varphi) \equiv \partial\alpha(\varphi)/\partial\varphi$, at the cosmologically determined value of the scalar field: say $\alpha_0 \equiv \alpha(\varphi_0)$, $\beta_0 \equiv \beta(\varphi_0)$ where the cosmological value φ_0 enters the theory as the boundary condition on the scalar field at spatial infinity. The square of α_0 determines the post-Newtonian parameter $\tilde{\gamma}$ through Eq. (2), while the other post-Newtonian parameter is given by [5]

$$\tilde{\beta} - 1 = \frac{1}{2}\frac{\beta_0\alpha_0^2}{(1+\alpha_0^2)^2} . \tag{5}$$

The spherically symmetric, static metric generated by an isolated, nonrotating neutron star can be written as

$$ds_*^2 = g^*_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(r)}dt^2 + \frac{dr^2}{1-2\mu(r)/r} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) . \tag{6}$$

The physical stress-energy tensor $\widetilde{T}^{\mu\nu} \equiv 2(\tilde{g})^{-1/2} \times \delta S_m/\delta\tilde{g}_{\mu\nu}$ takes the perfect-fluid form $\widetilde{T}^{\mu\nu} = (\tilde{\varepsilon} + \tilde{p})\tilde{u}^\mu\tilde{u}^\nu + \tilde{p}\tilde{g}^{\mu\nu}$, and is related to its g^*-frame counterpart by $T_*^\mu{}_\nu = A^4(\varphi)\widetilde{T}^\mu_\nu$. From the field equations (4a), (4b), and the stress-energy balance equation $(\widetilde{\nabla}_\nu \widetilde{T}^\nu_\mu = 0)$ one derives the following first-order differential system (with a prime denoting d/dr):

$$\begin{aligned}
\mu' &= 4\pi G_* r^2 A^4(\varphi)\tilde{\varepsilon} + \frac{1}{2}r(r-2\mu)\psi^2 ,\\
\nu' &= 8\pi G_* \frac{r^2 A^4(\varphi)\tilde{p}}{r-2\mu} + r\psi^2 + \frac{2\mu}{r(r-2\mu)} ,\\
\varphi' &= \psi ,\\
\psi' &= 4\pi G_* \frac{rA^4(\varphi)}{r-2\mu}[\alpha(\varphi)(\tilde{\varepsilon}-3\tilde{p}) + r\psi(\tilde{\varepsilon}-\tilde{p})]\\
&\quad -\frac{2(r-\mu)}{r(r-2\mu)}\psi ,\\
\tilde{p}' &= -(\tilde{\varepsilon}+\tilde{p})\left[4\pi G_* \frac{r^2A^4(\varphi)\tilde{p}}{r-2\mu} + \frac{1}{2}r\psi^2 + \frac{\mu}{r(r-2\mu)} \right.\\
&\quad \left. +\alpha(\varphi)\psi\right] .
\end{aligned} \tag{7}$$

Given some equation of state relating $\tilde{\varepsilon}$ and $\tilde{p}$, one can integrate the system (7) starting from the center, $r = 0$, with the initial conditions $\mu(0) = 0$, $\nu(0) = 0$, $\varphi(0) = \varphi_c$, $\psi(0) = 0$, and $\tilde{p}(0) = \tilde{p}_c$. Although the right-hand sides of Eqs. (7) do not vanish outside the matter, it is sufficient to integrate the system up to the surface of the star, $r = r_s$, where the pressure vanishes. Then one can match the interior solution $(g^*_{\mu\nu}, \varphi)$ [written using a Schwarzschild-like radial coordinate, Eq. (6)] to the exact general exterior solution, which is known in closed form when using a radial coordinate introduced by Just [5,8,9]. The latter general exterior solution contains two independent parameters: the total ADM mass of $g^*_{\mu\nu}$, say G_*m_A, and the total scalar charge ω_A such that $\varphi = \varphi_0 + G_*\omega_A/r + O(1/r^2)$ as $r \to \infty$. Actually, a more interesting scalar quantity is the dimensionless ratio $\alpha_A \equiv -\omega_A/m_A$. We here follow the notation of Ref. [5]; in particular the subscript A is used as a label to distinguish a particular star member of a multiple system (e.g., a binary pulsar system). From matching the interior and exterior solutions we find $\alpha_A = 2\psi_s/\nu'_s$ and

$$\varphi_0 = \varphi_s + \frac{2\psi_s}{(\nu_s'^2 + 4\psi_s^2)^{1/2}}\operatorname{arctanh}\left[\frac{(\nu_s'^2 + 4\psi_s^2)^{1/2}}{\nu'_s + 2/r_s}\right] , \tag{8}$$

$$m_A = \frac{r_s^2\nu'_s}{2G_*}\left(1 - \frac{2\mu_s}{r_s}\right)^{1/2} \exp\left[-\frac{\nu'_s}{(\nu_s'^2 + 4\psi_s^2)^{1/2}}\operatorname{arctanh}\left(\frac{(\nu_s'^2 + 4\psi_s^2)^{1/2}}{\nu'_s + 2/r_s}\right)\right] , \tag{9}$$

where the subscript s refers to quantities evaluated at the surface $r = r_s$ (and where, as above, φ_0 is the value of φ at infinity). Another quantity of direct physical interest is the total baryonic mass of the star A, say $\overline{m}_A = \tilde{m}_b \int \tilde{n}\sqrt{\tilde{g}}\tilde{u}^0 d^3x = \tilde{m}_b \int_0^{r_s} 4\pi\tilde{n}A^3(\varphi)r^2(1-2\mu/r)^{-1/2}dr$, where $\tilde{m}_b$ denotes, say, one atomic mass unit and $\tilde{n}$ the physical, proper baryonic number density.

It was shown in Ref. [5] that the equations of motion (at the post-Keplerian level), and the gravitational wave emission, of a system of N strongly self-gravitating bodies were determined by the values of the total inertial masses m_A (where $A = 1, \ldots, N$) together with the values of the dimensionless parameters $\alpha_A \equiv \partial \ln m_A/\partial\varphi_0$, $\beta_A \equiv \partial\alpha_A/\partial\varphi_0$, in which the derivatives are taken keeping $\overline{m}_A$ (and G_*) fixed. For instance, the Keplerian-order interaction energy between two stars is $-G_*m_Am_B(1 + \alpha_A\alpha_B)/r_{AB}$. The quantity $\alpha_A \equiv \partial \ln m_A/\partial\varphi_0$ is identical to $-\omega_A/m_A$ and plays the role of the effective coupling constant between the scalar field and the star A. In the limit of negligible internal gravity the parameters α_A and β_A tend toward $\alpha_0 \equiv \alpha(\varphi_0)$ and $\beta_0 \equiv \beta(\varphi_0)$, respectively. In Ref. [5] it was further shown that, when one works perturbatively in the strength $s_A \simeq G_*m_A/r_A$ of the self-gravity of body A, one obtains an expansion of the symbolic form $\alpha_A = \alpha_0[1 + a_1s_A + a_2s_A^2 + \cdots]$

with some coefficients a_1, $a_2, \ldots$ which stay finite when $\alpha_0 \to 0$ [and which are of order unity if the coupling function $A(\varphi)$ is smooth and involves no large parameters]. This expansion suggests that $\alpha_A \to 0$ when α_0 tends toward zero, even when s_A approaches unity (strong self-gravity). As all observable quantities associated to the exchange of a scalar interaction contain at least two α factors (e.g., in combinations such as $\alpha_A \beta_B \alpha_C$), this would mean that post-Newtonian experiments yield *a priori* constraints on possible strong-field effects observable in systems of compact objects. Actually, this conclusion is premature.

One can see heuristically in a simplified model how the infinite series of self-gravity contributions, $1 + a_1 s_A + a_2 s_A^2 + \cdots$, can compensate even a vanishingly small α_0. Let us consider the simple case where $A(\varphi) = \exp(\beta\varphi^2/2)$ [i.e., $\alpha(\varphi) = \beta\varphi$ and $\beta(\varphi) = \beta = \text{const}$], and let us approximate the scalar field equation (4b) by neglecting the curvature of $g^*_{\mu\nu}$ and replacing $-G_* T_* = G_* A^4(\tilde{\varepsilon} - 3\tilde{p})$ by a positive constant, $G_* m_A/(4\pi r_A^3/3) = 3 s_A/4\pi r_A^2$, all over the volume of the star ($r < r_A$). This yields the simple equation $\Delta\varphi = \text{sign}(\beta)\kappa^2\varphi$ where $\kappa^2 \equiv 3|\beta| s_A r_A^{-2}$ when $r < r_A$ and $\kappa^2 = 0$ when $r > r_A$. When β is negative, the solution of this equation in the interior of the star is $\varphi_{\text{in}}(r) = \varphi_c \sin(\kappa r)/\kappa r$ with $\varphi_c = \varphi_0/\cos(\kappa r_A) = \varphi_0/\cos[(3|\beta| s_A)^{1/2}] > \varphi_0$, so that the effect of the self-gravity of the star is to amplify the local value of $|\alpha(\varphi)| = |\beta|\varphi$ with respect to its cosmological value $|\alpha_0| = |\beta|\varphi_0$. When $|\beta|$ and the self-gravity of the star are such that $(3|\beta| s_A)^{1/2} = \pi/2$ this amplification mechanism can compensate even a vanishingly small α_0. (There appears a zero mode of φ, i.e., a nontrivial solution with vanishing boundary conditions at infinity.) As a typical ($1.4 m_\odot$) neutron star has $s_A \sim 0.2$, we expect that this nonperturbative amplification effect could take place when $\beta \lesssim -4$. By contrast, when β is positive the solution is obtained by the replacements sin $\to$ sinh, cos $\to$ cosh and one obtains a deamplification of the local value of $\alpha(\varphi)$ with respect to α_0 ($\alpha_c = \alpha_0/\cosh[(3\beta s_A)^{1/2}]$). In that case the strong self-gravity of neutron stars is expected to further quench deviations from general relativity.

By numerically integrating the exact system of equations (7) we have indeed shown the existence of a nonperturbative amplification mechanism of the coupling strength of the scalar field when the logarithm of the coupling function $A(\varphi)$ has a sufficiently negative curvature around φ_0: $\beta_0 \equiv \partial^2 \ln A/\partial\varphi_0^2 \lesssim -4$. The results of our numerical integrations are presented in Figs. 1–3.

To model the structure of a neutron star we considered polytropic equations of state, $\tilde{\varepsilon} = \tilde{n}\tilde{m}_b + K\tilde{n}_0\tilde{m}_b(\tilde{n}/\tilde{n}_0)^\Gamma/(\Gamma - 1)$, $\tilde{p} = K\tilde{n}_0\tilde{m}_b(\tilde{n}/\tilde{n}_0)^\Gamma$ with $\tilde{m}_b = 1.66 \times 10^{-24}$ g and $\tilde{n}_0 = 0.1\ \text{fm}^{-3}$, and with values of the parameters Γ and K adjusted to fit the curves (computed, within general relativity, from some realistic equations of state [10,11]) giving the fractional binding energy $f \equiv (\overline{m} - m)/m$ as a function of the baryonic mass $\overline{m}$ (see Fig. 1). In particular, we used $\Gamma = 2.34$ and $K = 0.0195$ to fit the equation of state II of Ref. [11], and $\Gamma = 2.46$ and $K = 0.009\,36$ to fit the equation of state A of Ref. [10].

Figure 1 illustrates the spectacular changes in the gravitational equilibrium configurations of a neutron star with a given nuclear equation of state when using, instead of general relativity, the tensor-scalar theory defined by $A(\varphi) = \exp(-3\varphi^2)$, with $\varphi_0 = 0.0043$, i.e., the maximum value consistent with the limits in Eq. (1). The maximum baryonic mass of a neutron star increases from $2.23 m_\odot$ in general relativity to $3.03 m_\odot$ [12]. Note the presence of a second branch of equilibrium configurations below the one continuously connected with the normal Newtonian configurations. This second branch is linked to the appearance, above a certain state of compactness,

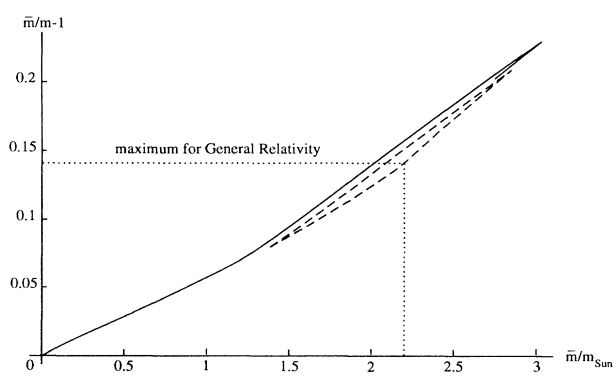

FIG. 1. Fractional binding energy vs baryonic mass for a neutron-star model (polytrope $\Gamma = 2.34$ with $K = 0.0195$) computed within the tensor-scalar theory $A(\varphi) = \exp(-3\varphi^2)$, with cosmological boundary condition $\varphi_0 = 0.0043$. The dashed curve represents a second, energetically less favorable, sequence of equilibrium configurations. For clarity, only the turning point of the binding energy curve within general relativity is indicated.

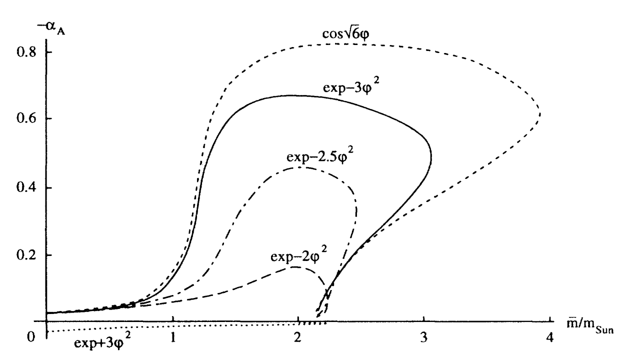

FIG. 2. Effective scalar coupling constant vs baryonic mass for the neutron star model of Fig. 1 computed within five different tensor-scalar theories. The labels indicate the corresponding coupling functions $A(\varphi)$. In each case, we chose the maximum value of φ_0 consistent with the two limits in Eq. (1).

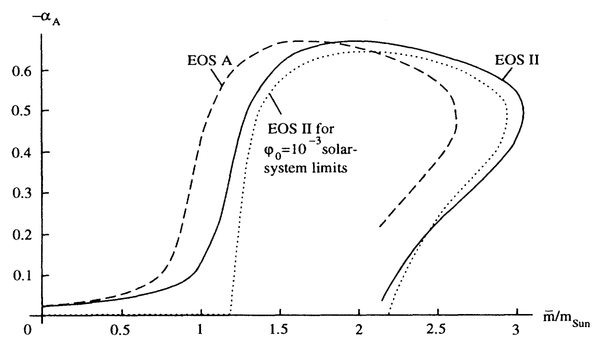

FIG. 3. Illustration of the weak dependence of the nonperturbative effects in α_A when reducing φ_0, keeping everything else fixed (dotted curve), and when changing the equation of state. The theory used is $A(\varphi) = \exp(-3\varphi^2)$; the labels EOS II and EOS A refer to the polytropic approximations discussed in the text.

of a nonlinear zero mode (exact, nontrivial configuration with zero scalar boundary condition). The fact that this second branch is energetically less favorable than the first and disconnected from it suggests that it is not associated to any physical pathologies. Figures 2 and 3 clearly exhibit the existence of strong-field deviations from general relativity (α_A of order unity) in tensor-scalar theories with $\beta_0 \equiv \partial^2 \ln A/\partial\varphi_0^2 \lesssim -4$. (Computation of β_A also exhibits clear strong-field effects for stars of baryonic mass $1.5m_\odot$ with $\alpha_A\beta_A\alpha_A < -2$ for $\beta_0 \leq -5$.) Two features illustrate the nonperturbative character of these strong-field effects. First, the large difference in Fig. 2 between the two cases $A_1(\varphi) = \exp(-3\varphi^2)$ and $A_2(\varphi) = \cos(\sqrt{6}\varphi)$ (two functions having the same curvature at the origin, $\beta_0 = -6$) shows that strong-field effects probe a large segment of the coupling function. Second, the small difference exhibited in Fig. 3 between the cases $\varphi_0 = 4.3 \times 10^{-3}$ and $\varphi_0 = 4.3 \times 10^{-6}$ when $\overline{m} \gtrsim 1.3m_\odot$ shows how strong-field effects free themselves from weak-field constraints when they develop. Note also in Fig. 3 the robustness of the strong-field effects against a change of nuclear equation of state. (However, an equation of state causing $\tilde{\varepsilon} - 3\tilde{p}$ to become negative over a sizable fraction of the radius of the star could introduce significant changes in our conclusions.) Finally, when β_0 is positive, we find that the strong gravitational field of neutron stars further quenches the small level of deviation from general relativity allowed by weak-field experiments [13].

In conclusion, a wide class of tensor-scalar theories can pass all the present solar-system tests and still exhibit large, strong-field-induced, observable deviations in systems involving neutron stars. The condition for the occurrence of such effects is a certain inequality, $\partial^2 \ln A/\partial\varphi_0^2 \lesssim -4$, which can be written in terms of the post-Newtonian parameters as $(\tilde{\beta}-1)/(\tilde{\gamma}-1) \gtrsim +1$ [14]. This provides new motivations for experiments probing the strong-field regime of relativistic gravity, notably binary pulsar experiments [15], which might reveal the existence of a scalar contribution to gravity too small to be detectable in solar-system experiments. Such strong-field effects could also have a significant impact on the emission of gravitational waves during supernova collapse, and neutron-star binary coalescence.

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[13] While preparing this work for publication we noticed a related study of H.W. Zaglauer [Astrophys. J. **393**, 685 (1992)]. When translated into our notation the latter work claims to find order-of-unity deviations from general relativity for $A(\varphi) = \cosh[\sqrt{\beta_0}\varphi]$, i.e., in a case where $\beta_0 > 0$. His results seem spurious and due (in part) to the artificial definitions he uses for the "sensitivity" of a neutron star.

[14] An example of a class of theories where φ_0 would be dynamically driven to a state satisfying this condition is a finite-range ($\lambda \gtrsim 10^6$ km), nonminimally coupled scalar field ($\frac{1}{2}\xi R_*\phi^2$) with $\xi \gtrsim +4$. On the other hand, strictly massless tensor-scalar theories may need some kind of fine tuning to be cosmologically driven toward a state where $\beta_0 \lesssim -4$. [T. Damour and K. Nordtvedt, this issue, Phys. Rev. Lett. **70**, 2217 (1993), find indeed that the minima of $A(\varphi)$ are natural cosmological attractors.]

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