Gravitational-wave bursts with memory: The Christodoulou effect

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The "memory" of a gravitational-wave burst is the permanent relative displacement that it imposes on free test masses, or more precisely, the permanent change in the burst's gravitational-wave field h_{jk}^{TT}. This memory, in general, is equal to the change, from before the burst to afterward, in the transverse-traceless (TT) part of the "$1/r$, Coulomb-type" gravitational field generated by the four-momenta of the source's various independent pieces. Christodoulou has recently identified a contribution to a burst's memory that arises from nonlinearities in the vacuum Einstein field equation. This paper shows that the Christodoulou memory is precisely the TT part of the "$1/r$, Coulomb-type" gravitational field produced by the burst's gravitons, and it therefore gets built up over the same length of time τ_{bwm} as it takes for the source to emit the gravitons. The sensitivity of broad-band gravitational-wave detectors such as LIGO to the Christodoulou memory is analyzed and discussed.

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I. GENERIC FORM OF THE MEMORY

Gravitational waves are often described [1] by a dimensionless, symmetric, second-rank tensor field $h_{jk}^{\mathrm{TT}}(t-z)$. This field is defined in the rest frame of a detector, it propagates (in this case) in the z direction with the speed of light, and it is purely spatial, transverse, and traceless (hence the "TT" superscript) — i.e., the indices j and k run over spatial coordinates x, y, z; and $h_{jz}^{\mathrm{TT}}=h_{zj}^{\mathrm{TT}}=\delta^{jk}h_{jk}^{\mathrm{TT}}=0$. If the detector consists of two free masses (or two masses that hang from overhead supports and move freely horizontally at frequencies high compared to the 1-Hz pendulum frequency), and if the masses have a vectorial separation l_k, then the gravitational-wave field changes their separation by $\delta l_j=\frac{1}{2}h_{jk}^{\mathrm{TT}}l^k$.

It has long been known [2,3] that a burst of gravitational waves of finite duration can produce permanent changes in the test-mass separations, i.e., it can have $h_{jk}^{\mathrm{TT}}=0$ before the burst arrives, and $h_{jk}^{\mathrm{TT}}=\Delta h_{jk}^{\mathrm{TT}}\neq 0$ after the burst has passed. The permanent change $\Delta h_{jk}^{\mathrm{TT}}$ is called [3] the burst's "memory."

It has also long been known (see, e.g., Ref. [4]) that the memory of a burst is equal to the change, from before the burst to afterward, in the transverse-traceless (TT) part of the "$1/r$, Coulomb-type" gravitational field generated by the four-momenta of the source's various independent pieces.

More specifically, before the burst is emitted and after it is finished, the source will consist of a set of freely moving systems that are gravitationally unbound from each other [e.g., two stars flying toward each other or apart, or a neutron-star binary (which is to be regarded as a single system rather than two independent stars, since the two stars are bound to each other gravitationally)]. Label these freely moving systems by an index $A=1,2,3,\ldots,N$. Then the memory, expressed in geometrized units ($G=c=1$), is [4]

$$\Delta h_{jk}^{\mathrm{TT}}=\Delta\sum_{A=1}^{N}\frac{4M_A}{r\sqrt{1-v_A^2}}\left[\frac{v_A^j v_A^k}{1-v_A\cos\theta_A}\right]^{\mathrm{TT}}. \qquad (1)$$

Here on the right-hand side Δ means the final value of the summation (after the burst) minus the initial value (before the burst), M_A is the mass of system A, and the other symbols denote the following quantities as measured in the rest frame of the detector: r is the distance from source to detector, v_A^j is the velocity of the center of mass of system A, and θ_A is the angle between v_A^j and the direction from the source to the detector. The superscript TT on the right-hand side means [5] "in the rest frame of the source project out the piece that is transverse to the line between source and detector and remove that piece's trace," i.e., "take the transverse, traceless or TT part." The right-hand side of Eq. (1) is precisely the TT part of the change in the source's $1/r$, Coulomb-type gravitational field.

II. THE CHRISTODOULOU PART OF THE MEMORY

The author (who is an advocate of simple physical explanations for important physical effects) has long thought he understood fully the memories of gravitational-wave bursts. It has been a salubrious experience, therefore, for the author to be shown by a mathematician (Christodoulou [6], who uses elegant mathematics that is rather far from the physics) that he (the author) had missed a very important physical effect: There is a contribution to the memory [7] that arises from nonlinearities in the Einstein field equation, a contribution that at first sight seems not to be included in Eq. (1).

The main purpose of this paper is to show that Christodoulou's effect, in fact, *is* included in the general expression (1) for the memory, but the author had missed

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it there due to his obtuseness. The Christodoulou effect, in fact, is the contribution to (1) from the gravitational-wave burst's gravitons. Each graviton can be regarded as an individual system and thus must be included in the final $\sum_A$, but, of course, not in the initial $\sum_A$. In the final sum, the quantity $M_A/\sqrt{1-v_A{}^2}$ for graviton A should be interpreted as the graviton's energy E_A as measured in the detector's rest frame, the graviton speed v_A must be set to 1, and correspondingly the gravitons' contribution to the memory (1) can be written as

$$\Delta h_{jk}^{\mathrm{TT}} = \frac{4}{r}\int \frac{dE}{d\Omega'}\left[\frac{\xi^{j'}\xi^{k'}}{1-\cos\theta'}\right]^{\mathrm{TT}} d\Omega' \ . \tag{2}$$

Here the integral is over solid angle $d\Omega'$ surrounding the source, $\xi^{j'}$ is a unit vector pointing from the source toward $d\Omega'$, and θ' is the angle between $\xi^{j'}$ and the direction to the detector. Equation (2) is the same, aside from notation, as one of the formulas that Christodoulou gives for his effect (the second equation following Eq. (13) in Ref. [6]) [8]; note that the first equation following Eq. (13), which describes that part of the memory not produced by gravitons, is the same as Eq. (1) above with the graviton contribution omitted).

In practical computations of the Christodoulou effect for specific sources, it may be helpful to rewrite Eq. (2) in the following way. Pick, at the detector, some transverse direction (call it the x direction) as an axis with respect to which to resolve h_{jk}^{TT} into its two polarizations, $h_+ \equiv h_{xx} = -h_{yy}$ and $h_\times \equiv h_{xy}$. Then, denoting by ϕ' the angle in the transverse x,y plane with respect to the x axis, rewrite expression (2) in the form

$$\Delta h_+ + i\Delta h_\times = \frac{2}{r}\int \frac{dE}{d\Omega'}(1+\cos\theta')e^{i2\phi'}d\Omega' \tag{3}$$

(with, of course, $d\Omega' = \sin\theta' d\theta' d\phi'$).

Christodoulou [6] has emphasized the following aspects of his effect, which show up clearly in the above formulas and their underlying physics. From Eq. (3) it should be clear that almost any anisotropy in $dE/d\Omega'$ will produce a nonzero result for the integral (3) and thus a nonzero memory. From the physical origin of the memory as the TT part of the "Coulomb-type" fields of the emitted gravitons, it should be clear that the memory gets built up continuously over the duration of the burst, and also that Eqs. (2) and (3) describe the gravitational-wave memory produced not only by a burst of gravitons, but also by a burst of any other kind of zero-rest-mass quanta, e.g., neutrinos [9].

III. ORDER OF MAGNITUDE OF THE CHRISTODOULOU EFFECT

In order of magnitude, the Christodoulou memory (3) is

$$\Delta h \sim (0.1 \text{ to } 1)E/r \ , \tag{4}$$

where E is the total energy emitted and r is the distance from source to detector. If the burst has amplitude h and wavelength λ and lasts for n cycles, then its energy will be $E \sim (\pi h/\lambda)^2 \lambda n r^2$; and correspondingly the quantity $h_c \equiv h\sqrt{n}$ ("characteristic amplitude"), which determines the signal-to-noise ratio in a detector [1], will be $h_c \sim \sqrt{\lambda E}/\pi r$. The ratio of the Christodoulou memory to the characteristic amplitude is thus [since $1/\pi \sim (0.1$ to $1)$]

$$\Delta h/h_c \sim \sqrt{E/\lambda} \ , \tag{5}$$

which is typically less than unity since $\lambda \gtrsim$ (size of source) $\gtrsim$ (gravitational radius of source) $\gtrsim E$. Only for the strongest of sources, e.g., the coalescence of a binary system made of two black holes, can $\Delta h/h_c$ be of order unity; for other sources it will be far smaller. (By contrast, the memory Δh due to motions of other, nongraviton parts of the source can easily be of order h_c even in weak sources; an example is the gravitational bremsstrahlung radiation emitted when two stars fly past each other [10].)

IV. EXPERIMENTAL STRATEGY AND DETECTOR SENSITIVITIES

Braginsky and Thorne [4] have discussed the experimental prospects for detecting the memories of gravitational-wave bursts. Their conclusion, in brief, was the following. In the real world, where gravitational-wave detectors are inevitably plagued by severe noise at frequencies lower than some cutoff, there is no hope to search for memories by integrating up a detector's signal for an arbitrarily long time. Instead, the best way to search for the memory of a short burst (duration τ_{bwm}) is to integrate up the signal for an integration time $\hat{\tau} \simeq 1/f_{\mathrm{opt}} \gtrsim \tau_{\mathrm{bwm}}$, where f_{opt} is that frequency below $1/\tau_{\mathrm{bwm}}$ at which one's detector has optimal amplitude sensitivity to bursts without memory. In such a search, the sensitivity $\Delta h_{3/\mathrm{yr}}$ to the memory Δh is approximately the same as the sensitivity $h_{3/\mathrm{yr}}(f_{\mathrm{opt}})$ to a broad-band burst without memory that has mean frequency f_{opt} and characteristic amplitude $h_c = \Delta h$:

$$\Delta h_{3/\mathrm{yr}} \simeq h_{3/\mathrm{yr}}(f_{\mathrm{opt}}) \ , \quad \text{where } f_{\mathrm{opt}} < 1/\tau_{\mathrm{bwm}} \ . \tag{6}$$

This optimal strategy implies that the best kind of detector to use in searching for the memory of a burst is one designed for the best achievable sensitivity to broad-band bursts without memory at any frequency $f_{\mathrm{opt}} \lesssim 1/\tau_{\mathrm{bwm}}$. The interferometric detectors [11] currently under development in the United States (the LIGO Project) and in Europe (the GEO and VIRGO projects) are of just this type.

The Appendix uses tools developed in Ref. [1] to make Eq. (6) more precise than heretofore. In particular, it is shown that Eq. (6) is correct to within a few tens of percent for interferometric detectors on Earth or in space, if $h_{3/\mathrm{yr}}$ and $\Delta h_{3/\mathrm{yr}}$ are interpreted as follows: $h_{3/\mathrm{yr}}(f_{\mathrm{opt}})$ is [1] the value of h_c for the weakest broad-band burst centered on frequency f_{opt} that can be detected with 90% confidence, in the presence of the detectors' Gaussian noise, using cross correlation of two identical detectors, if the burst arrives at a random time during an interval of 1/3 year; and $\Delta h_{3/\mathrm{yr}}$ is the weakest memory Δh that can be detected with 90% confidence from a burst with duration τ_{bwm}, using the same pair of detectors and using optimal signal processing, provided that the burst itself or a precursor is also detected so the time of arrival of the memory is already known. If the time of arrival of the memory is not known, then $\Delta h_{3/\mathrm{yr}}$ will be larger than $h_{3/\mathrm{yr}}(f_{\mathrm{opt}})$ by a factor of about 5 in the case of Earth-based interferometric detectors (LIGO, GEO, and VIRGO) and by about 3.5 for space-based interferometric detectors (e.g., LAGOS [11]).

V. PROSPECTS FOR DETECTING THE CHRISTODOULOU MEMORY

Prospects are good but not wonderful for detecting the strongest of all Christodoulou-type memories, that of a gravitational-wave burst from the coalescence of a black-hole binary.

Assuming (so as to maximize the memory's strength) that the two holes have comparable masses $M/2$, then the total energy emitted in the final coalescence will be $E \sim 0.1M$, and correspondingly the strength of the Christodoulou memory (4) will be in the range

$$\Delta h \sim \left[\frac{1}{10} \text{ to } \frac{1}{100}\right] \frac{M}{r} . \tag{7}$$

(Here and below Newton's gravitation constant and the speed of light are set equal to unity.) The duration of the final burst (the time over which most of the energy is emitted) will be $\tau_{\mathrm{bwm}} \sim 100M = 0.5\ \mathrm{msec}(M/M_{\odot})$, so the Earth-based LIGO, GEO, and VIRGO, with their $f_{\mathrm{opt}} \simeq 100$ Hz as presently planned, are optimally suited to detecting the memory so long as the binary's total mass is $M \lesssim 20M_{\odot}$, and the space-based LAGOS as presently conceived, with its $f_{\mathrm{opt}} \simeq 0.003$ Hz, is optimally suited for $M \lesssim 10^6 M_{\odot}$.

For such a binary, the characteristic amplitude of the precursor to the final burst (the waves produced near the detector's optimal frequency f_{opt} as the two holes spiral toward each other) will be [12]

$$h_c \simeq \frac{1}{3}\frac{M}{r} , \tag{8}$$

provided the time for the precursor to sweep in frequency from, say, $0.5f_{\mathrm{opt}}$ to $1.5f_{\mathrm{opt}}$ is shorter than the experimenters' integration time. (This will always be the case for Earth-based detectors such as LIGO and usually but not always for the lower-frequency space-based dectectors such as LAGOS [13].) Note that the precursor's characteristic amplitude (8) is stronger than the Christodoulou memory (7) by a factor 3 to 30, and correspondingly, if there is any hope to detect the memory, then the precursor will surely be detected [13]. (Recall [cf. Eq. (6) and the paragraph following it] that the sensitivities to the memory and the precursor are the same to within a few tens of percent, $\Delta h_{3/\mathrm{yr}} \simeq h_{3/\mathrm{yr}}$.)

As a specific example, the best present estimates of the rate of coalescence of binary black holes with masses $M/2 \sim 10M_{\odot}$ suggest an event rate of 3 per year at a distance of about 200 Mpc from Earth [14]. At this distance the strengths of the precursor and the Christodoulou memory will be $h_c \simeq 1.5\times10^{-21}$ and Δh between 5×10^{-23} and 5×10^{-22}. For comparison, the first detectors in LIGO are designed for sensitivities $h_{3/\mathrm{yr}}(f_{\mathrm{opt}}) \simeq \Delta h_{3/\mathrm{yr}} \simeq 2\times10^{-21}$ — near the level of the precursor and much too poor for the memory. More advanced detectors in LIGO, which are expected to operate soon after the turn of the century, are expected to have $h_{3/\mathrm{yr}}(f_{\mathrm{opt}}) \simeq \Delta h_{3/\mathrm{yr}} \simeq 1\times10^{-22}$, very easily good enough to detect the precursor and probably but not certainly good enough for the Christodoulou memory.

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APPENDIX: SENSITIVITY OF DETECTORS TO THE MEMORY OF A BURST

In this appendix the sensitivities of broad-band detectors to the memory of a gravitational-wave burst will be quantified using the techniques of Ref. [1].

Denote by $\Delta h \equiv \sqrt{(\Delta h_+)^2 + (\Delta h_\times)^2}$ the strength of the memory, and by (θ,ϕ,ψ) the direction to the source and the polarization angle of the memory as defined in Fig. 9.2 of Ref. [1]. Then if the experimenters, knowing they are searching for a memory, use optimal signal processing to fight the noise in their detector, the signal-to-noise ratio in their measurement will be, when a single detector is used,

$$\frac{S}{N} = \frac{\Delta h}{\pi\sqrt{2}} \left[\int_{-\infty}^{\ln(1/\tau_{\mathrm{bwm}})} \frac{d\ln f}{[h_n(f)]^2}\right]^{1/2} . \tag{A1}$$

Here $h_n(f)$ is the detector's rms noise for a burst without memory at frequency f and with bandwidth $\delta f = f$, coming from the same direction as the burst with memory, i.e.,

$$h_n = \frac{\sqrt{f S_h(f)}}{|F_+(\theta,\phi,\psi)|} , \tag{A2}$$

where $S_h(f)$ is the spectral density of the detector's strain noise at frequency f, and $F_+(\theta,\phi,\psi)$ is the detector's amplitude beam pattern function (Eq. (104a) of Ref. [1] for an interferometric detector). Expression (A1) follows directly from Eqs. (26), (28a), and (29) of Ref. [1], with $h_+(t) = \Delta h$ for $t > \tau_{\mathrm{bwm}}$ and 0 for $t < 0$, and with $h_\times(t) = 0$.

If the experimenters use two identical cross-correlated detectors and optimal signal processing to search for the memories of bursts, then the sensitivity of their detection system can be characterized by a quantity $\Delta h_{3/\mathrm{yr}}$ that answers the following question: *What is the strength* Δh *of a memory with a suffcently large* S/N *that, if it is seen* *three times per year (once each 10^7 seconds) by two identical detectors operating in coincidence, we can be 90% confident the detectors are not just seeing their own Gaussian noise?* The answer to this question, $\Delta h_{3/\mathrm{yr}}$, depends on whether or not the experimenters have also seen the burst or a precursor to it. If they *have* seen the burst or a precursor, then they know just when the memory should have arrived; if they have *not* seen the burst or a precursor, then they do not know when it should have arrived, so in a 1/3 year stretch of data they must search for the memory in approximately $f_{\mathrm{opt}}/10^{-8}$ Hz time bins. In the former case, for 90% confidence, each of the two detectors need only exhibit a signal-to-noise ratio of at least unity; in the latter, Gaussian statistics requires that they each exhibit a signal-to-noise ratio of at least $[\ln(f_{\mathrm{opt}}/10^{-8}\ \mathrm{Hz})]^{1/2}$. This, together with the type of analysis given in Eqs. (29)–(34) of Ref. [1], implies the following expressions for the sensitivity of the detection system:

$$\Delta h_{3/\mathrm{yr}}^{\mathrm{known}}=\frac{\sqrt{4\pi^2/3}}{\langle F_+^2(\theta,\phi,\psi)\rangle^{1/2}}\left[\int_{-\infty}^{\ln(1/\tau_{\mathrm{bwm}})}\frac{d\ln f}{fS_h(f)}\right]^{-1/2}, \tag{A3a}$$

$$\Delta h_{3/\mathrm{yr}}^{\mathrm{unknown}}=[\ln(f_{\mathrm{opt}}/10^{-8}\ \mathrm{Hz})]^{1/2}\Delta h_{3/\mathrm{yr}}^{\mathrm{known}}, \tag{A3b}$$

in the case that the arrival time is known, or unknown, respectively. Here the brackets $\langle\cdots\rangle$ denote an average over the sky (θ,ϕ) and over the polarization angle ψ.

The value of $\langle F_+^2\rangle^{1/2}$ for an interferometric detector is $1/\sqrt{5}$ (Eq. (110) of Ref. [1]). For Earth-based detectors such as LIGO or GEO or VIRGO [11], the optimal frequency to search for memories is roughly 100 Hz, and correspondingly the value of $[\ln(f_{\mathrm{opt}}/10^{-8}\ \mathrm{Hz})]^{1/2}$ is about 5. For interferometers that might fly in space (e.g., LAGOS [11]), f_{opt} is roughly 0.003 Hz and $[\ln(f_{\mathrm{opt}}/10^{-8}\ \mathrm{Hz})]^{1/2}$ is about 3.5. In both cases, Earth-based and space-based, the spectrum of the detector noise is [11] $fS_h\propto f^{-p}$ for $f<f_{\mathrm{opt}}$ and $fS_h\propto f^{+q}$ for $f>f_{\mathrm{opt}}$ with $p\simeq q\simeq 3$, and correspondingly the quantity $(\cdots)^{-1/2}$ in Eq. (A3a) is about $\sqrt{3/2}\sqrt{f_{\mathrm{opt}}S_h(f_{\mathrm{opt}})}$. When these values are inserted into Eqs. (A3), the result is the following memory sensitivity:

$$\Delta h_{3/\mathrm{yr}}=\beta\sqrt{f_{\mathrm{opt}}S_h(f_{\mathrm{opt}})}, \tag{A4a}$$

where

$$\beta=\begin{cases}10, & \text{Earth-based, arrival time known,}\\ 50, & \text{Earth-based, arrival time unknown,}\\ 10, & \text{space-based, arrival time known,}\\ 35, & \text{space-based, arrival time unknown.}\end{cases} \tag{A4b}$$

(If, as in the currently conceived space-based LAGOS, there is only one detector not two and nevertheless the noise is Gaussian, then $\Delta h_{3/\mathrm{yr}}$ will be larger than this by $\sqrt{2}$.) For comparison, the sensitivity to a broad-band burst that is centered on a frequency f_c is (Eqs. (32), (34), and (110) of Ref. [1])

$$h_{3/\mathrm{yr}}=\gamma\sqrt{f_cS_h(f_c)}, \tag{A5a}$$

where

$$\gamma=\begin{cases}11, & \text{Earth-based, arrival time unknown,}\\ 7, & \text{space-based, arrival time unknown.}\end{cases} \tag{A5b}$$

By comparing Eqs. (A4) and (A5) we see that, if the arrival time of the memory is known (by virtue of the burst or its precursor having been detected), then to within a few tens of percent

$$\Delta h_{3/\mathrm{yr}}^{\mathrm{known}}\simeq h_{3/\mathrm{yr}}(f_{\mathrm{opt}}), \quad \text{if } \tau_{\mathrm{bwm}}>1/f_{\mathrm{opt}}. \tag{A6}$$

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[1] See, e.g., K.S. Thorne, in *300 Years of Gravitation*, edited by S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1987).

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[5] Box 35.1 of C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[6] D. Christodoulou, Phys. Rev. Lett. **67**, 1486 (1991).

[7] It has been called to my attention that this Christodoulou memory was discovered independently by Luc Blanchet and Thibault Damour (L. Blanchet, Habilitation Thesis, University of Paris, 1990, pp. 205–213, where it is referred to as the "bundle term" in the "hereditary" part of the metric).

[8] In his expression for the graviton-induced memory, Christodoulou uses $dE/d\Omega'-(3/4\pi)P_j\xi^{j'}-E$ (where P_j and E are the total momentum and energy radiated) in place of $dE/d\Omega'$. (In his notation, he uses $F-F_{[1]}$ in place of F.) However, it is not necessary to subtract off the $(3/4\pi)P_j\xi^{j'}+E$ because this quantity gives a vanishing contribution to the TT part of the integral (and correspondingly $F_{[1]}$ gives a vanishing contribution to the full integral in Christodoulou's formula).

[9] R. Epstein, Astrophys. J. **223**, 1037 (1978); M.S. Turner, Nature (London) **274**, 565 (1978).

[10] See, e.g., S.J. Kovacs and K.S. Thorne, Astrophys. J. **224**, 62 (1978), and earlier references cited therein.

[11] For an overview of interferometric detectors see, e.g., Ref. [1] (where they are called "beam detectors"). For a brief description of the projected capabilities of the Earth-based LIGO and space-based LAGOS, see, e.g., K.S. Thorne, in *Recent Advances in General Relativity*, edited by A. Janis and J. Porter (Birkhauser, Boston, in press). For overviews of the LIGO Project, see R. Vogt, in *Proceedings of the Sixth Marcel Grossmann Meeting on General Relativity*, edited by T. Nakamura and H. Sato (World Scientific, Singapore, in press); and A. Abramovici, W. Althouse, R.

Drever, Y. Gürsel, S. Kawamura, F. Raab, D. Shoemaker, L. Sievers, R. Spero, K. Thorne, R. Vogt, R. Weiss, S. Whitcomb, and M. Zucker (in preparation).

[12] Equation (46b) of Ref. [1], multiplied by $\sqrt{2}$ to correct for an error in its derivation [in Eqs. (42) $2\pi f t$ should have been $2\pi \int (df/dt)dt$], and with $(Mf)^{1/6} \simeq 0.5$.

[13] Peter Bender (private communication) has pointed out the following possibly important exception: a binary with $M \ll 3000 M_{\odot}$, being studied by LAGOS. Such a binary will spend many years near the frequency $f_{\rm opt} \simeq 0.003$ Hz during its precursor, inspiral stage, so in a reasonable integration time (less than one year), LAGOS will not be able to build up the precursor signal to anywhere near the level of Eq. (8) — and LAGOS might not have even been in operation at the time the precursor passed through $f_{\rm opt}$. As a result, the memory may be easier for LAGOS to detect than the precursor. However, for such a binary, if the memory is detectable by LAGOS, then the final burst should be detectable by the higher-frequency, Earth-based LIGO.

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