ON THE ACCRETION THEORY OF STELLAR EVOLUTION

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1. *Introduction.*—During the last two years the writers have advanced a new theory of the rate of addition of mass to a star by the process of accretion of hydrogen.* The formula arrived at for the rate of change of mass when the motion has become steady may be written as

$$\frac{\partial M_A}{\partial t} = 18\gamma^2 M^2 \rho / \bar{v}^3, \tag{1}$$

where ρ is the density of the cloud in the neighbourhood of the star, M the mass of the star, γ the constant of gravitation and $\bar{v}$ the relative velocity of the star and cloud appropriately averaged to allow for the motion of the star in the galaxy. This formula involves a rate of increase of mass greater for many stars by a factor of the order of 10^6 than the rate arrived at on the basis of earlier formulæ. However, the advantages to theoretical astronomy of this large rate of accretion are manifold and a number of them have been explained by us in some detail.* In spite of these successes the derivation of this formula and its application to certain problems have been subjected to some criticism. Thus in two recent papers † discussing our work conclusions have been advanced by Atkinson that differ so widely from the views we have advocated that some statement of the position in regard to this problem as it now stands seems desirable. The present paper is accordingly concerned with setting out the main hypotheses upon which the derivation and application of formula (1) depend. It will be seen from the discussion that follows that the divergence between Atkinson's point of view and our own depends upon how far these hypotheses can be maintained and it will be shown in this paper that the assumptions adopted lie entirely within the range of valid argument and that our investigations are in accordance with accepted principles.

2. *The Derivation of the Accretion Formula.*—If a particle of mass m describes a hyperbolic orbit about a star and has velocity v relative to the star when moving at infinite distance, then the total energy of the particle at any stage in its motion is $\frac{1}{2}mv^2$, provided any internal energy possessed by the particle is neglected. (The mass of the particle is supposed infinitesimal compared with that of the star.) Thus the total energy remains positive and is constituted entirely of a mass motion of the centre of gravity of the particle together with its negative gravitational energy. It follows therefore that the only means by which such a single particle could be captured is by direct collision with the body of the star. If the cosmical cloud could be considered as being composed of independent particles, so that collisions could be neglected in the accretion process, then only those particles that directly collided with the star could be captured. It was in this way that the older formulæ for accretion were calculated.‡ In our first paper on this question it was shown, however, that the assumption that collisions may be neglected cannot be justified even as a first approximation in dealing with this problem and that despite the low density of the cloud collisions produced by the gravitational action of the star may introduce important dissipative processes. Thus the mass motion of gaseous material as it streams past a star does not vary in the

* *Proc. Camb. Phil. Soc.*, **35**, 405, 592, 1939; and **36**, 325 and 424, 1940.

† (1) *Proc. Camb. Phil. Soc.*, **36**, 313, 1940; and (2) *M.N.*, **100**, 500, 1940.

‡ Jeffreys, *M.N.*, **77**, 98, 1916; and Eddington, *Internal Constitution of the Stars*, p. 391, 1930.

same way as in the case of an assembly of independent particles and there occurs a transference of energy from organised mass motion of the material to random thermal energy. In addition it was found possible to estimate quantitatively the loss of energy of mass motion and to begin with we made the assumption that only the mass motion is of consequence in determining whether the material escapes from the gravitational field of the star. Although the energy possessed by the material consisted of three parts, viz. (i) the mass motion of the material plus its negative gravitational potential energy, (ii) the thermal energy produced by additional collisions due to the gravitational field of the star, (iii) the thermal energy possessed by the material when at infinity, together with any thermal energy developed on approaching the star by the absorption of radiation—it was assumed that only the portion (i) of this energy was of importance for assisting the escape of the material and the problem of how the portions (ii) and (iii) were concerned was then left over for future consideration.* The formula (1) had been deduced on this assumption and although the method was necessarily of an approximate nature the investigation could hardly introduce an error of much more than a factor of 2. Atkinson's first paper is almost entirely concerned with pointing out that the contributions (ii) and (iii) cannot be neglected and argues that since the sum of (i), (ii) and (iii) is positive the material must therefore of necessity evaporate away to infinity. In any case this objection does not seem valid, for in any random distribution of particles a finite fraction of them must have velocity less than an assigned escape velocity; thus, if $\bar{c}$ is the average velocity in a Maxwellian distribution of particles, the rate of effusion † into the surrounding empty space is the same as if the material moved outwards with velocity $\frac{1}{4}\bar{c}$. Accordingly $\bar{c}$ might be above the escape velocity and yet a considerable fraction of the material not escape. In other words, the motion once having become unorganised, it must remain so, and there can be no possibility of more than a fraction of the particles escaping.

But quite apart from this, the method we adopted would be completely justified if the energy contributions (ii) and (iii) are removed in some way, as, for instance, by the direct emission of radiation. Thus if some process of emission could be found that results in the loss of the energy contributions (ii) and (iii) in a time interval short compared with the time intervals that are significant in the accretion process, then the formula (i) could be accepted with considerable confidence. In his paper (1) Atkinson makes it clear that his criticisms of our initial treatment are based on the assumption that such radiative processes do not arise. Before the publication of either of Atkinson's papers we had already considered this aspect of the problem and, by making two assumptions about the structure of the hydrogen cloud, had been able to show that a highly effective emission process was forthcoming. These assumptions are:

(*a*) That the cosmical cloud in its regions of highest density contains an appreciable proportion of hydrogen molecules—10 per cent. by mass would suffice.

(*b*) That the cloud is not everywhere uniformly distributed, but possesses local irregularities.

The thermal energy (ii) and (iii) is then found to be dissipated by the quadrupole vibrational-rotational transitions of the $1'\Sigma$ state of the hydrogen molecules. A detailed account of the operation of this process has now been published.‡

The assumption (*b*) means that if a portion of the orbit of a star in the galaxy, say 1000 parsecs of its length, is considered and the average density of the cloud taken along this section of the track is ρ, then the star is not to be regarded as passing always through material of density ρ but as passing through comparatively short regions of much higher density separated by regions of much lower density. For example, the

* *Proc. Camb. Phil. Soc.*, **35**, 597, lines 24–35, 1939.
† Jeans, *Dynamical Theory of Gases*, p. 121 (Camb. Univ. Press, 1925).
‡ *Proc. Camb. Phil. Soc.*, **36**, 424, 1940.

star might spend $\frac{1}{10}$ of the time passing through material of density 10ρ and be in a region practically devoid of cloud for the remaining $\frac{9}{10}$ of the time. This assumption does not so far require any particular linear dimensions for the irregularities and they could even be small compared with the average interstellar distance. Thus the assumption concerns only the local distribution of the cloud and has no reference to the question of large scale distribution that will be considered in the next section of this paper. Accordingly it means that for two stars with widely separated orbits, no information is given as to whether the average densities are equal along the two orbits, the hypothesis merely requires the cloud to be patchily distributed along both orbits. The necessity for this assumption appears to arise in particular for the case of the stars of classes O and B, for it is found that hydrogen molecules can only reach the capture radius of these stars (apparently an essential requirement if accretion is to take place at the full rate given by the formula) if the density lies above a certain limit that depends on the surface temperature of the star and its total luminosity. For stars of very high surface temperature and great luminosity this lower limit to the density would seem to exceed the highest value that could reasonably be assigned to the density of the cloud averaged along the whole orbit of the star. The assumption of irregular distribution overcomes this difficulty.

3. *The Application of the Formula.*—The most immediate application of the formula (1) concerns the resolution of the difficulty of the short lifetimes of massive stars and it was the consideration of this problem that led to the view that the energy of the stars must be constantly resupplied from without. By supposing that the cloud is constituted mainly of hydrogen the lifetimes of the stars are not limited to the time required for the transmutation of the amount of hydrogen present in the stars at any given time. If $\partial M_T/\partial t$ represents the mass of hydrogen in grams transmuted within the star per second, then

$$\frac{\partial M_T}{\partial t} = 125\frac{L}{c^2} = 1\cdot4 \times 10^{-19}\,.\,L,$$

where L is the luminosity in ergs per second. If the star consists of a mass αM of hydrogen and a mass $(1-\alpha)M$ not hydrogen, the condition that the proportion of hydrogen is kept up to this value α by a rate of accretion $\partial M_A/\partial t$ is

$$\frac{\dfrac{\partial M_A}{\partial t} - 125\dfrac{L}{c^2}}{\alpha} \geqslant \frac{124\dfrac{L}{c^2}}{1-\alpha} \quad \text{or} \quad \frac{\partial M_A}{\partial t} \geqslant \frac{L}{c^2}\,\frac{125-\alpha}{1-\alpha}.$$

Thus if α corresponds to 25 per cent. hydrogen by mass the required condition may be written

$$18\gamma^2 M^2\rho/\bar{v}^3 \geqslant 1\cdot85 \times 10^{-19}\ \ L. \qquad (2)$$

If $\bar{v}$ is *estimated* from the observed peculiar radial velocities, then for any particular star of known M and L the minimum value of ρ satisfying (2) can be determined. This gives the order of magnitude of the minimum value of the average density of material along the orbit of the star. For stars of large mass the minimum value of ρ is about 2×10^{-21} gm. per c.c.* In paper (2) Atkinson has arrived at almost the same value as this.

A rough estimate for the average value of ρ throughout the galaxy can be obtained from Kepler's third law applied to the motion of the Sun. There is an uncertainty by a factor of order 10 in the result arrived at, however, due to the combined effect of uncertainties in such quantities as the linear velocity of the Sun in the galaxy, the distance of the Sun from the galactic centre, the general shape of the galaxy and the question of the existence of spiral arms. A discussion of this has been given in a recent

* *Proc. Camb. Phil. Soc.*, 36, 325, 1940.

paper *, where it was shown that a value of about $2 \cdot 10^{-22}$ gm. per c.c. would represent a reasonable upper limit for the average density of interstellar material lying within the orbit of the Sun. Atkinson in his paper (2) also considers an upper limit of this order, the actual value adopted being 10^{-22} gm. per c.c., and he concludes that no reasonable value of ρ can satisfy (2) for highly luminous stars. This conclusion does not seem to be warranted, however, for it assumes that the average value of ρ along the orbit of the particular star under consideration is of the same order as the average density throughout the whole of the galaxy lying within 10,000 parsecs of the centre. There is no dynamical reason why this should be the case and what observational evidence there is does not confirm it. It seems more reasonable to assume that highly luminous stars exist only in exceptional regions of the galaxy where the average value of ρ along their orbits is higher by a factor of order 10 than the average value for the whole galaxy. Thus in our view the whole, or at any rate a large part, of the orbit of a highly luminous star is to be regarded as lying in an exceptional region of the galaxy. As a simple model of the galaxy that illustrates how this could happen we may picture the cosmical cloud as being highly concentrated towards the galactic plane, so that in regions within 100 parsecs of this plane the density is of order 10^{-21} gm. per c.c. The distribution may of course involve other departures from uniformity in addition to this, such as the existence of spiral arms.

The requirements of the accretion theory as it is at present formulated may be summarised in the following three hypotheses:—

(*a*) That the cosmical cloud in its regions of highest density contains an appreciable proportion of hydrogen molecules—10 per cent. would suffice.

(*b*) That the cosmical cloud is not everywhere evenly distributed but possesses local small irregularities.

(*c*) That the cosmical cloud is irregularly distributed also on a large scale, and in particular it is strongly concentrated towards the galactic plane, where the density rises to a value of order 10^{-21} gm. per c.c.

4. *The Hypotheses* (*a*), (*b*) *and* (*c*).—In the first place, so far as investigations of the consequences of the accretion process have gone, none of these hypotheses introduces any astrophysical difficulty and no contradictions have been encountered, while the number of facts explained far exceeds the number of assumptions made. This in itself is regarded in science as sufficient justification for the introduction and adoption of a hypothesis. Moreover, in the present case, the denial of any of the hypotheses (*a*), (*b*) and (*c*) would automatically require the introduction of an alternative new hypothesis to take its place. This means that in making these assumptions the position of the theory is not weakened in respect of the principle that has been termed "the economy of postulates." Thus, for example, if the hypothesis (*b*) is denied in favour of a uniform distribution of the cloud, then (*b*) is merely replaced by the hypothesis that the density of the cloud is constant. (This is not to say that the hypotheses (*a*), (*b*) and (*c*) may not ultimately be shown to be consequences of other hypotheses of wider significance.) The case for the acceptance of (*a*), (*b*) and (*c*) then depends entirely on how much can be predicted and explained (correlated) by theory working from these assumptions. In the present problem the fact that the accretion theory promises to meet successfully the difficulties concerning the lifetime of the stars and the origin and evolution of binary systems, and in addition provides many other satisfactory results, indicates that these assumptions can be regarded with the completest confidence that can be accorded to any hypothesis not directly verifiable by observation. This aspect of the accretion theory has been dealt with in our earlier papers, so that although this argument forms the strongest evidence in favour of hypotheses (*a*), (*b*) and (*c*) it will not be necessary to give here any recapitulation of this work.

There are a number of further arguments that afford interesting confirmation of

* *Proc. Camb. Phil. Soc.*, 36, 325, 1940.

the conclusions arrived at in these earlier papers. For instance, the adoption of hypothesis (*a*) requires an appreciable proportion of the cosmical cloud to be in molecular form. In order to make an estimate of the probability of forming hydrogen molecules from separate atoms the cross-section for this process is required and this unfortunately is not at present available either from theory or experiment. On the other hand, it is of the greatest interest to find that direct observational confirmation of the occurrence of vibrational-rotational transitions in the molecules *CH* and *CN* in the cosmical cloud has recently been found by Adams * and McKellar.† Thus although our prediction that the regions of high density in the cloud are at low temperature, compared with the well-known estimate due to Eddington, might otherwise have been regarded with some reserve, McKellar's results afford strong observational support for this conclusion.

The assumption (*c*) requiring a density of the order of 10^{-21} gm. per c.c. in regions close to the galactic plane will probably be regarded as open to some question and we shall therefore give further considerations that lend additional support to this postulate. In the first place, this assumption yields the immediate result that the most massive and luminous stars may be expected to be strongly concentrated to the galactic plane. This agrees with observation and thereby obtains a result not previously accounted for by theoretical astronomy. Secondly, it is reasonable to suppose from dynamical considerations that in a rotating mass of gas the density will rise to maximum value on the plane of symmetry determined by its rotation axis.

The possible suggestion that the value 10^{-21} gm. per c.c. represents too high a density near the galactic plane is an objection that, if it applies to all, can apply only to the galaxy itself. For, by means of observation of a number of extra-galactic nebulæ of the spiral type, Jeans ‡ has obtained values of the order of 10^{-21} gm. per c.c. for the average central density of these bodies. Accordingly objections such as those urged by Atkinson in (2) could not in any case apply to the stars situated in these external galaxies. Since, however, the evolutionary property of the stars, so far as have been ascertained, seem to be much the same for our galaxy as for the extra-galactic nebulæ, this affords further support for the hypothesis (*c*). It would of course be possible to argue that the values of the velocities of the stars relative to the cloud differ systematically between the galaxy and the external nebulæ, but the adoption of this as an *ad hoc* assumption to avoid the resolution of an acknowledged theoretical difficulty would be not only extremely artificial but also contrary to scientific practice.

5. A number of other points of Atkinson's paper (2) seem to require comment. In the first place, although it is not explicitly stated, the general tenor of the paper gives the impression that the author believes that the accretion process is merely in the course of being considered as a possible way out of certain astrophysical difficulties. This in itself seems to be nothing more than a possibly unrecognised continuance of the attitude that characterised former researches on stellar evolution in which the origin and early history of the stars were vaguely passed over by ascribing them, somewhat arbitrarily, to condensation in parent nebulæ.§ In our view, however, the stars must not (*a priori*) be considered as completely autonomous bodies. Moreover, the accretion process is not by any means to be regarded merely as a further speculative hypothesis designed to extend the age of the stars, but is a process that must in any case take place on the scale that we have proposed, as will be shown in a moment. The only possible alternative would be in supposing that the stars have not condensed at all but have always been compact; such an assumption, however, achieving nothing more than its own statement, is so unplausible as to be outside the scope of serious discussion.

Returning now to the previous point, it can easily be seen from general considera-

* *Harvard Announcement Card*, 526, 1940.

† *P.A.S.P.*, **52**, 187, 1940; and *Mt. Wilson Annual Report*, p. 18, 1941.

‡ *Astronomy and Cosmogony*, p. 331 (1929 ed.).

§ For example, see *Internal Constitution of the Stars*, p. 17.

tions that the rate of increase of mass of the Sun, say, at some time in its history must have been of the same order as that given by our formula, at the very least. For, taking the present mass of the Sun as 2×10^{33} grams and the age of the universe as 10^{11} years, the average rate of increase of mass during this period is 2×10^{22} gm. per year or 6×10^{14} gm. per sec., while, according to our formula, the present rate of accretion must be of the order of 10^{14} gm. per sec., which establishes the result. Evidently then, unless the age of the universe is very much greater than 10^{11} years, other factors must have affected the accretion of the Sun. For example, the Sun may have possessed a companion body of comparable mass for a considerable part of its existence, and this could to some extent increase the accretion rate. More important still, owing to its application to the stars generally, is the possibility that the velocity relative to the cloud may in effect be less than the peculiar velocity. (See also below.) Furthermore, it cannot yet be decided whether the density of the cloud can be regarded as not changing with time and it is not inconceivable that it was different in the past.

As Atkinson has pointed out, an effective capture radius S may always be utilised irrespective of the actual mechanism of accretion and the accretion rate is then $\pi S^2 \rho v$. If we equate this to 10^{14} gm. per sec. for the Sun, a condition between the capture radius and density is thereby obtained, viz.:

$$S^2 \rho = 1 \cdot 6 \times 10^7.$$

It follows that if the density were of order 10^{-23} gm. per c.c. the capture radius would be of the order of 100 astronomical units; while if the capture radius were of the order of the radius of the Sun, as the other extreme, the density must be of the order of 10^{-15} gm. per c.c. Unless it can be shown how the capture radius could have such a high value, the first of these results rules out the value generally adopted for the density, while the second result shows that the capture process can have no direct relation to the radius of the star.

The foregoing remarks refer to increase of mass only, without regard to the composition of the matter added, and as such apply to all stars. For the very luminous and massive stars, however, the most pressing difficulty presented by them involves periods of time of a different order of magnitude and the need was for an additional source of energy. In solving this problem by supplying hydrogen from without, it is incidental that this involves also a rapid increase of mass, but it is this feature that, without further assumption, solves the problem of the evolution of binary systems. Thus two difficulties are overcome by a single step.

6. In discussing the dynamical evolution of binary systems Atkinson criticises our work from two standpoints, neither of which can be upheld. The first concerns the incorrect conclusion that the equation (2) can never be satisfied, and an instance is considered in which the accretion is so small that the rate of increase of mass by accretion is less than the rate of decrease of mass by radiation, *i.e.*

$$\frac{\partial M_A}{\partial t} < \frac{1}{125} \cdot \frac{\partial M_T}{\partial t}.$$

In such a case Atkinson then argues that no dynamical evolution can take place no matter how long a time-scale is taken ("whatever the time-scale and whatever the energy source"). This conclusion is incorrect. The actual state of affairs is that, given sufficient time, *no matter how slowly accretion takes place*, it can increase the mass of a star to any extent, whereas radiation, *no matter how fast it takes place*, can alter the mass of a star by no more than one part in 125 of the final mass arrived at by accretion. Atkinson's error arises on account of the unnoticed assumption that if the above inequality holds at a particular time then it holds for all time, whereas the inequality must always be reversed over some period in the case of a time-scale greater than 10^{10} years.

Secondly, Atkinson writes that it is "only at the price of extending the time-scale to 10^{10} years or more" that the requirements of binary evolution can be overcome by accretion. It is correct that the accretion theory does indicate an age for the stellar universe (in the galaxy) of order 5×10^{10} years, a value in excess of the usual estimates by a factor of at least 5. There is no mention in Atkinson's paper, however, of what "price" it is that would have to be paid in so extending the time-scale. Our calculation * of a lower limit of 3×10^{10} years for the age of the companion of Sirius, of such obvious importance in the present connection, is passed over without reference. There appears to be no cogent reason why the time-scale should not be extended in accordance with the indications of the accretion theory of stellar evolution.

Although the long time-scale as advocated by Jeans of 10^{12} or 10^{13} years has fallen out of favour the main idea behind Jeans' work remains as important as before—namely, that sufficient time must have elapsed to produce the extensive dynamical evolution that observations force us to regard the stars as having undergone. If the time-scale becomes so cramped that there is insufficient time for widespread dynamical evolution the "price" seems to be in the opposite sense to that used by Atkinson. Such questions as the dynamical evolution of binary systems as a result of accretion and the lifetime of the companion of Sirius provide strong evidence for a time-scale in excess of 10^{10} years.

7. It is further maintained by Atkinson that if the rate of accretion were given by a formula of the type proposed by us, an observable distinction should exist sharply separating stars of small velocity from those of slightly higher velocity (p. 509, lines 22–26), and hence, since this is not observed to be the case, that accretion cannot be given by our formula. In answering this objection several important points arise. In the first place, in making this suggestion Atkinson appears to have ignored our demonstration † that the velocity of a star relative to the cloud may vary by a factor of at least two as it pursues its orbit in the galaxy and hence that even if the velocity at any instant were known, its use for v in the formula might introduce an error of a factor of about 3 in the average accretion rate. For, if we denote the average rate of accretion by $\bar{A}$, our analysis shows that in the course of describing an orbit in the galaxy the rate varies between about $\frac{1}{3}\bar{A}$ and $\frac{8}{3}\bar{A}$, corresponding to the maximum and minimum values of the velocity. This is, of course, on the assumption of uniform density all along the orbit. Thus there is no reason to suppose that two stars having peculiar velocities of equal magnitude at a given time and situated in the same region of the galaxy would necessarily have equal average rates of accretion.

This alone would be sufficient answer to Atkinson's claim, but the question of the value of v to be adopted leads to other considerations of interest. Even on the assumption that the motion of the cloud is strictly in circular orbits, it is not certain that the peculiar velocity of a star, which depends on its motion relative to other stars, coincides with its velocity relative to the cloud, though there seems to be no reason to expect these velocities to be greatly different. However, the velocity appears cubed in the formula and quite a small error in estimating v could therefore lead to a more considerable error in the calculated accretion. Such an uncertainty may very well be involved in the velocities in any case, for it has to be remembered that the so-called K-term, which refers to stars of low velocity in particular and for these stars is of the same order as the peculiar velocities themselves, is not yet satisfactorily explained.

For stars of small mass having sufficient energy provision apart from future accretion, for periods of order 10^9 years or longer, the calculation of their total accretion would require a knowledge of v over such intervals and it would not be permissible to suppose that the value of the peculiar velocity would remain at its present value throughout. The origin of the peculiar velocities of the stars has not yet been fully explained; it is hoped to examine this question in subsequent work.

* *Proc. Camb. Phil. Soc.*, 35, 603, 1939. † *Proc. Camb. Phil. Soc.*, 35, 600, 1939.

There is the further point that if the cloud is not regarded as uniformly distributed, the denser regions would have their separate motions and might themselves be expected to possess velocities differing appreciably from the circular velocity. Bearing all these factors in mind, it is clear that in using the peculiar velocity of the star for the value of v in the formula, the result so obtained is not likely to give more than an order of magnitude estimate of the accretion rate. The uncertainties in the parameters so involved are unavoidable at the present time, but this cannot in any way be regarded as a defect of the theory itself.

We may in passing refer to a criticism of a minor nature that has been advanced concerning the application of the accretion theory to the question of changes of the solar radiation. It was shown * that a change of 10 per cent. in the Sun's radiation would occur for $v = 10$ km. per sec. and $\rho = 10^{-18}$ gm. per c.c., and it has been pointed out that the value 10^{-18} gm. per c.c. may not actually be attained in any nebulous cloud. It is clear at once, however, that if the maximum density available should be only 10^{-19} gm. per c.c., say, the same change in the Sun's radiation would be produced for a relative velocity of about 5 km. per sec. for the star and cloud. That such relative velocities may occur at times is confirmed by the measures of radial velocity in certain nebulæ.†

8. In conclusion we may review the general position and development of the accretion theory. Before its advent physical theory had progressed sufficiently to make practically certain that the process of transmutation of hydrogen must provide almost the whole of stellar energy, while the available astronomical evidence, particularly from double stars, led us to the view that a further potential source of energy must be introduced from outside the stars, either continually or intermittently replenishing the hydrogen in the star. Once this idea was clearly grasped there was little difficulty in conceiving the broader requirements of the accretion theory; the details, however, were not so readily forthcoming. The existence of interstellar matter in gaseous form, at any rate in certain regions of the galaxy, had already been established. However, the presence of such a cloud as this could provide no help in the problem. The reason that this is so is worth mentioning as it does not seem to have been appreciated. In the first place, the chief constituent of the cloud was regarded as calcium, a number of similar elements such as sodium possibly being present also, but the addition of such matter to a star and the consequent increase of mass could not extend the life of a star, but would if anything shorten it. Thus, in stating that it had been generally recognised by astronomers that the problem of the energy supply of bright stars could be solved by addition of matter to them, Dr. Atkinson draws attention to what must have been a widespread misunderstanding of the true nature of the problem. In Eddington's account of the process of the accretion of mass‡, however, this mistake is certainly not made and there is no suggestion of the process leading to any extension of the life of stars; this is further evidenced by the fact that the accreted material is taken as having atomic weight 10. In the event of material other than hydrogen being supplied to the star the less is accreted the better as far as the lifetimes are concerned. It is true that at the time of publication of Eddington's work on this question some doubt yet remained as to whether the source of energy were due to the complete annihilation of matter or to transmutation of hydrogen. So long as the speculation involved in the former of these possibilities was retained then of course accretion of mass would extend the lifetimes, but once the source of energy is ascribed to the second process then mere accretion of mass is useless, and accretion of hydrogen only is effective.

In our own discussions of the problem we simply postulate the hydrogen cloud and proceed to consider what conditions the cloud would have to satisfy. Unfortunately

* *Proc. Camb. Phil. Soc.*, 35, 412, 1939.
† Russell, Dugan and Stewart, vol. ii, 829.
‡ *I.C.S.*, pp. 391 and 392.

we were encountered at the outset with what seemed to be an insuperable obstacle to any progress with the problem. We refer, of course, to the question of the motion of the gas in the gravitational field of the star. As there was no hydrodynamical precedent to guide us, the only possibility seemed to lie in getting some simple picture of the process that would enable the order of magnitude of the rate of accretion to be calculated. Any such solution must necessarily involve some assumption additional to those that would lead to the rigorous solution of the problem and the justification of such a procedure would then rest on how much the solution turned out to be capable of explaining. After considerable attention had been given to the problem it was decided that a formula of the type we eventually proposed would be most likely to give the best estimate. The most striking feature of this formula was the enormous rate of increase of mass that it involved, even if the density of the cloud were no higher than the estimate of 2×10^{-23} gm. per c.c. given for the calcium, etc., cloud.

Now in so solving the problem of the source of energy of the bright stars, we had invoked a hypothesis that also solved the question of the dynamical evolution of binary stars. The situation with regard to this second problem was to some extent similar to that of the former, for here the attention of astronomers was largely engaged with the fission theory of the origin of binary systems. Thus at a single blow this new process resolved the chief difficulties of these hitherto widely separated problems and thereby unified the dynamical and physical evolution of the stars. Moreover, the process proposed cannot be regarded as speculative in the sense, for example, that the hypothesis of the complete annihilation of matter was speculative. The latter hypothesis at the time of its consideration was not only a process so far unobserved to occur, but even with its postulated properties it did not by any means fully and consistently explain the phenomena to which it referred. On the other hand, in the accretion process there is involved no question of a mechanism unknown to science, while as far as considerations of the consequences of the process have proceeded no contradictions have yet appeared.

The situation is, therefore, that so far as investigations have progressed we have developed within the limitations fixed by the inherent mathematical difficulties of the problems involved a consistent theory of stellar evolution. If this theory, including its hypotheses, were completely confirmed by the present observations of the entities involved, such as the density and structure of the cloud, the velocities, luminosities, compositions, etc., of the stars, then the work would have achieved nothing more than showing the logical interrelation of such quantities. But in reality our work is at present of greater potential value than this for it shows that (on the basis of the present accepted physical theory) a consistent theory can only be built if the parameters associated with certain factors take fairly definite values. For instance, the theory indicates that the density of hydrogen in space (which *must* be assigned some value), averages at least 10^{-22} gm. per c.c. throughout the main part of the galaxy, while rising to a value above 10^{-21} gm. per c.c. in some regions. Furthermore, as has been seen, there is nothing in the available evidence bearing on this question to show that such a requirement cannot be fulfilled. If, however, future investigations should show that these conditions cannot be accepted then it would be necessary to modify the theory without sacrificing its consistency, or, if this were not possible, to construct a new theory. It will be seen from these remarks that in criticising our work by maintaining that the required density is impossibly high (or higher than at present seems permissible) Atkinson is attacking the theory on a point where it is not exposed to any attack until definite evidence can be cited against the density adopted.*

9. The position may be illustrated very clearly by recalling some analogous questions. In attempting to account for the anomalous motion of Uranus, the postulate of additional material in the solar system was made by J. C. Adams, who then

* *Observatory*, **63**, 39, February 1940; and *Proc. Camb. Phil. Soc.*, **36**, 325, 1940.

endeavoured to calculate, as best as could be managed in such a difficult problem, the value that the density would have to take. In order to simplify the treatment as much as possible it was supposed that the whole matter could be regarded as concentrated in a sphere moving at a certain distance from the Sun, the density elsewhere being zero. On this basis the question of the density distribution resolved itself into a determination of the co-ordinates of the centre of the sphere. A critic of this entirely legitimate procedure could apparently have objected that the density taken by Adams was far beyond any that then seemed permissible, while it would also have been possible to criticise the analysis employed. Quite possibly these objections were actually advanced at the time, but Adams's investigations on the problem would have been equally justifiable whether Neptune had consequently been discovered or not.

In the same way, as a possible explanation of the peculiarity of the motion of Mercury outstanding from its Newtonian orbit, a distribution of material was again postulated and the required density calculated. In this case, however, for the theory to be logically consistent (after accepting the usual gravitational equations, etc.) the density was found to be quite discordant with more direct evidence from observations, the amount of matter theoretically required far exceeding that which could possibly be present. Accordingly the proposed explanation had to be discarded.

These examples show what the correct attitude towards the accretion theory of stellar evolution should be and that much of Atkinson's criticisms has no valid basis. The real need at present in this problem is for trustworthy observations leading to information of the density distribution and velocities of the stars relative to the cloud. At present, in regard to stellar evolution, the choice is between the consistent theory based on the idea of accretion and no theory at all. In making a decision between these two, before it is argued that the problem may be conditioned by some effect as yet unrecognised or unknown to science and that the agreements achieved by the accretion theory are due to pure coincidence, it must first be recalled that such an argument would dispose of each and every scientific theory yet invented. It is not within the claim of any theory to show that the same conclusions could or could not follow from other hypotheses at present unknown; the proper purpose of a theory is to construct a model that successfully predicts and correlates phenomena within what are called the errors of observation.

Summary

The discussion of accretion given in a recent paper by Atkinson and the criticism of our work involved in it are considered in some detail. It is shown that most of the arguments therein advanced had been dealt with by us before the publication of Atkinson's article, while several errors of method have led to a number of other invalid criticisms being put forward. It is shown that when these errors are set right the difficulties met with by their author are removed.