Superradiant instabilities in astrophysical systems

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Outline

1. Motivation
2. Massive scalar fields
3. Massive vector fields
4. Conclusions
Motivation
Superradiance effect

- Penrose process (Penrose '69, Christodoulou '70)
  - scattering of particles off Kerr BH
  $\Rightarrow$ reduction of BH mass

- superradiant scattering (Misner '72, Zeldovitch '71)
  - scattering of wave packet off Kerr BH
  - superradiance condition
    \[
    \omega < m\Omega_H = m\frac{a}{2Mr_+}
    \]
  $\Rightarrow$ extraction of energy and angular momentum off BH
  $\Rightarrow$ amplification of energy and angular momentum of wave packet
Superradiance instability

“black hole bomb” (Press & Teukolsky '72, Zeldovich '71, Cardoso et al '04)

- consider Kerr BH surrounded by mirror
- consider field with $\omega < m\Omega_H$
  $\Rightarrow$ superradiant scattering
- subsequeunt amplification of superradiant modes

$\Rightarrow$ exponential growth of modes
$\Rightarrow$ instability due to superradiant scattering
Superradiance instability in physical systems

natural mirror provided by

- anti-de Sitter spacetimes
- massive fields with mass coupling $M\mu$
  (Damour et al. ’76, Detweiler ’80, Zouros & Eardley ’79)

![Diagram of potential with ergo-region, barrier region, and potential well.](image)

Arvanitaki & Dubovsky ’11
growth rate of massive scalar fields

- Detweiler '80: $M \mu << 1$
  \[
  \tau \sim 24 \left( \frac{a}{M} \right)^{-1} (M \mu)^{-9} \left( \frac{GM}{c^3} \right)
  \]

- Zouros & Eardley '79 $M \mu >> 1$
  \[
  \tau \sim 10^7 \exp(1.84M \mu) \left( \frac{GM}{c^3} \right)
  \]

- for astrophysical BHs and known particles: $M \mu \sim 10^{18}$

  $\Rightarrow$ insignificant for astrophysical systems?
Superradiance instability in physical systems

- most promising mass range: $M_\mu \sim 1$
- ultralight bosons proposed by string theory compactifications: axions (Arvanitaki & Dubovsky ’10)

- formation of bosonic bound states around astrophysical BHs
- gravitational wave emission
- “bosenova”-like particle bursts (Yoshino & Kodama ’12)

(Arvanitaki & Dubovsky ’11)
Superradiance instability in physical systems

(Kodama & Yoshino '11)

- landscape of ultralight axions ⇒ “string axiverse”
- bosonic fields with $M\mu \sim 10^{-22}$ as dark matter candidates
- small, primordial BHs with $M \sim 10^{-18} M_\odot$
- bosonic cloud around SMBHs ($M \sim 10^9 M_\odot$) if $10^{-21} \leq M\mu \leq 10^{-16}$ ⇒ probe of photon mass (upper bound $\mu_\gamma \sim 10^{-18}$ (Nakamura et al.'10))
Massive scalar field
Massive scalar fields - recent results

- **Klein-Gordon equation**

  \[(\Box - \mu^2)\psi = 0, \quad \text{with} \quad \psi = \exp(i m \phi - i \omega t) S_{lm}(\theta) R_{lm}(r)\]

- **QNMs** (Dolan '07, Cardoso & Yoshida '05, Berti et al '09)

<table>
<thead>
<tr>
<th>$M \mu$</th>
<th>$a/M$</th>
<th>$M \omega_{11} (n = 0)$</th>
<th>$M \omega_{11} (n = 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.2929 - i0.09766</td>
<td>0.2645 - i0.3063</td>
</tr>
<tr>
<td>0.00</td>
<td>0.99</td>
<td>0.4934 - i0.03671</td>
<td>0.4837 - i0.09880</td>
</tr>
<tr>
<td>0.42</td>
<td>0.00</td>
<td>0.4075 - i0.001026</td>
<td>0.4147 - i0.0004053</td>
</tr>
<tr>
<td>0.42</td>
<td>0.99</td>
<td>0.4088 + i1.504 \cdot 10^{-7}</td>
<td>0.4151 + i5.364 \cdot 10^{-8}</td>
</tr>
</tbody>
</table>

- **Tails**

  - massless field (Price '71, Leaver '86, Ching et al '95)
    \[\psi \sim t^{-2l-3}\]

  - massive fields (Koyama & Tomimatsu '01,'02, Burko & Khanna '04)
    \[\psi \sim t^{-l-3/2} \sin(\mu t), \quad \text{at intermediate times}\]
    \[\psi \sim t^{-5/6} \sin(\mu t), \quad \text{at very late times}\]
Massive scalar fields - recent results

- bound states: maximum instability growth rate for (Dolan '07, Cardoso & Yoshida '05)

\[ l = m = 1, \quad a/M = 0.99, \quad M\mu = 0.42 : \quad \frac{1}{\tau} = \omega_I \sim 1.5 \cdot 10^{-7} \left( \frac{GM}{c^3} \right)^{-1} \]

- numerical results:
  - Strafuss & Khanna '05: \( M\omega_I \sim 2 \cdot 10^{-5} \) (Gaussian initial data)
  - Kodama & Yoshino '12: \( M\omega_I \sim 3.2 \cdot 10^{-7} \) (bound state initial data)
Massive scalar fields - Code setup

- goal: study time evolution of massive scalar field in Kerr background
  - Kerr background in Kerr-Schild coordinates $\rightarrow$ excision of BH region
  - Klein-Gordon equation $(\Box - \mu^2)\psi = 0$ as $3 + 1$ time evolution problem

\[ \begin{align*}
  d_t \psi &= -\alpha \Pi \\
  d_t \Pi &= -\alpha (D^i D_i \psi - \mu^2 \psi - K \Pi) - D^i \alpha D_i \psi
\end{align*} \]

- initial data:
  - Gaussian wave packet with $r_0 = 12M$, $w = 2M$
  - quasi-bound state

- $4^{th}$ finite differences in space, $4^{th}$ order Runge-Kutta time-integrator
- extraction of scalar field at fixed $r_{ex}$, mode decomposition

\[ \psi_{lm}(t) = \int d\Omega \psi(t, \theta, \phi) Y_{lm}^*(\theta, \phi) \]
Code tests I - space dependent mass coupling

- consider spherically symmetric scalar field with *unphysical* mass

\[(M_\mu)^2 = -\frac{10}{r^4}\]

- theoretical prediction: \(\psi \sim \exp(0.071565 \, t)\)
- numerical result: \(\psi \sim \exp(0.07161 \, t) \Rightarrow \text{agreement within 0.06\%}\)
QNM frequencies

\[
a/M = 0.0 \quad M\omega_{11} = 0.294 - i0.096 \quad (0.2929 - i0.09766)
\]

\[
a/M = 0.99 \quad M\omega_{11} = 0.493 - i0.0368 \quad (0.4934 - i0.03671)
\]

tail: \( \psi_{00} \sim t^{-3.04} \) \( (\psi_{00} \sim t^{-3}) \)

numerical error: \( \Delta \psi_{00}/\psi_{00} \leq 3\% \), \( \Delta \psi_{11}/\psi_{11} \leq 8\% \)
Massive scalar fields in Schwarzschild

consider mass coupling \( M\mu = 0.1, 0.42, 1 \)

- tails in agreement with (Koyama & Tomimatsu '02, Burko & Khanna '04):
  
  \[
  \begin{align*}
  M\mu &= 0.1 & M\omega_{11} &= 0.293 - i0.036 & \psi_{11} &\sim t^{-2.543} \sin(0.1t) \\
  M\mu &= 1.0 & M\omega_{11} &= 0.965 - i0.0046 & \psi_{11} &\sim t^{-0.873}
  \end{align*}
  \]
Massive scalar field with $M\mu = 0.42$ in Schwarzschild

- dependent on location of measurement
- beating of modes:
  - similar frequency $M\omega_R$ of fundamental and overtone mode
    $\omega_{R,n} = \omega_{R,0} + \delta, \delta \ll 1$

$$\psi \sim A_0 \exp(-i\omega_0 t) + A_n \exp(-i\omega_n t)$$

$$\sim \exp(-i\omega_{R,0} t)(B_0 + B_n \cos(\delta t) - iB_n \sin(\delta t))$$

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Massive scalar field in Kerr - bound states

Evolution of bound-state scalar field with $M\mu = 0.42$, $a/M = 0.99$

- localized in the vicinity of the BH
- node of overtone at $x = 26.5 \ M$
- by construction $\Psi_{11} \Psi_{11}^* \sim \exp(-\omega t) \exp(\omega t) = \text{const.}$

**Initial data**

![Initial data graph]

**Evolution**

![Evolution graph]
Massive scalar field in Kerr - Gaussian

Evolution of a scalar field with $M\mu = 0.42$ in Kerr $a/M = 0.99$ with Gaussian initial data

- beating of modes if $\omega_{R,n} = \omega_{R,0} + \delta$, $\delta << 1$

$$\psi \sim A_0 \exp(-\omega_0 t) + A_n \exp(-\omega_n t)$$
$$\sim \exp(-\omega_{R,0} t)(B_0 + B_n \cos(\delta t) - iB_n \sin(\delta t))$$

- amplitude of modes dependent on location of measurement
- possible explanation for result by Strafuss & Khanna ’05
Massive vector fields
massive hidden $U(1)$ vector fields from string theory compactification (e.g., Jaeckel & Ringwald ’10)

expected: superradiance effect stronger than in scalar field case

rich phenomenology

studied by Galt’sov et al ‘84, Konoplya ‘06, Konoplya et al ‘07, Herdeiro et al ‘11, Rosa & Dolan ‘11

vector field eqs. in Kerr non-separable $\Rightarrow$ challenging problem
Rosa & Dolan ’11:

- Proca field equations
  \[ \nabla_\nu F^{\mu\nu} + \mu A_{\mu} = 0 \quad F_{\mu} = \nabla_\mu A_{\nu} - \nabla_\nu A_{\mu} \]

- Lorenz condition has to be satisfied \( \nabla_\mu A^\mu = 0 \)  
  \( \Rightarrow \) scalar mode gains physical meaning

- decomposition of \( A_\mu \) in vector spherical harmonics \( Z^{(i)lm}_{\mu} \)

- continued fraction method and forward integration
Massive vector fields in Schwarzschild background

Rosa & Dolan '11: QNM spectrum

- for given $l$, $n$:
  - 2 even parity modes (scalar and vector field modes),
  - 1 odd parity mode (vector field mode)
- in electromagnetic limit ($M_{\mu A} \to 0$):
  - scalar mode = gauge mode
  - even and odd vector mode degenerate
- field mass - breaking of degeneracy
- distinct frequencies of even parity modes
goal: study time evolution of Proca field in Kerr background (work in progress)

- Kerr background in Kerr-Schild coordinates $\rightarrow$ excision of BH region
- Proca equation $\nabla_\nu F^{\mu\nu} + \mu^2 A^\mu = 0$
  Lorenz condition $\nabla_\mu A^\mu = 0$
- define $A_\mu = A_\mu + n_\mu \varphi, \ E_\mu = F_{\mu\nu} n^\nu$
  $\Rightarrow$ formulation as $3 + 1$ time evolution problem
- initial data: gaussian wave packet
- $4^{th}$ finite differences in space, $4^{th}$ order Runge-Kutta time-integrator
- extraction of Newman-Penrose scalar $\Phi_2$ at fixed $r_{ex}$, mode decomposition

$$\Phi_{2,lm}(t) = \int d\Omega \Phi_2(t, \theta, \phi) Y_{lm}^*(\theta, \phi)$$
Massless vector field in Kerr

vector field with $\mu_A = 0$ in Kerr with $a/M = 0.99$

- QNM frequencies: $M\omega_{11} = 0.461 - i0.041$ ($0.463 - i0.031$)
- in agreement with theoretical prediction (Berti et al, ’09)
- improving with increasing resolution
vector field with $\mu_A = 0.01, 0.42, 1$ in Kerr with $a/M = 0.99$

QNM frequencies

$$M\mu_A = 0.01 \quad \omega_{11} M = 0.462 - i0.041$$
$$M\mu_A = 0.40 \quad \omega_{11} M = 0.415$$
$$M\mu_A = 1.0 \quad \omega_{11} M = 0.959 - i0.004$$
Proca field in Kerr

Evolution of a Proca field with $M\mu = 0.40$ in Kerr $a/M = 0.99$

- beating of modes
- growth of the mode - signature of instability?
massive fields in Kerr spacetimes exhibit extremely rich spectra

evolution of scalar field wave packets
  - extensive code testing
  - beating of fundamental and overtone modes

first evolutions of massive vector fields in Kerr background
  - low mass fields $M_{\mu A} = 0.01$ damped
  - beating effect for $M_{\mu A} = 0.42$
  - growing of $l = m = 1$ mode $\rightarrow$ possible signature of instability
  - still in its infantry $\Rightarrow$ more results to come soon
Thank you!

http://blackholes.ist.utl.pt